LOW-COMPLEXITY VARIABLE FORGETTING FACTOR MECHANISM FOR BLIND ADAPTIVE CONSTRAINED CONSTANT MODULUS ALGORITHMS

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Prior Work & Motivation

- Blind constrained minimum variance (CMV) algorithms:
  - CMV receiver in AWGN channels (1995 [1])/multipath channels (2001 [2]).
  - A novel variable step-size mechanism for CMV-SG algorithms (2006 [3]).
  - Novel reduced-rank algorithms based on CMV (2009 [4], 2010 [5]).
  - Drawback: They are sensitive to uncertainties.
- Blind constrained constant modulus (CCM) algorithms:
  - Blind CCM-SG receiver in multipath channels (2002 [6]).
  - Variable step-size mechanisms (2002 [7], 2009 [8]).
  - Blind CCM receiver with RLS implementation (2005 [9]).
  - Novel reduced-rank algorithms based on CCM (2008 [10], 2011 [11]).
- Motivation
  - RLS algorithm is one of the fastest and most effective methods.
  - It is impractical to compute a predetermined value for the forgetting factor in nonstationary wireless environments.
  - No work with blind variable forgetting factor techniques using CM criterion.

Contributions

- A low-complexity variable forgetting factor mechanism combined with blind CCM-RLS algorithms is introduced for multipath DS-CDMA channels.
- We extend the conventional GVFF mechanism to blind adaptive algorithms with the CM criterion.
- The convergence analysis of the adaptive CCM-RLS receiver with the proposed TAVFF mechanism is carried out.
- We derive formulas to predict the steady-state MSE and analyze the complexity of the blind GVFF and proposed TAVFF mechanisms.
- Simulations show that the TAVFF mechanism with the CCM-RLS receiver obtains significant gains in performance over existing schemes.

Signal Model

- DS-CDMA linear signal model:

$$\mathbf{r}(i) = \sum_{k=1}^{K} \left(A_k b_k(i) \mathbf{C}_k \mathbf{h}(i) + \boldsymbol{\eta}_k(i) \right) + \mathbf{n}(i),$$

where

$\boldsymbol{C}_k$ is an M x L matrix that contains one chip shift versions of the signature sequence ,

h(i) is the L x 1 channel vector ,

n(i) is an M x 1 the noise vector,

(i) denotes the time instant and $\boldsymbol{A}_k$ is the amplitude of user k,

where M=N+L-1, N is the spreading gain.

- The signal processing scheme observes r(i) and performs linear filtering.

Blind CCM-RLS Algorithm and Problem Statement

- We determine a FIR filter with M coefficients that provide an estimate of the desired symbol as follows

$$z_k(i) = \mathbf{w}_k^H(i)\mathbf{r}(i),$$

- Design: minimize $\sum_{n=1}^{i} \gamma^{i-n}\left(|\mathbf{w}_k^H(i)\mathbf{r}(n)|^2 - 1\right)^2$

  subject to $\mathbf{w}_k^H(i)\mathbf{C}_k\mathbf{h}(i) = \nu,$

- The CCM-RLS algorithm is given as

$$\mathbf{w}_k(i) = \mathbf{Q}_k^{-1}(i)\Big(\mathbf{d}_k(i) - \big(\mathbf{h}^H(i)\mathbf{C}_k^H\mathbf{Q}_k^{-1}(i)\mathbf{C}_k\mathbf{h}(i)\big)^{-1}\big(\mathbf{h}^H(i)\mathbf{C}_k^H\mathbf{Q}_k^{-1}(i)\mathbf{d}_k(i)\mathbf{C}_k\mathbf{h}(i) - \nu\mathbf{C}_k\mathbf{h}(i)\big)\Big).$$

$$\mathbf{s}_k(i) = \frac{\mathbf{Q}_k^{-1}(i-1)\mathbf{u}_k(i)}{\gamma + \mathbf{u}_k^H(i)\mathbf{Q}_k^{-1}(i-1)\mathbf{u}_k(i)}$$

$$\mathbf{Q}_k^{-1}(i) = \gamma^{-1}\mathbf{Q}_k^{-1}(i-1) - \gamma^{-1}\mathbf{s}_k(i)\mathbf{u}_k^H(i)\mathbf{Q}_k^{-1}(i-1)$$

$$\mathbf{d}_k(i) = \gamma\mathbf{d}_k(i-1) + z_k^*(i)\mathbf{r}(i).$$

- How to devise a cost-effective mechanism to adjust γ ?

