Blind Adaptive Code-Constrained Constant Modulus Algorithms for CDMA Interference Suppression in Multipath Channels

Rodrigo C. de Lamare and Raimundo Sampaio-Neto

Abstract— **A code-constrained constant modulus (CCM) design criterion for linear receivers is investigated for direct sequence code division multiple access (DS-CDMA) in multipath channels based on constrained optimization techniques. A computationally efficient recursive least squares (RLS) type algorithm for jointly estimating the parameters of the channel and the receiver is developed in order to suppress multiaccess (MAI) and intersymbol interference (ISI). An analysis of the method examines its convergence properties and simulations under non-stationary environments show that the novel algorithms outperform existent techniques.**

Index Terms— **Interference suppression, multiuser detection, DS-CDMA, constant modulus cost function, blind adaptive algorithms.**

I. Introduction

BLIND adaptive linear receivers [1]-[4] for DS-CDMA systems are promising techniques for interference suppression as they offer an attractive trade-off between performance and complexity and can be used in situations where a receiver loses track of the desired signal and a training sequence is not available. A blind adaptive minimum variance (MV) detector was introduced by Honig *et al.* in [1] and trades off the need for a training sequence in favor of the knowledge of the desired user's spreading code. A disadvantage of the method in [1] is that it suffers from the problem of signature mismatch and thus cannot work in multipath environments. Constrained MV (CMV) adaptive algorithms were proposed by Xu and Tsatsanis [2] in order to operate in multipath channels. They developed stochastic gradient (SG) and RLS algorithms based on constrained optimization techniques that jointly estimate the channel and suppress MAI and ISI. Recently, a modified SG algorithm using the constant modulus (CM) cost function was reported in [3] and then extended for the multipath case in [4]. These CM-based SG algorithms are quite effective for ISI and MAI rejection and outperform their MV counterparts. In this letter, we present a CCM solution for the design of linear receivers and develop a computationally efficient blind adaptive RLS-type algorithm to jointly estimate the channel and the parameters of the receiver.

Manuscript received September 1, 2004. The associate editor coordinating the review of this letter and approving it for publication was Dr. Philippe Ciblat. This work was supported by CNPq.

The authors are with CETUC/PUC-RIO, 22453-900, Rio de Janeiro, Brazil (e-mail: delamare@infolink.com.br, raimundo@cetuc.puc-rio.br).

Digital Object Identifier 10.1109/LCOMM.2005.04022.

II. DS-CDMA System Model

Let us consider the downlink of a synchronous BPSK DS-CDMA system with K users, N chips per symbol and L_p paths. Assuming that the channel is constant during each symbol and the receiver is synchronized with the main path, the received signal after filtering by a chip-pulse matched filter and sampled at chip rate yields the $(M = N + L_p - 1) \times 1$ received vector

$$\mathbf{r}(i) = \sum_{k=1}^{K} A_k b_k(i) \mathbf{g}(i) \star \mathbf{s}_k + \boldsymbol{\eta}(i) + \mathbf{n}(i) \tag{1}$$

where $\mathbf{n}(i) = [n_1(i) \ \dots \ n_M(i)]^T$ is the complex Gaussian noise vector with $E[\mathbf{n}(k)\mathbf{n}^H(i)] = \sigma^2 \mathbf{I}$, where $(.)^T$ and $(.)^H$ denote transpose and Hermitian transpose, respectively, $E[.]$ stands for ensemble average, $b_k(i) \in \{\pm 1 + j0\}$ is the symbol for user k, $j^2 = -1$, $\boldsymbol{\eta}(i)$ is the ISI, the amplitude of user k is A_k, the channel vector is $\mathbf{g}(i) = [g_0(i) \dots g_{L_p-1}(i)]^T$, symbol $\star$ denotes convolution and $\mathbf{s}_k = [a_k(1) \dots a_k(N)]^T$ is the signature sequence for user k.

