Joint Transmit Diversity Optimization and Relay Selection for Multi-Relay Cooperative MIMO Systems Using Discrete Stochastic Algorithms

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Abstract—**We propose a joint discrete stochastic optimization based transmit diversity selection (TDS) and relay selection (RS) algorithm for decode-and-forward (DF), cooperative multiple-input multiple-output (MIMO) systems with a non-negligible direct path. TDS and RS are performed jointly with continuous least squares channel estimation (CE) without the need for inter-relay communication and linear minimum mean square error (MMSE) receivers are used at all nodes. The performance of the proposed scheme is evaluated via bit-error rate (BER) comparisons and diversity analysis, and is shown to converge to the optimum exhaustive solution.**

Index Terms—**MIMO relaying, transmit diversity, cooperative systems, relay selection.**

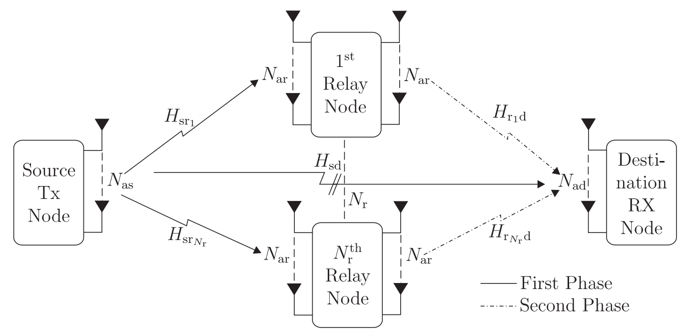

Fig. 1. MIMO multi-relay system model.

I. Introduction

COOPERATIVE multiple-input multiple-output (MIMO) networks have significant benefits over non-cooperative networks in terms of diversity and robustness. Consequently, they have been presented as a topology for the next generation of mobile networks [1], leading to antenna selection, relay selection (RS) and diversity maximization becoming central themes in MIMO relaying literature [2]–[4]. However, current approaches to these topics are often limited to stationary and single relay systems which assume a negligible direct path [3].

In this letter, the problems of transmit diversity selection (TDS) and RS are formulated as a joint discrete optimization problem where RS refines the set from which TDS is made. Low-complexity iterative discrete stochastic algorithms (DSA) with mean square error (MSE) cost functions are employed to obtain a solution and improvements in convergence, performance and complexity result. Continuous recursive least squares (RLS) channel estimation (CE) is introduced to form a combined framework where adaptive RS and TDS are performed jointly with no forward channel state information (CSI). The proposed algorithms are implemented and bit error-rate (BER) and diversity comparisons given against the exhaustive solution and the standard cooperative system.

II. System Model

We consider a quadrature phase shift keying (QPSK), two-phase, decode-and-forward (DF), multi-relay MIMO system with half-duplex relays. Linear minimum mean square error (MMSE) receivers are used at all nodes and an error-free control channel is assumed [2], [4]. All channels between antenna pairs are flat fading, have a coherence time equal to the period of an N symbol packet and are represented by a complex gain. The direct path is non-negligible and has an expected gain which is a fraction of the indirect paths. The maximum spatial multiplexing gain and diversity advantage simultaneously available in the system are $r^* = N_{as}$ and $d^* = N_{ad}(1 + (N_r N_{ar}/N_{as}))$, respectively [5], [6]. An outline system model is given by Fig. 1. The system comprises N_r intermediate relay nodes which lie between single source and destination nodes that have N_{as} and N_{ad} antennas, respectively. Each relay has N_{ar} antennas, where N_{ar} is an integer multiple of N_{as} in order to reduce feedback requirements. The transmitted data consists of N_{as} independent data streams which are allocated to the correspondingly numbered antennas at the source and relay nodes. The source node transmits to the relay and destination nodes during the first phase and the second phase involves the relay nodes decoding and forwarding their received signal to the destination. The maximum spatial multiplexing gain and diversity advantage simultaneously available in the system are $r^* = N_{as}$ and $d^* = N_{ad}(1 + (N_r N_{ar}/N_{as}))$, respectively [5], [6]. The $N_{ad} \times 1$ and $N_{ar} \times 1$ first phase received signals at the destination and the n^{th} relay are given by

$$\mathbf{r}_{sd}[i] = \mathbf{H}_{sd}[i] A_s \mathbf{T}_s \mathbf{s}[i] + \eta_{sd}[i] \tag{1}$$

and

$$\mathbf{r}_{sr_n}[i] = \mathbf{H}_{sr_n}[i] A_s \mathbf{T}_s \mathbf{s}[i] + \eta_{sr_n}[i], \tag{2}$$

respectively. The matrices $\mathbf{H}_{sd}$ and $\mathbf{H}_{sr_n}$ are the $N_{ad} \times N_{as}$ source - destination and $N_{ar} \times N_{as}$ source - n^{th} relay channel matrices, respectively where the subscripts s, d and r_n refer to the source, destination and n^{th} relay nodes, respectively. The quantity η is a vector of zero mean additive white Gaussian noise, $\mathbf{s}$ is the $N_{as} \times 1$ data vector, and A_s is the scalar transmit power