Blind GVFF Scheme in Multipath Channels

- By taking the gradient of the instantaneous CM cost function with respect to the variable forgetting factor we obtain :

$$\gamma(i+1) = \Big[\gamma(i) - \mu \frac{\partial\big((|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1)^2\big)}{\partial\gamma}\Big]_{\gamma^-}^{\gamma^+},$$

$$\frac{\partial\big((|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1)^2\big)}{\partial\gamma} = (|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1)\Re[\mathbf{Y}_k^H(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_k(i)],$$

where $\mathbf{Y}_k(i) = \frac{\partial \mathbf{w}_k(i)}{\partial\gamma}$, $[.]_{\gamma^-}^{\gamma^+}$ denotes truncation to the limits of the range.

- By computing $\mathbf{Y}_k(i)$ we generate $\frac{\partial \mathbf{Q}_k^{-1}(i)}{\partial\gamma}$, $\frac{\partial \mathbf{d}_k(i)}{\partial\gamma}$ and $\frac{\partial \mathbf{s}_k(i)}{\partial\gamma}$. Please check the paper for the details.

- The GVFF mechanism is implemented by using the updated equations of $\frac{\partial \mathbf{Q}_k^{-1}(i)}{\partial\gamma}$, $\frac{\partial \mathbf{d}_k(i)}{\partial\gamma}$ $\frac{\partial \mathbf{s}_k(i)}{\partial\gamma}$ and $\mathbf{Y}_k(i)$ with initial values.

Proposed TAVFF Scheme

- Motivated by the VSS mechanism for an LMS algorithm proposed in [12]. We have devised

$$\phi(i) = \delta_1 \phi(i-1) + \delta_2 (|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1)^2,$$

- $\phi(i)$ denotes an updated component that is controlled by the instantaneous CM cost function .
- $\phi(i)$ is a small value, it changes rapidly as the instantaneous value of the cost function.
- The use of $\phi(i)$ has the potential to provide a suitable indication of the evolution of the cost function. The forgetting factor should vary in an inversely proportional way to the cost function:

$$\gamma(i) = \left[\frac{1}{1+\phi(i)}\right]_{\gamma^-}^{\gamma^+}$$

- δ_1 is close to 1, and δ_2 is a small positive value.

Proposed TAVFF Scheme (cont.)

- We assume $\lim_{i\rightarrow\infty} E[(|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1)^2] = \xi_{min} + \xi_{ex}(\infty)$.

  where ξ_{min} denotes the minimum value of the cost function and $\xi_{ex}(\infty)$ denotes the steady-state excess error of the CM cost function.

- Assuuming that $\xi_{min} \gg \xi_{ex}(\infty)$ the steady-state statistical properties of the forgetting factor value are given by

$$E[\gamma(\infty)] \approx \frac{1-\delta_1}{1+\delta_2\xi_{min}-\delta_1}$$

$$E[\gamma^2(\infty)] \approx \frac{(1-\delta_1)^2(1+\delta_1)}{(1-\delta_1)^2(1+\delta_1)+2\delta_2(1-\delta_1)(1+\delta_1)\xi_{min}+2\delta_1\delta_2\xi_{min}^2}$$

Computational complexity

- The TAVFF mechanism alone requires 4 multiplications and 2 additions,
- The blind GVFF algorithm alone requires $10M^2 + 16M + 7$ multiplications and $10M^2 + 6M - 1$ additions.

