Adaptive Decision Feedback Detection with Parallel Interference Cancellation and Constellation Constraints for Multiuser MIMO systems

Peng Li, Rodrigo C. de Lamare, Jingjing Liu

***Abstract*—In this paper, a novel low-complexity adaptive decision feedback detection with parallel decision feedback and constellation constraints (P-DFCC) is proposed for multiuser MIMO systems. We propose a constrained constellation map which introduces a number of selected points served as the feedback candidates for interference cancellation. By introducing a reliability checking, a higher degree of freedom is introduced to refine the unreliable estimates. The P-DFCC is followed by an adaptive receive filter to estimate the transmitted symbol. In order to reduce the complexity of computing the filters with time-varying MIMO channels, an adaptive recursive least squares (RLS) algorithm is employed in the proposed P-DFCC scheme. An iterative detection and decoding (Turbo) scheme is considered with the proposed P-DFCC algorithm. Simulations show that the proposed technique has a complexity comparable to the conventional parallel decision feedback detector while it obtains a performance close to the maximum likelihood detector at a low to medium SNR range.**

***Index Terms*—RLS, multiuser detection, MIMO, adaptive receivers, iterative (Turbo) processing.**

I. Introduction

MULTI-user detection (MUD) [1] algorithms have shown that they can be applied to the uplink of 3G and next generation multi-antenna communication systems. MUD can also be applied to spatially multiplexed multi-input multi-output (MIMO) wireless communication systems to form a spatial division multiple access (SDMA) scheme. In such systems, multiple users are operated within the same frequency band simultaneously and the spatial dimension is exploited which can significantly increase the bandwidth efficiency. In order to successfully restore the signals from the received signal combination, pre-coding [2], [3] and decoding [4], [5] techniques are developed at the transmitter side and receiver side respectively. Due to the fact that for a multiple access uplink scenario, it is difficult for each user equipment (UE) to know the channel state information (CSI) of others, in this paper, we focus on the decoding part.

Several detection techniques have been developed for use at the receiver to suppress the multi-access interference (MAI), recover the simultaneously transmitted signals and increase the throughput for the served UEs [4]. The optimal maximum likelihood detection (MLD) [1] scheme has exponential complexity with the number of data streams and the modulation level, which is impractical for systems even with a moderate number of UEs. The cost effective ML solutions such as sphere decoders (SD) [6] [7] approach the optimal performance with reduced complexity [8], however, they still have a lower bound complexity which is polynomial or exponential depending on the number of UEs as well as the signal-to-noise ratio (SNR) [9]. In order to avoid the high complexity of ML or near-ML detectors, linear detectors which are based on minimum mean square error (MMSE) or zero-forcing receiver filters have been investigated. Generally, linear detectors experience a performance loss and achieve a lower capacity. A decision feedback receiver with successive decision feedback (S-DF) [10] or with parallel decision feedback (P-DF) [12] can be employed to achieve a higher capacity. These DF receiver structures [4], [13], [15], [16], are preferred as they offer an attractive performance and complexity trade-off, which is usually a key concern in multiple access systems.

The S/P-DF architectures are able to provide high spectral efficiencies when multiple transmit antennas are deployed [3]. However, the application to systems with time-varying channels is difficult due to the excessive computational load for updating the receive filter coefficients and tracking the channel [17]. The estimation of the receive filter weights and the CSI requires matrix inversions and other operations that lead to a significant number of computations.

As an alternative, the training aided adaptive techniques may be deployed for multiuser systems in time-varying channels [18]. Adaptive algorithms can be used to track the channels and to avoid excessive computations when the channels are varying. In [18], the authors developed a low-complexity data-aided adaptive technique for detecting the time-varying channels based on the GDF [13] structure, the weight vectors are updated using the recursive least squares (RLS) based algorithm. The multiple access interference introduced by spatial multiplexing can be suppressed in a serial or parallel manner and the transmitted symbols are estimated at each stage. Despite its many benefits, there is a large performance loss when one compares the performance of a DF based receiver with that of the optimal detector. This is due to the fact that (1) the DF structure can not provide the full receive diversity order achieved by the optimal MLD in spatially multiplexed systems. (2) The average performance of S/P-DF is dominated by the data stream with the lowest SINR and the effect of error propagation is inevitable [14]. (3) With the adaptive solution, the receiver filter is directed by the decisions made in the previous time instance. Therefore, erroneous decisions lead to unreliable filter weights.

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To address these problems, an adaptive multiuser decision feedback solution is proposed for time-varying multiple access MIMO channels. The so-called adaptive decision feedback detection with parallel interference cancellation and constellation constraints (P-DFCC) algorithm proposes a constrained constellation map which introduces a list which serves as the feedback candidates for P-DF detection. By calculating the bit and symbol reliability, a higher degree of freedom is introduced to refine the unreliable estimates in the cancellation stage. The proposed algorithm is able to significantly improve the performance for a traditional adaptive S-DF or P-DF detector and close the gap from the MLD. Thanks to the reliability calculation, the proposed algorithm obtains the combination list at a small additional computational cost.