III. Code-Constrained Constant Modulus Receivers

Consider the received vector $\mathbf{r}(i)$, the $M \times L_p$ constraint matrix that contains one-chip shifted versions of the signature sequence for user k and the $L_p \times 1$ vector $\mathbf{g}(i)$ with the multipath components to be estimated:

$$\mathbf{C}_k = \begin{bmatrix} a_k(1) & & \mathbf{0} \\ \vdots & \ddots & a_k(1) \\ a_k(N) & & \vdots \\ \mathbf{0} & \ddots & a_k(N) \end{bmatrix}, \mathbf{g}(i) = \begin{bmatrix} g_0(i) \\ \vdots \\ g_{L_p-1}(i) \end{bmatrix} \tag{2}$$

The CCM linear receiver design is equivalent to determining an FIR filter $\mathbf{w}_k$ with M coefficients that provide an estimate of the desired symbol $\hat{b}_k(i) = sgn(\Re(\mathbf{w}_k^H(i)\mathbf{r}(i)))$, where $sgn(.)$ is the signum function, $\Re(.)$ selects the real component and $\mathbf{w}_k$ is optimized according to the CM cost function:

$$J_{CM}(\mathbf{w}_k) = E\Big[(|\mathbf{w}_k^H \mathbf{r}|^2 - 1)^2\Big] \tag{3}$$

subject to the constraints given by $\mathbf{C}_k^H \mathbf{w}_k = \nu \ \mathbf{g}$, where $\mathbf{C}_k \mathbf{g} = \mathbf{g} \star \mathbf{s}_k$, $\mathbf{g}$ is the vector that contains the multipath gains that are to be determined and ν is a constant to ensure the convexity of (3), as will be shown later. Given $\mathbf{g}$ let us now consider the problem through an unconstrained cost function given by $J'_{CM}(\mathbf{w}_k) = E[(|\mathbf{w}_k^H\mathbf{r}|^2 - 1)^2] + \Re[(\mathbf{C}_k^H\mathbf{w}_k - \nu\ \mathbf{g})^H\boldsymbol{\lambda}]$, where $\boldsymbol{\lambda}$ is a complex vector of Lagrange multipliers. By taking the gradient terms of J'_{CM} with respect to $\mathbf{w}_k$ and setting them to zero we have $\nabla J'_{CM} = 2E[(|\mathbf{w}_k^H\mathbf{r}|^2 - 1)\mathbf{r}\mathbf{r}^H\mathbf{w}_k] + \mathbf{C}_k\boldsymbol{\lambda} = 0$, then rearranging the terms we obtain $E[|z_k|^2\mathbf{r}\mathbf{r}^H]\mathbf{w}_k = E[z_k^*\mathbf{r}] - \mathbf{C}_k\boldsymbol{\lambda}/2$ and then $\mathbf{w}_k = \mathbf{R}_k^{-1}[\mathbf{d}_k - \mathbf{C}_k\boldsymbol{\lambda}/2]$, where $z_k = \mathbf{w}_k^H\mathbf{r}$, $\mathbf{R}_k = E[|z_k|^2\mathbf{r}\mathbf{r}^H]$, $\mathbf{d}_k = E[z_k^*\mathbf{r}]$ and the asterisk denotes complex conjugation. Using the constraint $\mathbf{C}_k^H\mathbf{w}_k = \nu\ \mathbf{g}$ we arrive at the expression for $\boldsymbol{\lambda} = 2(\mathbf{C}_k^H\mathbf{R}_k^{-1}\mathbf{C}_k)^{-1}(\mathbf{C}_k^H\mathbf{R}_k^{-1}\mathbf{d}_k - \nu\ \mathbf{g})$. By substituting $\boldsymbol{\lambda}$ into $\mathbf{w}_k = \mathbf{R}_k^{-1}[\mathbf{d}_k - \mathbf{C}_k\boldsymbol{\lambda}]$ we obtain an iterative expression for the CCM linear receiver:

$$\mathbf{w}_k = \mathbf{R}_k^{-1}\left[\mathbf{d}_k - \mathbf{C}_k(\mathbf{C}_k^H\mathbf{R}_k^{-1}\mathbf{C}_k)^{-1}\Big(\mathbf{C}_k^H\mathbf{R}_k^{-1}\mathbf{d}_k - \nu\ \mathbf{g}\Big)\right] \quad (4)$$

The solution in (4) assumes the knowledge of the channel parameters. However, in several applications where multipath is present these parameters are not known and thus channel estimation is required. Here, we adopt the blind channel estimation procedure based on the power method described in [5]:

$$\hat{\mathbf{g}} = \arg\min_{\mathbf{g}}\ \mathbf{g}^H\mathbf{C}_k^H\mathbf{R}^{-m}\mathbf{C}_k\mathbf{g} \quad (5)$$

subject to $||\hat{\mathbf{g}}|| = 1$, where $\mathbf{R} = E[\mathbf{r}\mathbf{r}^H]$ and m is a finite power. The solution is the eigenvector of the $L_p \times L_p$ matrix corresponding to the minimum eigenvalue of $\mathbf{C}_k^H\mathbf{R}^{-1}\mathbf{C}_k$ through singular value decomposition (SVD). Here, we use $\mathbf{R}_k$ in lieu of $\mathbf{R}$ to avoid the estimation of both $\mathbf{R}$ and $\mathbf{R}_k$, and which shows no performance loss as verified in our studies. The values of m are restricted to 1 even though the channel estimator and consequently the receiver can be improved by increasing m.