	Number of operations per symbol	
Algorithm	Multiplications	Additions
TAVFF	$6M^2 + LM + 10M + 5$	$5M^2 + LM + M$
Blind GVFF	$16M^2 + LM + 26M + 8$	$15M^2 + LM + 7M - 3$

Convergence Analysis of the Proposed Algorithm

- We derive the steady-state MSE expression of the proposed blind adaptive algorithm in the scenario of time-invariant channels.
- The final result of the steady-state MSE expression is given as follows: (for the detailed derivation please see the paper.)

$$\begin{aligned}\lim_{i\to\infty} \xi_{mse}(i) &= \lim_{i\to\infty} E[|A_k b(i) - \mathbf{w}_k^H(i)\mathbf{r}(i)|^2] \\ &\approx \lim_{i\to\infty} (\Xi(i) + A_k^2 - A_k^2 \mathbf{w}_0^H \mathbf{C}_k \mathbf{h} - A_k^2 \mathbf{h}^H \mathbf{C}_k^H \mathbf{w}_0) \\ &= \lim_{i\to\infty} \Xi(i) + (1-2\nu)A_k^2,\end{aligned}$$

where $\mathbf{w}_0^H \mathbf{C}_k \mathbf{h} = \nu$ and when i becomes a large number $\Xi(i) \approx \bar{\zeta}_4 + \Xi_{ex}(i)$

and $\Xi_{ex}(i) = tr[\mathbf{R}\boldsymbol{\Theta}(i)]$ denotes the steady-state excess MSE.

$$\Xi_{ex}(\infty) \approx \frac{(1-E[\gamma(\infty)])^2}{1-E[\gamma^2(\infty)]}\{\bar{\zeta}_1 tr[\mathbf{R}\mathbf{w}_0\mathbf{h}^H\mathbf{C}_k^H\mathbf{R}^{-1}\mathbf{C}_k\mathbf{h}\mathbf{w}_0^H] - \bar{\zeta}_2\nu - \bar{\zeta}_3\nu + \bar{\zeta}_4 M\}$$

Convergence Analysis of the Proposed Algorithm (cont.)

where we have

$$\bar{\zeta}_1 = E[|\mathbf{v}_0^H \mathbf{r}(i)|^2] = \mathbf{v}_0^H \mathbf{R} \mathbf{v}_0$$

$$\bar{\zeta}_2 = E[\mathbf{w}_0^H \mathbf{r}(i) \mathbf{r}^H(i) \mathbf{v}_0] = \mathbf{w}_0^H \mathbf{R} \mathbf{v}_0$$

$$\bar{\zeta}_3 = E[\mathbf{v}_0^H \mathbf{r}(i) \mathbf{r}^H(i) \mathbf{w}_0] = \mathbf{v}_0^H \mathbf{R} \mathbf{w}_0$$

$$\bar{\zeta}_4 = E[|\mathbf{w}_0^H \mathbf{r}(i)|^2] = \mathbf{w}_0^H \mathbf{R} \mathbf{w}_0$$