Manuscript received November 19, 2010. The associate editor coordinating the review of this letter and approving it for publication was D. Michalopoulos.

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Digital Object Identifier 10.1109/LCOMM.2011.082611.102262

allocation. The TDS matrix, $\mathbf{T}_s$, is a $N_{as} \times N_{as}$ diagonal matrix where each element on the main diagonal specifies whether the correspondingly numbered antenna is active. The received signal of the second phase at the destination is the sum of the forwarded signals from the N_r relays and is given by

$$\mathbf{r}_{rd}[i] = \boldsymbol{\mathcal{H}}_{rd}[i] A_r \boldsymbol{\mathcal{T}}_r[i] \hat{\tilde{\mathbf{s}}}[i] + \eta_{rd}[i], \quad (3)$$

where $\boldsymbol{\mathcal{T}}_r = \text{diag}[\mathbf{T}_{r_1}\ \mathbf{T}_{r_2} ... \mathbf{T}_{r_{N_r}}]$ is the $N_{ar}N_r \times N_{ar}N_r$ relay TDS matrix, $\hat{\tilde{\mathbf{s}}}[i] = [\hat{\mathbf{s}}^T_{r_1}[i] \overset{N_{ar}}{\underset{N_{as}}{\cdots}} \hat{\mathbf{s}}^T_{r_1}[i] ... \hat{\mathbf{s}}^T_{r_{N_r}}[i] \overset{N_{ar}}{\underset{N_{as}}{\cdots}} \hat{\mathbf{s}}^T_{r_{N_r}}[i]]$ is the $N_{ar}N_r \times 1$ estimated data vector and $\boldsymbol{\mathcal{H}}_{rd}[i] = [\mathbf{H}_{r_1 d}[i]\ \mathbf{H}_{r_2 d}[i] ... \mathbf{H}_{r_{N_r} d}[i]]$ is the $N_{ad} \times N_{ar}N_r$ channel matrix.

The linear MMSE receiver at the n^{th} relay is given by

$$\mathbf{W}_{sr_n}[i] = \arg\min_{\mathbf{W}_{sr_n}} E\left[\left\|\mathbf{s}[i] - \mathbf{W}^H_{sr_n}[i]\mathbf{r}_{sr_n}[i]\right\|^2\right], \quad (4)$$

which results in $\mathbf{W}_{sr_n} = \mathbf{R}^{-1}_{sr_n}\mathbf{P}_{sr_n}$, where $\mathbf{R}_{sr_n} = E[\mathbf{r}_{sr_n}[i]\mathbf{r}^H_{sr_n}[i]]$ and $\mathbf{P}_{sr_n} = E[\mathbf{r}_{sr_n}[i]\mathbf{s}^H[i]]$ are the autocorrelation and cross correlation matrices, respectively. At the destination the received signals are stacked to give $\mathbf{r}_d[i] = [\mathbf{r}^T_{sd}[i]\mathbf{r}^T_{rd}[i]]^T$, whose associated linear MMSE filter is given by

$$\mathbf{W}_d[i] = \arg\min_{\mathbf{W}_d} E\left[\left\|\mathbf{s}[i] - \mathbf{W}^H_d[i]\mathbf{r}_d[i]\right\|^2\right], \quad (5)$$

where $\mathbf{W}_d = \mathbf{R}^{-1}_d\mathbf{P}_d$, $\mathbf{R}_d = E[\mathbf{r}_d[i]\mathbf{r}^H_d[i]]$ and $\mathbf{P}_d = E[\mathbf{r}_d[i]\mathbf{s}^H[i]]$. A QPSK slicer follows MMSE reception at all nodes; the output of which is taken as the symbol estimate [7]. Using (4) and (5), the MSE at the n^{th} relay and destination are given by $\sigma^2_{\mathbf{s}} - \text{trace}(\mathbf{P}_{sr_n}\mathbf{R}^{-1}_{sr_n}\mathbf{P}_{sr_n})$ and $\sigma^2_{\mathbf{s}} - \text{trace}(\mathbf{P}_d\mathbf{R}^{-1}_{d,}\mathbf{P}_d)$, respectively, where $\sigma^2_{\mathbf{s}} = E[\mathbf{s}^H[i]\mathbf{s}[i]]$.