IV. Blind Adaptive Code-Constrained CM RLS-Type (CCM-RLS) Algorithm

Given the solution for $\mathbf{w}_k$ in (4) we develop an algorithm that estimates the matrices $\mathbf{R}_k^{-1}$ and $(\mathbf{C}_k^H\mathbf{R}_k^{-1}\mathbf{C}_k)^{-1}$ recursively, reducing the computational complexity. Using the matrix inversion lemma and Kalman RLS recursions [6] we have:

$$\mathbf{G}_k(i) = \frac{\alpha^{-1}\hat{\mathbf{R}}_k^{-1}(i-1)z_k(i)\mathbf{r}(i)}{1 + \alpha^{-1}\mathbf{r}^H(i)z_k(i)\hat{\mathbf{R}}_k^{-1}(i-1)z_k^*(i)\mathbf{r}(i)} \quad (6)$$

$$\hat{\mathbf{R}}_k^{-1}(i) = \alpha^{-1}\hat{\mathbf{R}}_k^{-1}(i-1) - \alpha^{-1}\mathbf{G}_k(i)z_k^*(i)\mathbf{r}^H(i)\hat{\mathbf{R}}_k^{-1}(i-1) \quad (7)$$

where $\mathbf{G}_k$ is the Kalman gain vector with dimension $M \times 1$, $\hat{\mathbf{R}}_k$ is the estimate of the matrix $\mathbf{R}_k$ and $0 < \alpha \leq 1$ is the forgetting factor. At each processed symbol, the matrix $\hat{\mathbf{R}}_k^{-1}(i)$ is updated and we employ another recursion to estimate $(\mathbf{C}_k^H\mathbf{R}_k^{-1}(i)\mathbf{C}_k)^{-1}$ as described by:

$$\boldsymbol{\Gamma}_k^{-1}(i) = \frac{\boldsymbol{\Gamma}_k^{-1}(i-1)}{1-\alpha} - \frac{\boldsymbol{\Gamma}_k^{-1}(i-1)\boldsymbol{\gamma}_k(i)\boldsymbol{\gamma}_k^H(i)\boldsymbol{\Gamma}_k^{-1}(i-1)}{\frac{(1-\alpha)^2}{\alpha} + (1-\alpha)\boldsymbol{\gamma}_k^H(i)\boldsymbol{\Gamma}_k^{-1}(i)\boldsymbol{\gamma}_k(i)} \quad (8)$$

where $\boldsymbol{\Gamma}_k(i)$ is an estimate of $(\mathbf{C}_k^H\mathbf{R}_k^{-1}(i)\mathbf{C}_k)$ and $\boldsymbol{\gamma}_k(i) = \mathbf{C}_k^H\mathbf{r}(i)z_k(i)$. To estimate the channel and avoid the SVD on $\mathbf{C}_k^H\mathbf{R}_k^{-1}(i)\mathbf{C}_k$, we estimate the matrix $\hat{\mathbf{V}}_k(i) = \mathbf{C}_k^H\hat{\mathbf{R}}_k^{-1}(i)\mathbf{C}_k$ and employ the variant of the power method introduced in [7]:

$$\hat{\mathbf{g}}(i) = (\mathbf{I} - \gamma(i)\hat{\mathbf{V}}_k(i))\hat{\mathbf{g}}(i-1) \quad (9)$$

where $\gamma(i) = 1/tr[\hat{\mathbf{V}}_k(i)]$, $tr[.]$ stands for trace and we make $\mathbf{g}(i) \leftarrow \mathbf{g}(i)/||\mathbf{g}(i)||$ to normalize the channel. The CCM linear receiver is then designed as described by:

$$\hat{\mathbf{w}}_k(i) = \hat{\mathbf{R}}_k^{-1}(i)\left[\hat{\mathbf{d}}_k(i) - \mathbf{C}_k\boldsymbol{\Gamma}_k^{-1}(i)\Big(\mathbf{C}_k^H\hat{\mathbf{R}}_k^{-1}(i)\hat{\mathbf{d}}_k(i) - \nu\ \hat{\mathbf{g}}(i)\Big)\right] \quad (10)$$

where $\hat{\mathbf{d}}_k(i+1) = \alpha\hat{\mathbf{d}}_k(i) + (1-\alpha)z_k^*(i)\mathbf{r}(i)$ corresponds to an estimate of $\mathbf{d}_k(i)$. In terms of computational complexity, the CCM-RLS algorithm requires $O(M^2)$ to suppress MAI and ISI and $O(L_p^2)$ to estimate the channel, against $O(M^3)$ and $O(L_p^3)$ required by (4) and (5), respectively.