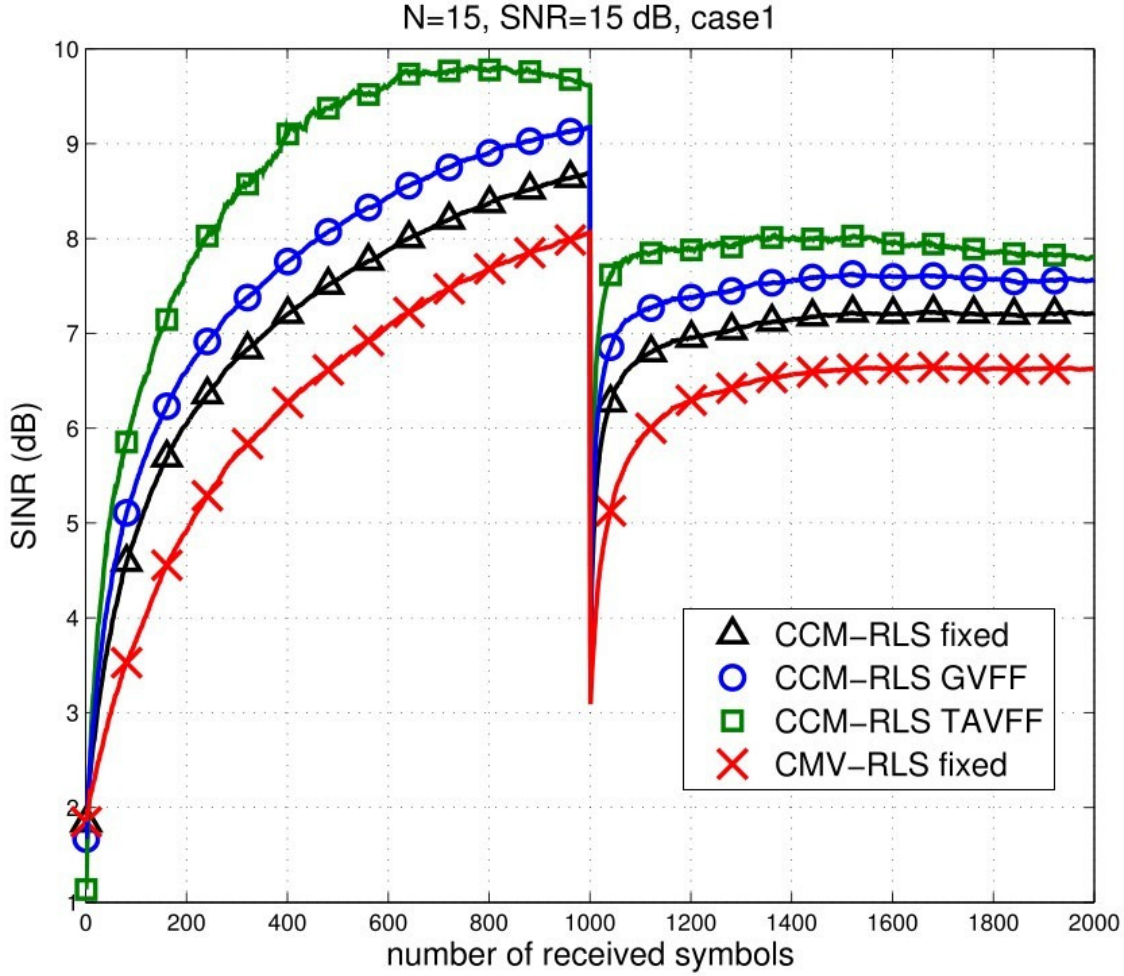
where $\mathbf{w}_0$ is the optimum CCM receiver and $\mathbf{v}_0$ is the optimum minimum variance receiver.

$$\mathbf{v}_0 = \frac{\mathbf{R}^{-1} \mathbf{C}_k \mathbf{h}}{\mathbf{h}^H \mathbf{C}_k^H \mathbf{R}^{-1} \mathbf{C}_k \mathbf{h}}$$

Simulations: Scenario and Parameters

- DS-CDMA system with Gold spreading codes, processing gain N=15, channel with delay spread of L chips, the channel is computed with the Jakes' model.
- We compare the TAVFF with the GVFF mechanism using the CCM-RLS and the CCM-RLS and CMV-RLS with fixed forgetting factor mechanisms.
- We show the following simulation results:
  - (1) Nonstationary case 1, 5 users (1 3dB above)->5+3 users (2 3dB+1 6dB above).
  - (2) Nonstationary case 2, 5 users ->5+3 users, equal power users.
  - (3) Forgetting factor versus symbols for case 1 & 2.
  - (4) BER performance versus SNR & number of users (K).
  - (5) Analytical results: a comparison between the steady-state analysis and simulation results.
- The channels are modelled by an FIR filter and the Jakes model, have L=3 paths with powers equal to 0, -6 and -10 dB and spacing given by 1 chip.
- We employ the steady-state filter weights of the CCM-RLS algorithm asthe optimum CCM receiver.