We also consider a spatially multiplexed multiuser iterative detection and decoding (IDD) scheme incorporated with the proposed structure. In this coded system, the soft-input soft-output (SISO) detector is required to produce soft-decision values in terms of log-likelihood ratio (LLR). The proposed SISO detector uses the produced combination list to compute the likelihood of each transmitted bit, the probability of the decision is conveyed. This SISO detector is further concatenated with a SISO channel decoder to form a turbo structure which allows a lower SNR requirement for the adaptive MUD receiver. Computer simulations indicate that the proposed P-DFCC algorithm significantly outperforms the conventional S/P-DF schemes (i.e. [18]) and approaches the optimal performance with very low additional detection complexity.

The main contributions of this paper are:

- An adaptive decision feedback based algorithm is developed for data detection in time-varying MIMO channels.
- A P-DF receiver structure is investigated with the adaptive scheme, the constellation constraints (CC) is incorporated in the receiver to enhance the performance of interference cancellation.
- The error performance and the detection complexity of the proposed algorithm are compared with several popular existing S/P-DF and optimal detection schemes.
- A SISO detector is developed as a component of a multiuser IDD receiver structure.

The organization of this paper is as follows. Section II gives the multiuser spatial multiplexing MIMO system model as well as the conventional S/P-DF detector and optimal detection criterion. The proposed P-DFCC and its implementation are described in section III and followed by a complexity comparison in section IV. The iterative detection and decoding structure is introduced in Section V. The simulation results are given in Section VI and Section VII concludes the paper.

II. System and Data model

Let us consider a model of an uplink MU-MIMO system with K UEs and an access point (AP). Each UE is equipped with a single antenna. At the receiver of the AP, N_R receive antennas are available for collecting and processing the signals. Throughout this paper, the complex baseband notation is used while vectors and matrices are written in lower-case and upper-case boldface, respectively. We assume that the signals

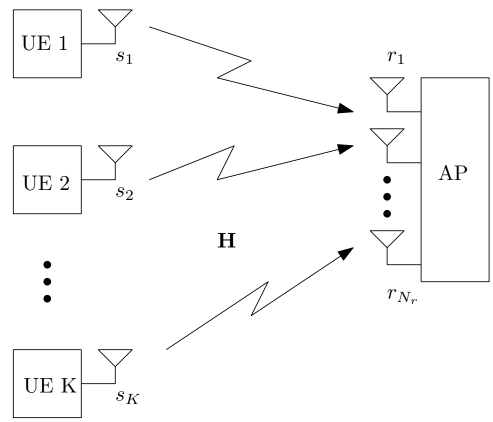


Fig. 1. Spatially multiplexed multiple access system. We assume the transmitted signal from K UEs are spatially uncorrelated and $K \leq N_R$.

of the UEs are perfectly synchronized at the AP, at each time instant $[i]$ K users simultaneously transmit K symbols which are organized into a vector $\boldsymbol{s}[i] = \big[s_1[i],\ s_2[i],\ \ldots,\ s_K[i]\big]^T$, where $(\cdot)^T$ denotes the transpose operation, and whose entries are chosen from a complex C-ary constellation set $\mathcal{X} = \{a_1,\ a_2,\ \ldots,\ a_C\}$. The symbol vector $\boldsymbol{s}[i]$ is transmitted over time-varying channels and the received signal is processed by the receiver at the AP with N_R spatially uncorrelated antennas. The received signal is collected to form an $N_R \times 1$ vector with sufficient statistics for detection

$$\begin{aligned} \boldsymbol{r}[i] &= \sum_{k=1}^{K} \boldsymbol{h}_k[i] s_k[i] + \boldsymbol{v}[i] \\ &= \boldsymbol{H}[i]\boldsymbol{s}[i] + \boldsymbol{v}[i], \end{aligned} \tag{1}$$

where the $N_R \times 1$ vector $\boldsymbol{v}[i]$ represents a zero mean complex circular symmetric Gaussian noise with covariance matrix $E\big[\boldsymbol{v}[i]\boldsymbol{v}^H[i]\big] = \sigma_v^2 \boldsymbol{I}$, σ_v^2 is the noise variance and $\boldsymbol{I}$ is the identity matrix, $E[\cdot]$ stands for the expected value and $(\cdot)^H$ denotes the Hermitian operator. The symbol vector $\boldsymbol{s}[i]$ has zero mean and a covariance matrix $E\big[\boldsymbol{s}[i]\boldsymbol{s}^H[i]\big] = \sigma_s^2 \boldsymbol{I}$, where σ_s^2 is the signal power for all transmitting UEs. Furthermore, the elements of $\boldsymbol{H}[i]$ are the time-varying complex channel gains from the k-th UE to the n_R-th receive antenna, which follow the Jakes' model [20]. The $N_R \times 1$ vector $\boldsymbol{h}_k[i]$ includes the channel coefficients of user k such that $\boldsymbol{H}[i]$ is formed by the channel vectors of all users. As the optimal SINR-based nulling and cancellation order (NCO) [18] requires a high computational complexity, we determine the NCO by computing the norms of the column vectors corresponding to all users and we detect them in decreasing order of their norms.