III. Problem Statement

In this section, we formulate the joint TDS and RS task as a discrete combinatorial MSE problem. The TDS optimization problem is given by

$$\begin{aligned}\boldsymbol{\mathcal{T}}^{opt}_r &= \arg\min_{\boldsymbol{\mathcal{T}}_r \in \Omega_T} C[i, \boldsymbol{\mathcal{T}}_r, \hat{\boldsymbol{\mathcal{H}}}_{rd}, \hat{\boldsymbol{\mathcal{H}}}_{sd}] \\ &= \arg\min_{\boldsymbol{\mathcal{T}}_r \in \Omega_T} E\left[\left\|\mathbf{s}[i] - \mathbf{W}_d[i, \boldsymbol{\mathcal{T}}_r, \hat{\boldsymbol{\mathcal{H}}}_{rd}, \hat{\boldsymbol{\mathcal{H}}}_{sd}]\mathbf{r}_d[i]\right\|^2\right], \quad (6)\end{aligned}$$

where Ω_T is the candidate TDS matrix set of cardinality $|\Omega_T| = \binom{N_{ar}N_r}{N_{asub}}$ and N_{asub} is the number of active relay antennas.

The performance and complexity of solutions to (6) depend on $|\Omega_T|$ and therefore we limit $|\Omega_T|$ whilst ensuring a minimum level of diversity by fixing $N_{asub} < N_{ar}N_r$. However, $|\Omega_T|$ is significant even at modest levels of antennas and relays, e.g. $N_r \geq 4$ and $N_{as} \geq 2$. Further improvements can be achieved by a process we term RS. By removing one or more relays from consideration by TDS based on their DF MSE performance, $|\Omega_T|$ and its quality can be improved without overly restricting the second-phase channels available to the TDS process. TDS using this refined set then effectively optimizes both phases.

The selection of the single highest MSE relay can be expressed as a discrete maximization problem given by

$$\begin{aligned}r^{opt}_n &= \arg\max_{r_n \in \Omega_R} \mathcal{F}[i, r_n, \hat{\boldsymbol{\mathcal{H}}}_{sr_n}] \\ &= \arg\max_{r_n \in \Omega_R} E\left[\left\|\mathbf{s}[i] - \mathbf{W}^H_{sr_n}[i, r_n, \hat{\boldsymbol{\mathcal{H}}}_{sr_n}]\mathbf{r}_{sr_n}[i]\right\|^2\right], \quad (7)\end{aligned}$$

TABLE I
Proposed discrete stochastic joint TDS and RS algorithm

Step
1. Initialization
choose $r[1] \in \Omega_R$, $r^W[1] \in \Omega_R$, $\boldsymbol{\pi}_R[1, r[1]] = 1$, $\boldsymbol{\pi}_R[1, \tilde{r}] = 0$ for $\tilde{r} \neq r[1]$
2. For the time index $i = 1, 2, ..., N$
choose $r^C[i] \in \Omega_R$
3. Comparison and update of the worst performing relay
if $\mathcal{F}[i, r^C[i]] > \mathcal{F}[i, r^W[i]]$ then $r^W[i+1] = r^C[i]$
otherwise $r^W[i+1] = r^W[i]$
4. State occupation probability (SOP) vector update
$\boldsymbol{\pi}_R[i+1] = \boldsymbol{\pi}_R[i] + \mu[i+1](\mathbf{v}_{r^W[i+1]} - \boldsymbol{\pi}_R[i])$ where $\mu[i] = 1/i$
5. Determine largest SOP vector element and select the optimum relay
if $\boldsymbol{\pi}_R[i+1, r^W[i+1]] > \boldsymbol{\pi}_R[i+1, r[i]]$ then $r[i+1] = r^W[i+1]$
otherwise $r[i+1] = r[i]$
6. TDS Set Reduction
remove members of Ω_T which utilize $r[i+1]$ ($\Omega_T \rightarrow \bar{\Omega}_T$)

where Ω_R is the set of candidate relays. Extension to the selection of multiple relays is then possible by populating Ω_R with sets of candidate relays and summing their MSE. This results in $|\Omega_R| = \binom{N_r}{N_{rem}}$ where N_{rem} is the number of relays to be removed. Once RS optimization is complete, a refined subset, $\bar{\Omega}_T \in \Omega_T$, is generated by removing members of Ω_T which involve transmission from r^{opt}_n. TDS then operates with this subset, where $|\bar{\Omega}_T| = \binom{N_{ar}(N_r - N_{rem})}{N_{asub}}$.