(1) SINR X Symbols, Case 1, normalized Doppler freq. 0.0001



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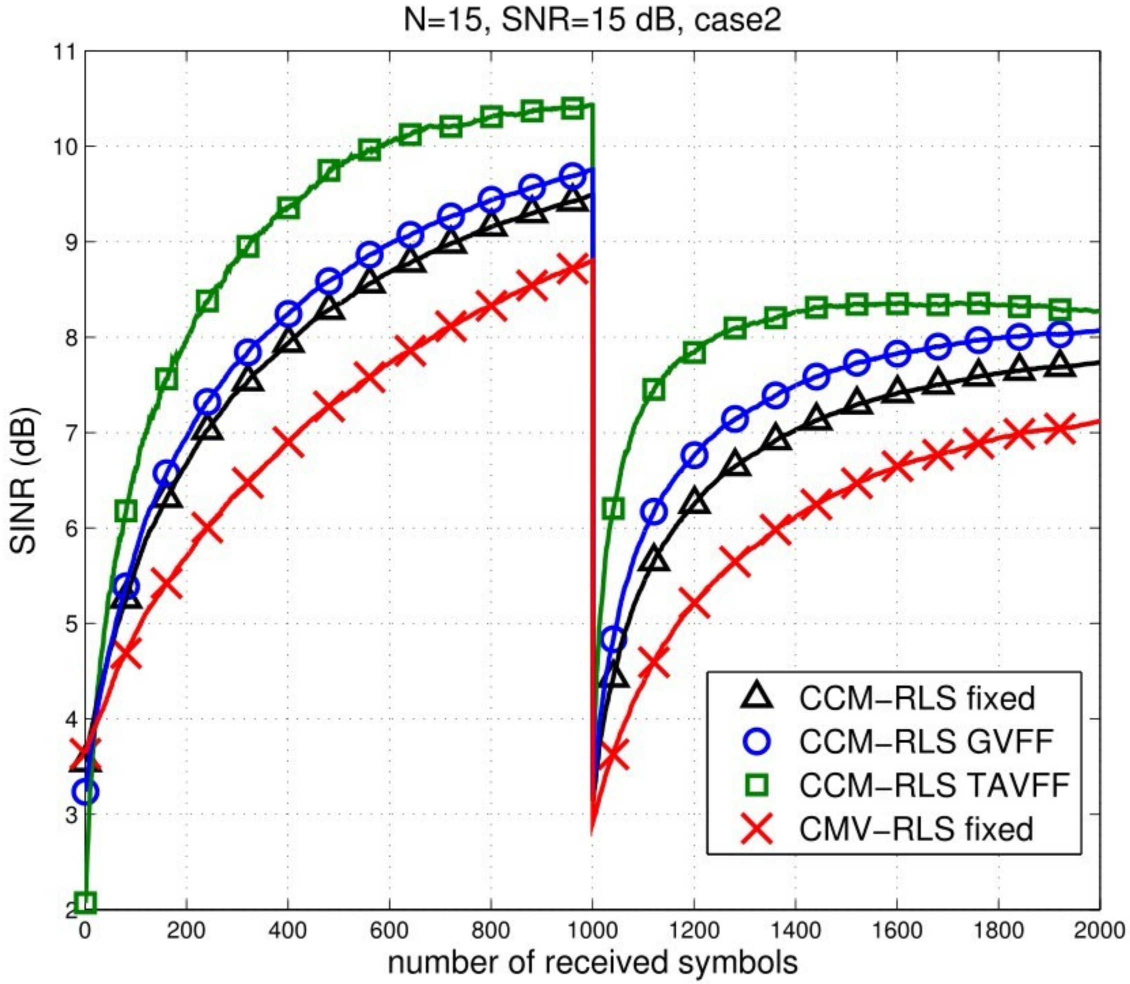
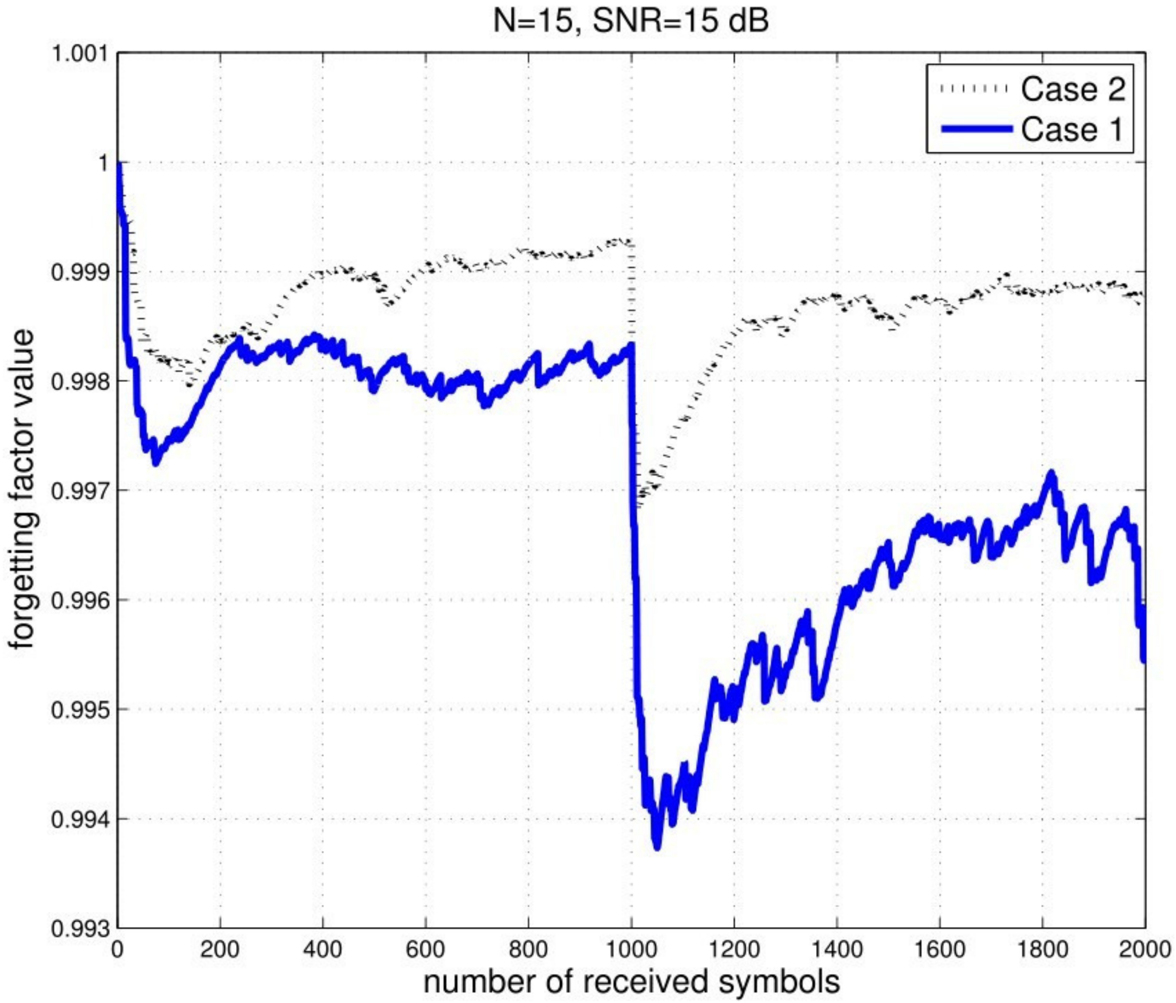