A. Optimal Detection

The optimal ML detection algorithm tries all the possible transmitted signal vectors with the given channel $\boldsymbol{H}$, the detector computes the Euclidean distance by $\mathcal{J}(s)_{\text{Euclidean}} = \|\boldsymbol{r} - \boldsymbol{H}\hat{\boldsymbol{s}}\|^2$, the signal vector with the minimum Euclidean distance is determined as the estimate of the transmitted signal:

$$\begin{aligned}\hat{\boldsymbol{s}}_{ML} &= \arg\max_{\boldsymbol{s}\in\mathcal{X}^K} P(\boldsymbol{r}|\boldsymbol{s}) \\ &= \arg\max_{\boldsymbol{s}\in\mathcal{X}^K} \frac{1}{(\pi\sigma_v^2)^K}\exp(-\frac{\|\boldsymbol{r}-\boldsymbol{H}\boldsymbol{s}\|^2}{\sigma_v^2}) \\ &= \arg\min_{\boldsymbol{s}\in\mathcal{X}^K} \mathcal{J}(s)_{\text{Euclidean}},\end{aligned} \quad (2)$$

Similarly to MAP detection, the algorithm requires an exhaustive search of $|\mathcal{X}|^K$ equations in (2) . The high complexity of the metric calculation prevents the actual application of these detectors in the real world, except for very small systems and constellations.

B. Successive and Parallel DF Receivers

Let $\hat{\boldsymbol{s}}[i] = \left[\hat{s}_1[i], \hat{s}_2[i], \ldots, \hat{s}_K[i]\right]^T$ represent the detected symbol vector. The soft symbol estimates $u_k[i]$ are obtained by calculating the difference between the output of the forward receive filter and the output of backward receive filter as described in [18] and given by

$$u_k[i] = \boldsymbol{\omega}_{f,k}^H[i]\boldsymbol{r}[i] - \boldsymbol{\omega}_{b,k}^H[i]\boldsymbol{d}_k[i], \quad (3)$$

where the column vector $\boldsymbol{\omega}_{f,k}[i] \in \mathbb{C}^{N_R\times 1}$ denotes the forward receive filter. The column vector $\boldsymbol{\omega}_{b,k}[i]$ indicates a backward receive filter with the dimension $k-1$ for successive decision feedback (S-DF) detection, or $K-1$ for parallel decision feedback (P-DF) detection.

1) S-DF: S-DF detection is illustrated in Fig.2(a), where the backward receive filter $\boldsymbol{\omega}_{b,k}[i] \in \mathbb{C}^{k-1}$ has k weight elements, and the size of $\boldsymbol{\omega}_{b,k}[i]$ increases as k raises. The forward filters $\boldsymbol{\omega}_{f,k}[i]$ act as the nulling vectors of the V-BLAST algorithm. Then for each data stream $k = 1, \ldots, K$, the decisions are accumulated and cancelled by the $(k-1)$-dimensional filter $\boldsymbol{\omega}_{b,k}[i]$. The backward receive filter is initialized by $\boldsymbol{\omega}_{b,1} = \boldsymbol{0}$ for the first user. For the following users, the $(k-1)$-dimensional detected symbol vector is obtained as

$$\boldsymbol{d}_k[i] = \left[\hat{s}_1, \hat{s}_2, \ldots, \hat{s}_{k-1}\right]^T. \quad (4)$$

The S-DF detection can provide a diversity order of $N_R - K + k$ for each user k assuming that perfect interference cancellation is performed by the receiver.

2) P-DF: By assuming perfect interference cancellation, P-DF is able to provide a higher diversity order compared to the S-DF based detection algorithms. Similar to S-DF scheme, the P-DF first processes the received signal $\boldsymbol{r}[i]$ by the forward receive filter $\boldsymbol{\omega}_{f,k}[i] \in \mathbb{C}^{N_R\times 1}$. However, as shown in Fig.2(b), the backward receive filter is different from S-DF, which is given as $\boldsymbol{\omega}_{b,k}[i] \in \mathbb{C}^{(K-1)\times 1}$, and the decision feedback symbol vector is defined as

$$\boldsymbol{d}_k[i] = \left[\hat{s}_1, \ldots, \hat{s}_{k-1}, 0, \hat{s}_{k+1} \ldots, \hat{s}_K\right]^T. \quad (5)$$

where the decisions for user $\hat{s}_k = \mathrm{Q}\{u_k[i]\}$ are obtained by applying a slicer represented by $\mathrm{Q}\{\cdot\}$.

For notational convenience, the forward and backward filters can be concatenated together as [18]

$$\tilde{\boldsymbol{\omega}}_k[i] = \begin{cases}\boldsymbol{\omega}_{f,k}[i], & k=1\\ \left[\boldsymbol{\omega}_{f,k}^T[i], \boldsymbol{\omega}_{b,k}^T[i]\right]^T, & k=2,\ldots,K.\end{cases} \quad (6)$$

The input can also be concatenated as

$$\tilde{\boldsymbol{r}}_k[i] = \begin{cases}\boldsymbol{r}[i], & k=1\\ \left[\boldsymbol{r}^T[i], \boldsymbol{d}_k^T[i]\right]^T, & k=2,\ldots,K.\end{cases} \quad (7)$$

Then, we can rewrite the soft estimates (3) as

$$u_k[i] = \tilde{\boldsymbol{\omega}}_k^H[i]\tilde{\boldsymbol{r}}_k[i]. \quad (8)$$

The forward and backward filters can be jointly optimized by using an MMSE criterion or solving a lest squares problem. For the sake of computational complexity, in the proposed structure the recursive least squares (RLS) algorithm is adopted for the design of the forward and backward filters. It should be noted that other advanced parameter estimation algorithms such as reduced-rank techniques [12], [21] can also be used.