IV. Proposed Algorithm

We propose a low-complexity DSA which jointly optimizes RS and TDS in accordance with (6) and (7), and converges to the optimal exhaustive solution. The RS portion of the DSA is given by the algorithm of Table I. At each iteration the MSE of a randomly chosen candidate relay (r^C, step 2) and that of the worst performing relay currently known (r^W) are calculated (step 3). Via a comparison, the higher MSE relay is designated r^W for the next iteration (step 3). The current solution and the relay chosen for removal (r) is denoted as the current optimum and is the relay which has occupied r^W most frequently over the course of the packet up to the i^{th} time instant; effectively an average of the occupiers of r^W. This averaging/selection process is performed by allocating each member of Ω_R a $|\Omega_R| \times 1$ unit vector, $\mathbf{v}_l$, which has a one in its corresponding position in Ω_R i.e. $\mathbf{v}_{r^W}[i]$ is the label of the worst performing relay at the i^{th} iteration. The current optimum is then chosen and tracked by means of a $|\Omega_R| \times 1$ state occupation probability (SOP) vector, $\boldsymbol{\pi}_R$. This vector is updated at each iteration by adding $\mathbf{v}_{r^W}[i+i]$ and subtracting the previous value of $\boldsymbol{\pi}_R$ (step 4). The current optimum is then determined by selecting the largest element in $\boldsymbol{\pi}_R$ and its corresponding entry in Ω_R (step 5). Through this process, the current optimum converges towards and tracks the exhaustive solution [8]. An alternative interpretation of the proposed algorithm is to view the transitions, $r^W[i] \rightarrow r^W[i+1]$, as a Markov chain and the members of Ω_R as the possible transition states. The current optimum can then be defined as the most visited state.

Once RS is complete at each time instant, set reduction ($\Omega_T \rightarrow \bar{\Omega}_T$, step 6) takes place followed by TDS using a modified version of steps 1-5. The considered set is replaced, $\Omega_R \rightarrow \bar{\Omega}_T$; the structure of interest is replaced, $r \rightarrow \boldsymbol{\mathcal{T}}_r$; the best performing matrix is sought $r^W \rightarrow \boldsymbol{\mathcal{T}}^B_r$; the SOP vector

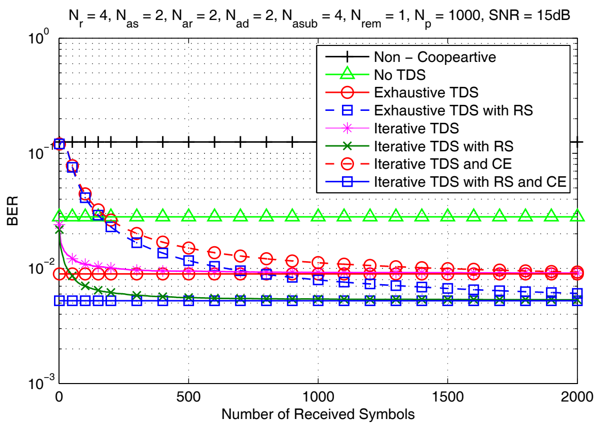

Fig. 2. BER performance versus the number of received symbols.

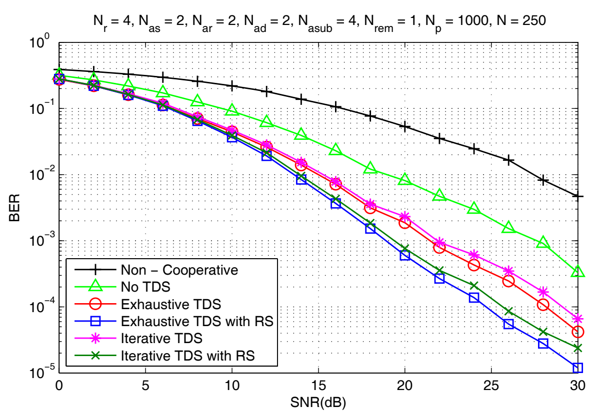

Fig. 3. BER performance versus SNR.

is replaced, $\boldsymbol{\pi}_{\mathrm{R}} \rightarrow \boldsymbol{\pi}_{\mathcal{T}}$; and $\mathcal{C} \rightarrow \mathcal{F}$ from (6). Finally, the inequality of step 3 is reversed to enable convergence to the lowest MSE TDS matrix, the index of which is fed back to the relays over the control channel.