V. Convergence Properties

Let us express the cost function in (3) as $J_{CM} = (E[|z_k|^4] - 2E[|z_k|^2] + 1)$, drop the time index (i) for simplicity, assume a stationary scenario and that b_k, k=1,…,K are statistically independent i.i.d complex random variables with zero mean and unit variance, b_k and $\mathbf{n}$ are statistically independent. Let us also define $\mathbf{x} = \sum_{k=1}^{K} A_k b_k\tilde{\mathbf{s}}_k$, $\mathbf{C}_k\mathbf{g} = \tilde{\mathbf{s}}_k$, $\mathbf{Q} = E[\mathbf{x}\mathbf{x}^H]$, $\mathbf{P} = E[\boldsymbol{\eta}\boldsymbol{\eta}^H]$, $\mathbf{R} = \mathbf{Q} + \mathbf{P} + \sigma^2\mathbf{I}$. Consider user 1 as the desired one, let $\mathbf{w}_1 = \mathbf{w}$ and define $u_k = A_k^*\tilde{\mathbf{s}}_k^H\mathbf{w}$, $\mathbf{u} = \mathbf{A}^H\tilde{\mathbf{S}}^H\mathbf{w} = [u_1 \ldots u_K]^T$, where $\tilde{\mathbf{S}} = [\tilde{\mathbf{s}}_1 \ldots \tilde{\mathbf{s}}_K]$, $\mathbf{A} = diag(A_1 \ldots A_k)$ and $\mathbf{b} = [b_1 \ldots b_K]^T$. Using the constraint $\mathbf{C}_1^H\mathbf{w} = \nu\hat{\mathbf{g}}$ we have for the desired user the condition $u_1 = (A_1^*\tilde{\mathbf{s}}_1^H)\mathbf{w} = A_1^*\mathbf{g}\mathbf{C}_1^H\mathbf{w} = \nu A_1^*\mathbf{g}^H\hat{\mathbf{g}}$. In the absence of noise and neglecting ISI, the (user 1) cost function can be expressed as $J_{CM}(\mathbf{w}) = E[(\mathbf{u}^H\mathbf{b}\mathbf{b}^H\mathbf{u})^2] - 2E[(\mathbf{u}^H\mathbf{b}\mathbf{b}^H\mathbf{u})] + 1 = 8(\sum_{k=1}^{K} u_k u_k^*)^2 - 4\sum_{k=1}^{K}(u_k u_k^*)^2 - 4\sum_{k=1}^{K} u_k u_k^* + 1 = 8(D + \sum_{k=2}^{K} u_k u_k^*)^2 - 4D^2 - 4\sum_{k=2}^{K}(u_k u_k^*)^2 - 4D - 4\sum_{k=2}^{K}(u_k u_k^*) + 1$, where $D = u_1 u_1^* = \nu^2|A_1|^2|\hat{\mathbf{g}}^H\mathbf{g}|^2$. To examine the convergence properties of the optimization problem in (3), we proceed similarly to [3]. Under the constraint $\mathbf{C}_1^H\mathbf{w} = \nu\hat{\mathbf{g}}$, we have:

$$\tilde{J}_{CM}(\bar{\mathbf{u}}) = 8(D + \bar{\mathbf{u}}^H\bar{\mathbf{u}})^2 - 4(D^2 + \sum_{k=2}^{K}(u_k u_k^*)^2) - 4(D + \bar{\mathbf{u}}^H\bar{\mathbf{u}}) + 1 \quad (11)$$

where $\bar{\mathbf{u}} = [u_2, \ldots, u_K]^T = \mathbf{B}\mathbf{w}$, $\mathbf{B} = \mathbf{A}'^H\tilde{\mathbf{S}}'^H$, $\tilde{\mathbf{S}}' = [\tilde{\mathbf{s}}_2 \ldots \tilde{\mathbf{s}}_K]$ and $\mathbf{A}' = diag(A_2 \ldots A_K)$. To evaluate the convexity of $\tilde{J}_{CM}(.)$, we compute its Hessian ($\mathbf{H}$) using the rule $\mathbf{H} = \frac{\partial}{\partial\bar{\mathbf{u}}^H}\frac{\partial(\tilde{J}_{CM}(\bar{\mathbf{u}}))}{\partial\bar{\mathbf{u}}}$ that yields:

$$\mathbf{H} = 16\Big[(D - 1/4)\mathbf{I} + \bar{\mathbf{u}}^H\bar{\mathbf{u}}\mathbf{I} + \bar{\mathbf{u}}\bar{\mathbf{u}}^H - diag(|u_2|^2 \ldots |u_K|^2)\Big] \quad (12)$$

Specifically, $\mathbf{H}$ is positive definite if $\mathbf{a}^H\mathbf{H}\mathbf{a} > 0$ for all nonzero $\mathbf{a} \in \boldsymbol{C}^{K-1\times K-1}$ [6]. The second, third and fourth terms of (12) yield the positive definite matrix $16\Big(\bar{\mathbf{u}}\bar{\mathbf{u}}^H + diag(\sum_{k=3}^{K}|u_k|^2 \quad \sum_{k=2,k\neq3}^{K}|u_k|^2 \ldots \sum_{k=3,k\neq K}^{K}|u_k|^2)\Big)$, while the first term provides the condition $\nu^2|A_1|^2|\hat{\mathbf{g}}^H\mathbf{g}|^2 \geq 1/4$ that ensures the convexity of $\tilde{J}_{CM}(.)$ in the noiseless case. Because $\bar{\mathbf{u}} = \mathbf{B}\mathbf{w}$ is a linear function of $\mathbf{w}$ then $\tilde{J}_{CM}(\bar{\mathbf{u}})$ being a convex function of $\bar{\mathbf{u}}$ implies that $J_{CM}(\mathbf{w}) = \tilde{J}_{CM}(\mathbf{B}\mathbf{w})$ is a convex function of $\mathbf{w}$. Since the extrema of the cost

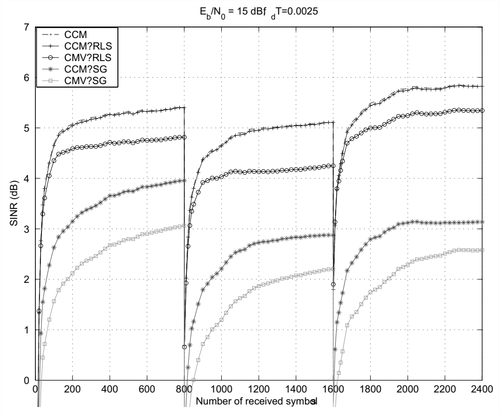

Fig. 1. SINR performance in a non-stationary environment.

function can be considered for small σ^2 a slight perturbation of the noise-free case [3], the cost function is also convex for small σ^2 when $\nu^2|A_1|^2|\hat{\mathbf{g}}^H\mathbf{g}|^2 \geq 1/4$. Interestingly, if we assume ideal channel estimation ($|\hat{\mathbf{g}}^H\mathbf{g}| = 1$) and $\nu = 1$, our result reduces to $|A_1|^2 \geq 1/4$, which is the same found in [8]. For larger values of σ^2, we remark that the term ν can be adjusted in order to make the cost function J_{CM} in (3) convex, as pointed out in [3].

VI. Simulations

The novel CCM-RLS algorithm is evaluated in different situations and compared with the CCM method of (4) and (5) (that requires $O(M^3)$ and $O(L_p^3)$) , the CMV-RLS reported in [2], the SG algorithms CMV-SG [2] and CCM-SG [4], that work with the linear multiuser receiver [1]-[4] and the RAKE single user receiver [9]. The DS-CDMA system employs Gold sequences of length $N = 31$. Since the channel length is not known a priori, we will assume that $L_p = 6$ is an upper bound for all scenarios. The channel coefficients for the users are $h_l(i) = p_l\alpha_l(i)$, where $\alpha_l(i)$, $l = 0, 1, 2$, is obtained with Clarke's model [9]. We show the results in terms of the normalized Doppler frequency f_dT (cycles/symbol) and adopt three-path channels with relative powers given by 0, -3 and -6 dB, where in each run the second and third paths delays are uniformly distributed between one and five chips. The phase ambiguity derived from channel estimation is eliminated in our simulations by using the phase of $\mathbf{g}(0)$ as a reference to remove the ambiguity. We employ $\nu = 1$, $|A_1|^2 = 1$, $\alpha = 0.998$, $\mathbf{R}(0) = 0.01\mathbf{I}$ and $f_dT = 0.0025$.