III. Adaptive P-DF with Constellation Constraints

A. Computation of P-DF filters

As a result, the structure and the signal processing model of the proposed DF detector are depicted in Fig.3. We denote the receive filter of each user as $\tilde{\boldsymbol{\omega}}_k[i]$ $(k = 1, \ldots, K)$, and the value of each entry can be obtained by solving the standard least squares (LS) problem. The LS cost function with an exponential window is given by

$$\mathcal{J}_k[i] = \sum_{\tau=1}^{i}\lambda^{i-\tau}\left|\hat{s}_k[\tau] - \tilde{\boldsymbol{\omega}}_k^H[i]\tilde{\boldsymbol{r}}_k[\tau]\right|^2, \quad (9)$$

where $0 \ll \lambda < 1$ is the forgetting factor, the scalar $\hat{s}_k[\tau]$ denotes the detected signal in the time index τ or the known pilots where $\hat{s}_k[\tau] = s_k[\tau]$. The optimal tap weight minimizing $\mathcal{J}_k[i]$ is given by

$$\tilde{\boldsymbol{\omega}}_k[i] = \boldsymbol{\Phi}_k^{-1}[i]\boldsymbol{p}_k[i], \quad (10)$$

where the time-averaged cross correlation matrix is obtained by $\boldsymbol{\Phi}_k[i] = \sum_{\tau=1}^{i}\lambda^{i-\tau}\tilde{\boldsymbol{r}}_k[\tau]\tilde{\boldsymbol{r}}_k^H[\tau]$ and $\boldsymbol{\Phi}_k[0] = \boldsymbol{0}$, the time-averaged cross correlation vector is defined by $\boldsymbol{p}_k[i] = \sum_{\tau=1}^{i}\lambda^{i-\tau}\tilde{\boldsymbol{r}}_k[\tau]\hat{s}_k^*[\tau]$.

Using the recursive least squares (RLS) algorithm [19], the optimal weights in (10) can be calculated recursively as follows:

$$\boldsymbol{q}_k[i] = \boldsymbol{\Phi}_k^{-1}[i-1]\boldsymbol{r}_k[i], \quad (11)$$

$$\boldsymbol{k}_k[i] = \frac{\lambda^{-1}\boldsymbol{q}_k[i]}{1+\lambda^{-1}\boldsymbol{r}_k^H[i]\boldsymbol{q}_k[i]}, \quad (12)$$

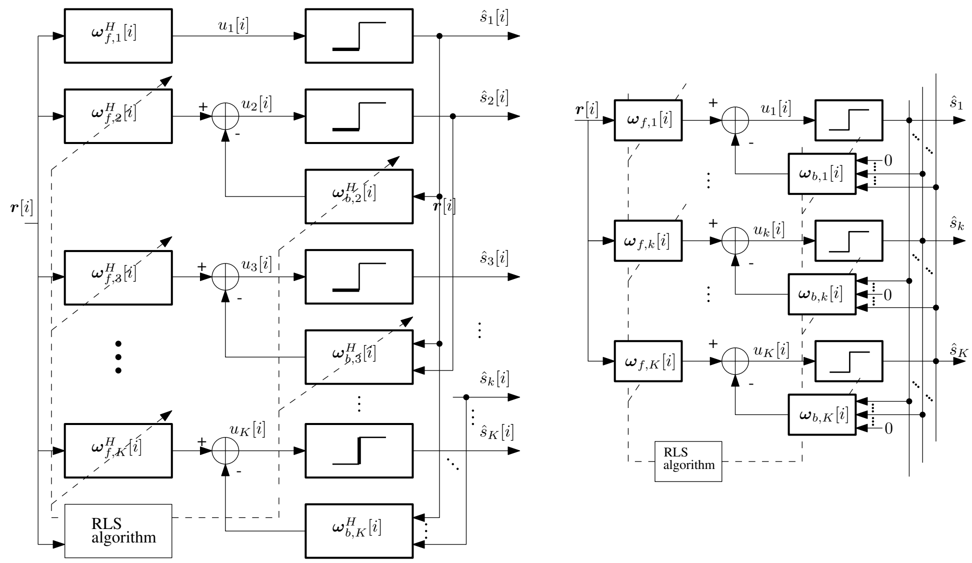
$$\boldsymbol{\Phi}_k^{-1}[i] = \lambda^{-1}\boldsymbol{\Phi}_k^{-1}[i-1] - \lambda^{-1}\boldsymbol{k}_k[i]\boldsymbol{q}_k^H[i], \quad (13)$$

$$\tilde{\boldsymbol{\omega}}_k[i] = \tilde{\boldsymbol{\omega}}_k[i-1] + \boldsymbol{k}_k[i]\xi_k^*[i], \quad (14)$$

where

$$\xi_k[i] = \begin{cases} s_k[i] - \tilde{\boldsymbol{\omega}}_k^H[i-1]\tilde{\boldsymbol{r}}_k[i], & \text{Training Mode,}\\ \hat{s}_k[i] - \tilde{\boldsymbol{\omega}}_k^H[i-1]\tilde{\boldsymbol{r}}_k[i], & \text{Decision-directed Mode.}\end{cases} \quad (15)$$