Convergence of the proposed algorithm to the optimal exhaustive solution is dependent on the independence of the cost function observations and the satisfaction of $\Pr\{\mathcal{F}[r^{\mathrm{opt}}[i]] > \mathcal{F}[r[i]]\} > \Pr\{\mathcal{F}[r[i]] > \mathcal{F}[r^{\mathrm{opt}}[i]]\}$ and $\Pr\{\mathcal{F}[r^{\mathrm{opt}}[i]] > \mathcal{F}[r^{\mathrm{C}}[i]]\} > \Pr\{\mathcal{F}[r[i]] > \mathcal{F}[r^{\mathrm{C}}[i]]\}$ for RS and TDS (with the afore mentioned modifications). In this work, to minimize complexity, independent observations are not used and therefore the proof of convergence is intractable. However, excellent convergence has been observed under these conditions in [4] and throughout the simulations conducted for this work.

Significant complexity savings result from the proposed algorithm; savings which increase with N_{as}, N_{ar}, N_{ad}, N_{r} and N_{rem}. When $N_{\mathrm{r}} = 10$, $N_{\mathrm{as}} = N_{\mathrm{ar}} = N_{\mathrm{ad}} = 2$, N_{rem} and $N_{\mathrm{asub}} = 4$, the number of complex multiplications for MMSE reception and exhaustive TDS, exhaustive TDS with RS, iterative TDS and iterative TDS with RS are 5.8×10^8, 1.7×10^8, 1.8×10^5 and 5.9×10^4, respectively, for each time instant.

V. Simulations

In this section, simulations of the proposed algorithm (Iterative TDS with RS) are presented and comparisons drawn against the optimal exhaustive solutions (Exhaustive TDS with RS), the standard system (No TDS), and the direct transmission (Non-Cooperative). Plots of the schemes with TDS only (Exhaustive TDS, Iterative TDS) are also included to illustrate the performance improvement obtained by RS. Equal power allocation is maintained in each phase, where $A_{\mathrm{r}} = 1/\sqrt{N_{\mathrm{asub}}}$ when TDS is employed and $A_{\mathrm{r}} = 1/\sqrt{N_{\mathrm{ar}} N_{\mathrm{r}}}$ for the standard system. For the RLS CE, $\mathbf{P}_{\hat{\mathcal{H}}_{\mathrm{rd}}}$, $\mathbf{P}_{\hat{\mathbf{H}}_{\mathrm{sr}_n}}$ and $\mathbf{P}_{\hat{\mathbf{H}}_{\mathrm{sd}}}$ are initialized as identity matrices and the exponential forgetting factor is 0.9. The initial values of $\hat{\mathcal{H}}_{\mathrm{rd}}$, $\hat{\mathbf{H}}_{\mathrm{sr}_n}$ and $\hat{\mathbf{H}}_{\mathrm{sd}}$ are zeros matrices. Each simulation is averaged over 1000 packets (N_{p}) each made up N pilot symbols.

Fig. 2 gives the BER convergence performance of the proposed algorithm. The iterative TDS with RS algorithm rapidly converges to the improved optimal exhaustive BER as does TDS with RS and CE, albeit in a delayed fashion due to the CE. These results and the interdependence between elements of the algorithm confirm that both the RS and TDS portions of the algorithm converge well and the probability conditions of Section IV are satisfied.

Fig. 3 shows the BER versus SNR performance of the proposed and conventional algorithms. Increased diversity and improved interference mitigation have been achieved whilst maintaining r^*, illustrating that although the maximum available diversity advantage decreases with RS with TDS to $d^* = N_{\mathrm{ad}}(N_{\mathrm{asub}}/N_{\mathrm{ar}} + 1)$, the diversity achieved has increased. However, due to the use of linear receivers it is not possible to achieve the full receive diversity on offer [6]. The improvements obtained can be attributed to the removal of transmissions over poor paths but also the increase in transmit power over the remaining paths. The largest gains are present in 5dB-25dB region and begin to diminish above this region as relay decoding becomes increasingly reliable and lower power paths become more viable for transmission.

VI. Conclusions

This work presented a joint DSA which combines TDS and RS along with continuous CE for multi-relay cooperative MIMO systems.The scheme exceeds the performance of systems which lack TDS and matches that of the optimal exhaustive solution whilst saving considerable computational expense, making it ideal for real-time mobile use.

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