In Fig. 1 we assess the algorithms in a non-stationary environment where users enter and exit the system. The system starts with four interferers with 7 dB above the desired user's power level and three interferers with the same power level of the desired one, which corresponds to $E_b/N_0 = 15$ dB. At 800 symbols, two interferers with 10 dB above the desired signal power level and two interferers with the same power level enter the system, whereas two interferers with 7 dB above the desired signal power level leave it. At 1600 symbols, one interferer with 10 dB above, one interferer with 7 dB above, and three interferers with the same power level of the desired signal exit the system, while one interferer with 15 dB above the desired user enters the system. The results for 200 runs show that the proposed CCM-RLS approach converges to a higher SINR than the other methods and matches the CCM performance. Note that in a near-far scenario the eigenvalue spread of the covariance matrix of the received vector $\mathbf{r}(i)$ is large, affecting the convergence performance of the SG algorithms that are subject to this phenomenon, whereas the rate of convergence of RLS-type techniques is invariant to such situation [6].

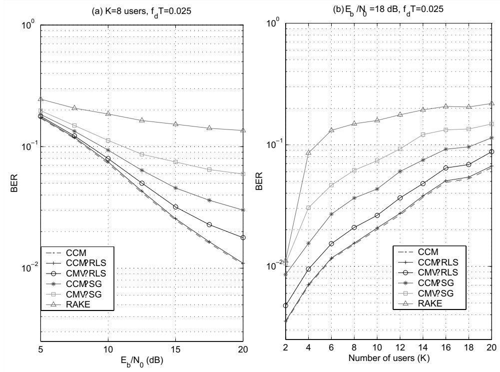

Fig. 2. BER versus (a) E_b/N_0 and (b) Number of users (K).

The BER performance versus E_b/N_0 and the number of users (K) is illustrated in Fig. 2. The receivers process 2000 symbols, averaged over 200 runs, and the parameters of the SG algorithms are optimized for each scenario. In these experiments, we have two interferers whose power levels are 5 and 10 dB (for $K > 2$) above the desired user, resulting in a near-far situation. The curves show that the new CCM-RLS algorithm outperforms the CMV-RLS reported in [2], the SG algorithms CMV-SG [2] and CCM-SG [4], saving transmission power for the same BER performance and increasing the capacity of the system.

VII. Conclusions

We presented and analyzed a CCM solution for linear receivers based on constrained optimization techniques and developed an RLS-type algorithm for jointly estimating the channel and the receiver parameters for MAI and ISI rejection.

References

[1] M. Honig, U. Madhow, and S. Verdu, "Blind adaptive multiuser detection," *IEEE Trans. Inform. Theory,* vol. 41, pp. 944-960, July 1995.

[2] Z. Xu and M.K. Tsatsanis, "Blind adaptive algorithms for minimum variance CDMA receivers," *IEEE Trans. Commun.*, vol. 49, pp. 180-194, Jan. 2001.

[3] C. Xu, G. Feng, and K. S. Kwak, "A modified constrained constant modulus approach to blind adaptive multiuser detection," *IEEE Trans. Commun.*, vol. 49, pp. 1642-1648, Sept. 2001.

[4] Z. Xu and P. Liu, "Code-constrained blind detection of CDMA signals in multipath channels," *IEEE Signal Processing Lett.*, vol. 9, pp. 389-392, Dec. 2002.

[5] X. G. Doukopoulos and G. V. Moustakides, "Blind channel estimation for downlink CDMA systems," in *Proc. IEEE ICC*, 2003, pp. 2416-2420.

[6] S. Haykin, *Adaptive Filter Theory*, 3rd edition. Englewood Cliffs, NJ: Prentice-Hall, 1996.

[7] X. G. Doukopoulos and G. V. Moustakides, "Power techniques for blind adaptive channel estimation in CDMA systems," in *Proc. IEEE Globecom*, 2003, pp. 2330-2334.

[8] J. P. de Villiers and L. P. Linde, "On the convexity of the LCCM cost function for DS-CDMA blind multiuser detection", *IEEE Commun. Lett.*, vol. 8, pp. 351-353, June 2004.

[9] T. S. Rappaport, *Wireless Communications*. Englewood Cliffs, NJ: Prentice-Hall, 1996.