As indicated in (15), this adaptive detection algorithm works in two modes. The first one is employed with the training sequence, while the second one is the decision-directed mode



(a) S-DF (b) P-DF

Fig. 2. Block diagram of the conventional (a) S-DF scheme and (b) P-DF scheme. An RLS algorithm is employed to iteratively obtain the filter weights.

that is switched on after the filter weights converge. In the decision-directed mode, the mean square error (MSE) of the estimated symbols has a major impact on the performance of adaptive DF algorithms. This is because the detection error of the current user may propagate throughout the detection of the following users. Moreover, in time-varying channels a poor $\xi_k[i]$ can easily damage the $\tilde{\boldsymbol{\omega}}_k[i]$ in equation (14) resulting in burst errors.

B. P-DF with Constellation Constraints

In order to address this problem, the proposed P-DF with constellation constraints (P-DFCC) structure introduces a number of selected constellation points as the candidate decisions when the filter output $u_k[i]$ is determined unreliable. After the system is switched to the decision-directed mode, the concatenated filter output $u_k[i]$ is checked by the CC device which is illustrated in Fig. 4. The CC structure is defined by the threshold distance d_{th}, which can be a constant or determined in terms of SNR. The reliability of the estimated symbol is determined by the Euclidean distance between the symbol estimates and its nearest constellation points, which are given by

$$d_k = \min_{a_c \in \mathcal{X}} \{|u_k[i] - a_c|\}, \tag{16}$$

where a_c denotes the constellation point which is the nearest to the soft estimation $u_k[i]$ of the k-th symbol. The CC device distinguishes the reliable estimation from the unreliable ones, which allows the P-DFCC to avoid redundant processing with reliable feedbacks and maintain the complexity at the same level of the conventional P-DF structure. The following is devoted to describe the detection of $\hat{s}_k[i]$ for the k-th user.

Let us define two regions for the QPSK constellation map: (1) The region inside the square obtained by connecting four a_c, the a_c are assumed to have the form, $a_c = \Big(\pm\epsilon/2, \pm(\epsilon/2)j\Big)$, where ϵ is the distance between two nearest

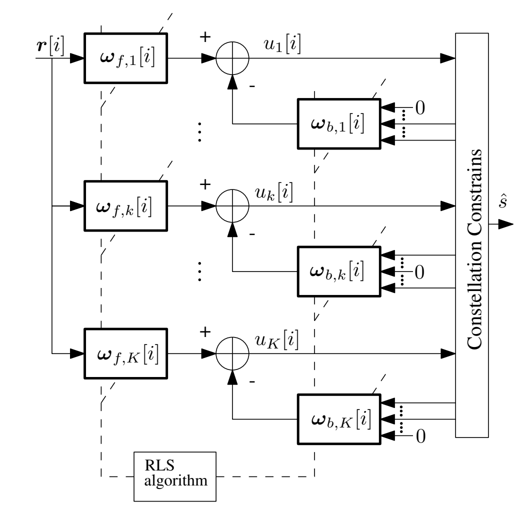


Fig. 3. Block diagram of the proposed P-DFCC multi-user detector. There are $K-1$ interference symbols for each user's backward filter.

constellation symbols. The estimate $u_k[i]$ is considered inside the square if the following equations hold

$$\begin{cases} \big|\Re\{u_k[i]\}\big| \le \epsilon/2 \\ \big|\Im\{u_k[i]\}\big| \le \epsilon/2. \end{cases} \tag{17}$$

where $\Re\{\cdot\}$ and $\Im\{\cdot\}$ denote the real part and the imaginary part of a complex-valued quantity, respectively.

(2) Otherwise, the estimate is in the region outside the square obtained.

1) CASE 1 inside the square: In the first case, the estimate $u_k[i]$ is considered as unreliable if the following equation holds

$$d_k > d_{\text{th}}. \tag{18}$$

where d_k denotes the distance between the estimated symbol $u_k[i]$ and its nearest constellation point and a_c is each element

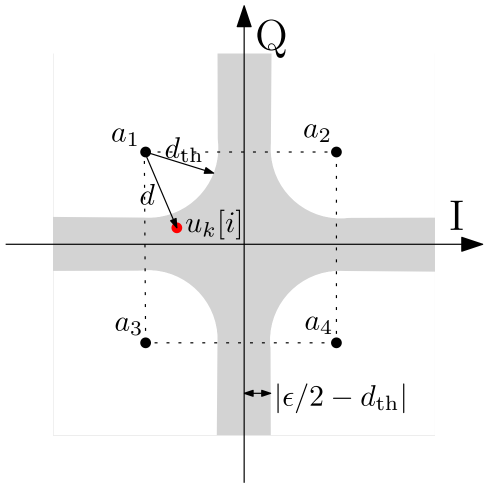


Fig. 4. The constellation constraints (CC) device. The CC procedure is invoked as the soft estimates $u_k[i]$ dropped into the shaded area. Parameter ϵ denotes the distance between 2 nearest constellation symbols.

of the constellation points. Otherwise, the estimated symbol is closer to the constellations and the decision is considered as reliable.