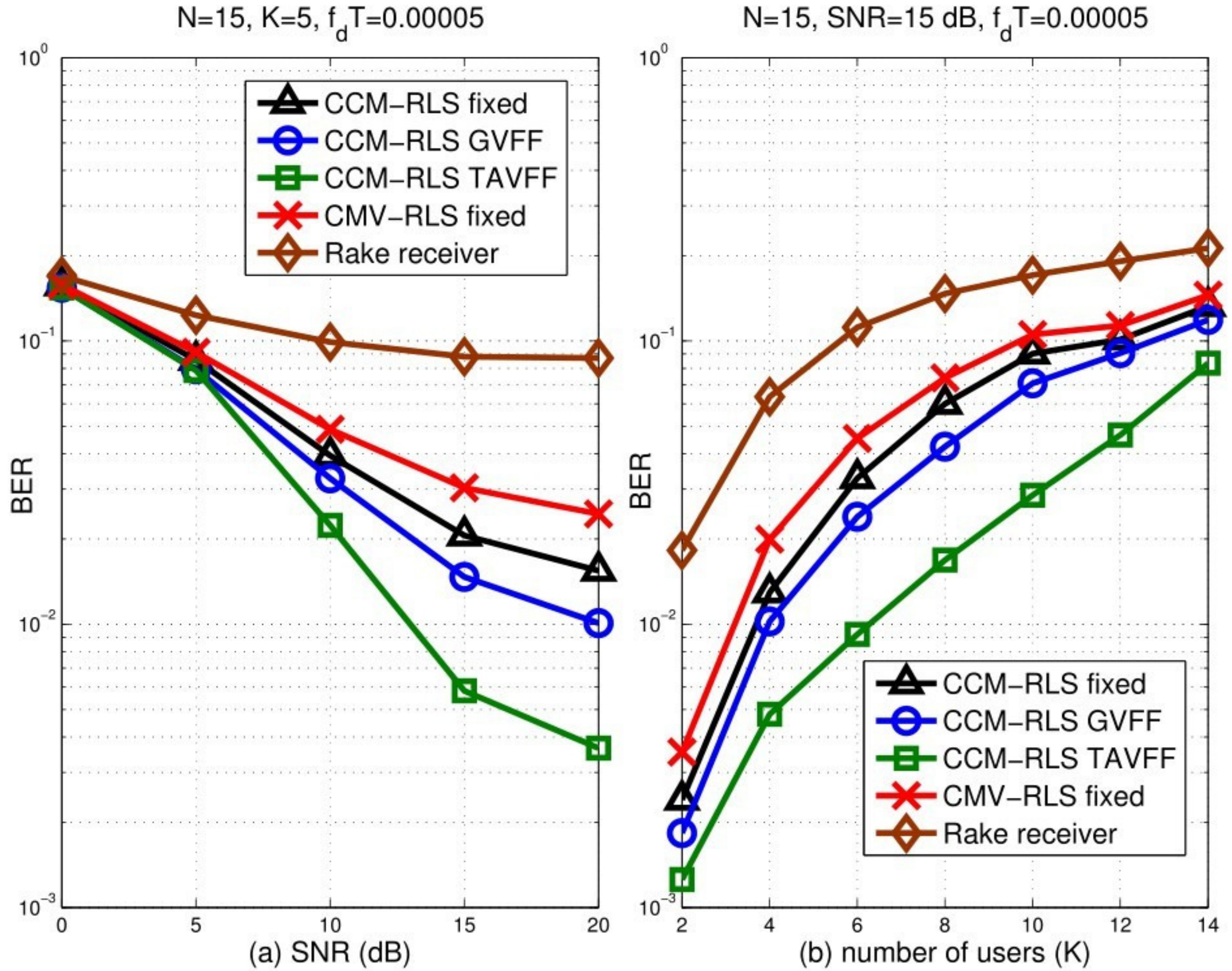
(2) SINR X Symbols, Case 2, normalized Doppler freq. 0.00005