2) CASE 2 outside the square: In this case, the equations of (17) do not hold, where the estimated symbol $u_k[i]$ is outside the square. In this case, the decision is determined unreliable if the distance from $u_k[i]$ to I(in-phase)-axis and Q(quadrature)-axis is small. Therefore, the estimate is unreliable if any of the following equations holds

$$\left|\Re\{u_k[i]\}\right| < \epsilon/2 - d_{\rm th}, \tag{19}$$

$$\left|\Im\{u_k[i]\}\right| < \epsilon/2 - d_{\rm th}. \tag{20}$$

Otherwise, the estimated symbol is far away from the axis borders and the estimate is considered as reliable.

This can be further extended to multi-tier constellations (eg.16-QAM) where the outer-tier would be similar to CASE 2, but we should also include two additional equations in addition to (19) and (20) which are given as

$$\min\left|\Re\{u_k[i]\} \pm \epsilon\right| < \epsilon/2 - d_{\rm th}, \tag{21}$$

$$\min\left|\Im\{u_k[i]\} \pm \epsilon\right| < \epsilon/2 - d_{\rm th}. \tag{22}$$

where $\left|\Re\{u_k[i]\} \pm \epsilon\right|$ are the distances between $u_k[i]$ and two vertical lines across the points $(0, \pm\epsilon)$, respectively. The matrices $\left|\Im\{u_k[i]\} \pm \epsilon\right|$ are similarly defined as the distances between $u_k[i]$ and two horizontal lines across points $(\pm\epsilon, 0)$, respectively. Therefore, for 16-QAM constellations, the estimate is considered as unreliable if any one of the four equations above (19-22) holds. On the other hand, for the inner-tier constellations, if

$$\min\left|a_k[i] - u_k[i]\right| \geq d_{\rm th} \qquad \forall c, \tag{23}$$

was true, the estimate is considered as unreliable. The CC device distinguishes the reliable feedback signals from the unreliable ones, which allows the P-DFCC to maintain the complexity at the same level of the conventional DF structure.

Reliable: If the filter output $u_k[i]$ is dropped into the lighted area of the constellation map, the decision is considered reliable. The tentative decision of $s_k[i]$ is obtained by

$$\mathcal{L}_k[i] = \arg_{a_c} \min\left|a_c - u_k[i]\right| \tag{24}$$

Unreliable: If it is the case that $u_k[i]$ is determined unreliable, we proceed by organizing the Euclidean distance obtained by (16) in decreasing order, a list of tentative decisions of $s_k[i]$ is obtained as given by

$$\mathcal{L}_k[i] \triangleq \{c_1, c_2, \ldots, c_\tau\}_k, \tag{25}$$

where $1 \leq \tau \leq |\mathcal{X}|$, and

$$\text{de}[c_1] \leq \text{de}[c_2] \leq \ldots, \text{de}[c_\tau], \tag{26}$$

where de$[\cdot]$ denotes the Euclidean distances between u_k and c_τ.

Therefore, for each user we obtain a tentative decision list $\mathcal{L}_k$. By listing all the combinations of the elements across K users, a length $\mathit{\Gamma}$ tentative decision list is formed. Each column vector on the list denotes a possible transmission symbol vector $\boldsymbol{s}'_l$ where $l = 1, \ldots, \mathit{\Gamma}$. The size of the list is obtained by

$$\mathit{\Gamma} = \prod_{k=1}^{K} |\mathcal{L}_k|, \qquad 1 \leq \mathit{\Gamma} \ll |\mathcal{X}|^K, \tag{27}$$

where $|\cdot|$ denotes cardinality. In order to obtain an improved performance, the maximum likelihood (ML) rule can be used to select the best among the $\mathit{\Gamma}$ candidate symbol vectors. The cost function for the ML selection criterion, which is equivalent to the minimum Euclidean distance criterion and the selected vector is given by

$$\boldsymbol{s}'_{\rm ML} = \arg\min_{l=1,\ldots,\mathit{\Gamma}} \left\|\boldsymbol{r}[i] - \boldsymbol{H}\boldsymbol{s}'_l[i]\right\|^2, \tag{28}$$

where $\boldsymbol{s}'_{\rm ML}$ is the ML selected vector, which can be used as the feedback symbols as well as the decision vector.

The number of $\mathit{\Gamma}$ could be considered as a reflection of the trade-off between complexity and performance. By assuming a large threshold $d_{\rm th}$, the proposed scheme is able to tolerate a higher error probability and results in a smaller $\mathit{\Gamma}$ but suffers from a performance loss. For an extreme case where $d_{\rm th} = \inf$, the proposed detector is equivalent to a conventional D-DF detector. In contrast, if $d_{\rm th} = 0$, we have $\mathit{\Gamma} = |\mathcal{X}|^K$ which means the proposed receiver performs DF detection with an ML rule that allows the search for an ML solution for each user. It is also worth to mention that a maximum $\tau_{\rm max}$ can be set to guarantee $1 \leq \mathit{\Gamma} \ll |\mathcal{X}|^K$, which prevent high complexity in very low SNR range.

By introducing a constellation constraint, (a) the detection diversity is directly related to the threshold $d_{\rm th}$: a decrease in the value of $d_{\rm th}$ could result in a longer list which may increase the diversity order. (b) In the low SNR region, it is also likely to obtain a longer list than that in a high SNR region, hence the diversity order tends to be higher. On the other hand, for the high SNR region, all the symbol estimates are considered reliable and the diversity order tends to be the same of a conventional P-DF (this is similar to increasing the threshold $d_{\rm th}$). This implies that the gain provided by P-DFCC is higher for a small to medium region of SNR.

C. Channel Estimation

As we discussed in the previous sections, the MIMO channel state information is required for the ML rule (28) and for generating the cancellation ordering codebook for ordered processing [9]. The LS channel estimation algorithm has been investigated in [22]. Based on a weighted average of error squares, the estimated channel minimizes the cost function whose expression at time instant i is defined as

$$\mathcal{J}_{\hat{\boldsymbol{H}}}[i] = \sum_{\tau=1}^{i} \lambda^{i-\tau} \Big| \boldsymbol{r}[\tau] - \hat{\boldsymbol{H}}[i]\boldsymbol{s}[\tau] \Big|^2, \tag{29}$$

where $\hat{\boldsymbol{H}}[i]$ is the channel matrix estimate at time instant i. The quantities $\boldsymbol{r}[\tau]$ and $\boldsymbol{s}[\tau]$ are the received signal and the pilot symbol vectors at the time instant τ, respectively.

To minimise the cost function, the gradient of the cost function with regard to the estimated channel matrix should be equated to a zero matrix as

$$\nabla_{\hat{\boldsymbol{H}}[i]} \mathcal{J}_{\hat{\boldsymbol{H}}}[i] = \mathbf{0}_{N_R,K}. \tag{30}$$

By solving the above equation, the LS estimate of the channel matrix is obtained as

$$\begin{aligned}\hat{\boldsymbol{H}}[i] &= \Big(\sum_{\tau=1}^{i} \lambda^{i-\tau} \boldsymbol{r}[\tau]\boldsymbol{s}^H[\tau] \Big) \Big(\sum_{\tau=1}^{i} \lambda^{i-\tau} \boldsymbol{s}[\tau]\boldsymbol{s}^H[\tau] \Big)^{-1} \\ &= \boldsymbol{D}[i]\boldsymbol{\Phi}^{-1}[i].\end{aligned} \tag{31}$$

In order to avoid the matrix inversion operation $(\cdot)^{-1}$, a recursive algorithm is developed. Let us define

$$\boldsymbol{\Phi}^{-1}[i] = \boldsymbol{P}[i], \tag{32}$$

where $\boldsymbol{D}[i]$ can be obtained iteratively by

$$\boldsymbol{D}[i] = \lambda \boldsymbol{D}[i-1] + \boldsymbol{r}[i]\boldsymbol{s}[i]^H \tag{33}$$

and $\boldsymbol{P}[i]$ is calculated iteratively by using the matrix inversion lemma,

$$\boldsymbol{P}[i] = \lambda^{-1}\boldsymbol{P}[i-1] - \frac{\lambda^{-2}\boldsymbol{P}[i-1]\boldsymbol{s}[i]\boldsymbol{s}[i]^H\boldsymbol{P}[i-1]}{1+\lambda^{-1}\boldsymbol{s}[i]^H\boldsymbol{P}[i-1]\boldsymbol{s}[i]}. \tag{34}$$

The initial state of the parameters are set as $\boldsymbol{D}[0] = \mathbf{0}_{N_R,K}$ and $\boldsymbol{P}[0] = \delta_c^{-1}\boldsymbol{I}$, where δ_c is a small constant.

IV. Iterative Detection and Decoding

In the previous section, we have introduced the concept of constellation constraints and its implication for an uncoded multi-user detection algorithm. In order to reduce the SNR requirement for a MIMO receiver, error-control coding is essential for the system. Iterative detection and decoding (IDD) has been recognized as central technique for solving a large number of decoding and detection problems in wireless communications. In this section, the we are interested in developing IDD algorithms for spatially multiplexed multi-user data streams.

For a multi-user MIMO IDD transmission system, the message is first encoded by an encoder, the coded bits are then interleaved and the coded bits are mapped to symbols before radiating from a transmitting antenna. At the receiver side, the P-DFCC detector is applied to detect the transmitted symbols and convert the symbol probability to bit probability in the form of LLRs. The *extrinsic information* $L_e(\cdot)$ is then exchanged between the detector and the decoder with several iterations. The *a posteriori* probability of the transmitted bits are then finally obtained at the output of the decoder.