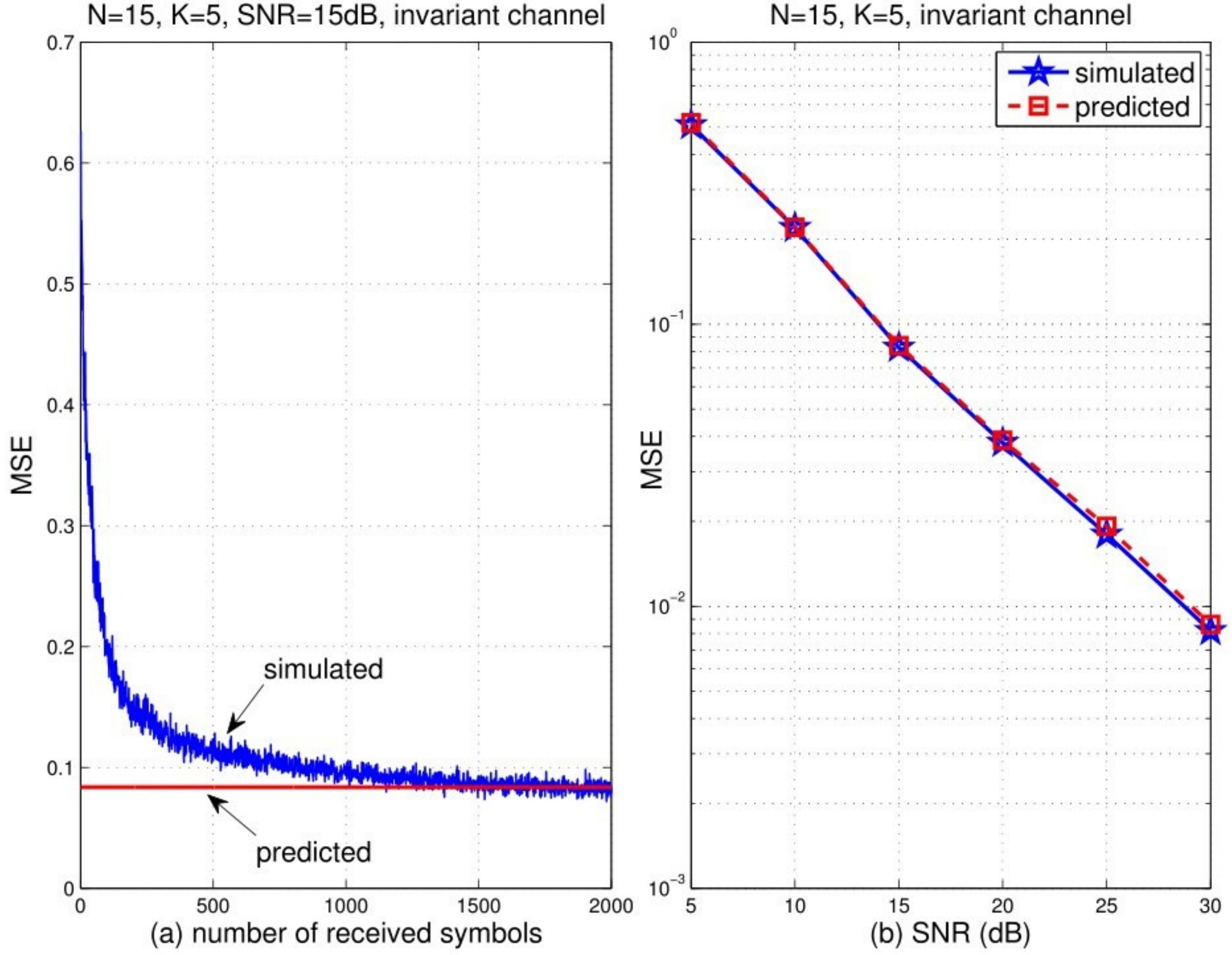
(3) Forgetting factor X symbols



(4) BER performance



(5) Analytical results



Conclusions

- A low-complexity variable forgetting factor mechanism for estimating the parameters of linear CDMA receivers that operate with RLS algorithms.
- We extended the conventional GVFF scheme to the blind CCM-RLS receiver in multipath fading channels.
- We compared the computational complexity of the new algorithm with the existent methods and further investigated the convergence analysis of the proposed TAVFF scheme.
- We also derived expressions to predict the steady-state MSE of the adaptive CCM-RLS algorithm with the TAVFF mechanism.
- The simulation results verify the analytical results and show that the proposed scheme significantly outperforms existing algorithms and supports systems with higher loads.

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