On one hand, the encoder and decoder blocks are considered as the outer code of a serially concatenated structure, when a non-systematic convolutional coded (NSC) is applied, the BCJR [23] based MAP or log-MAP decoding algorithm can be applied as well as the lower complexity alternative named soft-output Viterbi algorithms (SOVA) [24]. Instead of using a convolutional code as the channel code, turbo codes and LDPC [26] codes along with advanced decoding algorithms [27] can also be used in this structure to obtain a near-capacity performance [7] [28]. On the other hand, the mapping and MIMO detection blocks are considered as the inner component of the serially concatenated structure. In general, MAP is the optimal algorithm used as the SISO detection component in the IDD receiver. The MAP detector provide the optimal BER performance, however, the complexity is extreme. In order to solve this problem, a "list" version of SD was developed by Hochwald and ten Brink without significant loss of performance [7]. The complexity of the MIMO detection is further brought down by introducing soft parallel interference cancellation (SPIC) in [25], [25] at the cost of a performance loss. In this section, we adapt the proposed P-DFCC detection algorithm into the IDD structure.

In the coded systems, the model in (1) is used repeatedly to describe transmit streams of data bits which are separated into blocks. For a given block, the symbol vector $\boldsymbol{s}$ is obtained by mapping $\boldsymbol{b} = [b_1, ..., b_j, ..., b_{K\cdot J}]$ coded bits. The quantity J is the number of bits per constellation symbol. For coded transmissions, the vector $\boldsymbol{b}$ is designated as the output of a forward error-correction code of rate $R < 1$ that introduces redundancy. The transmission rate is then RKJ bits perreceived vector. In the IDD processing, the detector makes decisions by using the knowledge of correlations across time instants $[i], i = 0, 1, \ldots, I$ provided by the channel decoder, and the channel decoder needs to decode the bit information by using the likelihood information on all blocks obtained from the soft output detector.

For each user, a block of received signals $\boldsymbol{r}[i]$ is used to compute the *a posteriori* probability in the form of log-likelihood-ratios (LLRs), with P-DF, the MIMO input-output relation (1) has been transformed in to K parallel data streams. By assuming these K streams are statistically independent, we may approximate the *intrinsic a posteriori* LLRs as [30]

$$\Lambda_1^p[b_{j,k}[i]] \approx \log \frac{P[b_{j,k}[i]=+1|u_k[i]]}{P[b_{j,k}[i]=-1|u_k[i]]} \qquad \forall j,k, \tag{35}$$

where the equation can be solved by using Bayes' theorem and we leave the details to the references [7], [25]. We denote the *intrinsic* information provided by the decoder as $\Lambda_2^p[b_{j,k}[i]]$ and the bit probability is obtained as

$$P[b_{j,k}[i]] = \log \frac{P[b_{j,k}[i]=+1]}{P[b_{j,k}[i]=-1]} \qquad \forall j,k. \tag{36}$$

From [25], the bit-wise probability is obtained by

$$\begin{aligned} P[b_{j,k}[i] = \bar{b}_j] &= \frac{\exp\left(\bar{b}_j \Lambda_2^p[b_{j,k}[i]]\right)}{1+\exp\left(\bar{b}_j \Lambda_2^p[b_{j,k}[i]]\right)}, \\ &= \frac{1}{2}\left[1+\bar{b}_j \tanh\left(\frac{1}{2}\Lambda_2^p[b_{j,k}[i]]\right)\right]. \end{aligned} \tag{37}$$

where $\bar{b}_j = \{+1,-1\}$. Let us simplify the notation $P\big[s_k[i]\big] := P\big[s_k[i] = c_q\big]$ where c_q is an element chosen from the constellation $\mathcal{X} = \{c_1, \ldots, c_q, \ldots, c_A\}$. The symbol probability $P[s_k[i]]$ is obtained from the corresponding bit-wise probability, and assuming the bits are statistically independent, we have

$$\begin{aligned} P\big[s_k[i]\big] &= \prod_{j=1}^{J} P\big[b_{j,k}[i] = \bar{b}_j\big], \\ &= \frac{1}{2^J}\prod_{j=1}^{J}\left[1+\bar{b}_j \tanh\left(\frac{1}{2}\Lambda_2^p[b_{j,k}[i]]\right)\right]. \end{aligned} \tag{38}$$

From (37) and (38) we can conclude that $\sum_{|\mathcal{X}|} P\big[s_k[i]\big] = 1$. By organizing the probabilities obtained by (38) in decreasing order of values, a list of tentative decisions of $s_k[i]$ is obtained in each stream as given by

$$\mathcal{L}_k^{\text{IDD}}[i] \triangleq \{c_1, c_2, \ldots, c_\tau\}_k, \tag{39}$$

where $1 \leq \tau \leq |\mathcal{X}|$ and

$$Pr[c_1] \geq Pr[c_2] \geq \ldots, Pr[c_\tau], \tag{40}$$

and

$$Pr[c_q] \triangleq P\big[s_k[i] = c_q \big| u_k\big]. \tag{41}$$

For the IDD coded structure we replace (25) with (39), thanks to the error correction, for a moderate SNR, the size of $\mathcal{L}_k^{\text{IDD}}$ is significantly smaller than that value in (39). The pseudo-code for implementing the proposed P-DFCC with IDD structure is detailed in Algorithm. 1.

V. Simulations

In this section, several numerical examples are given to demonstrate the overall system performance of using our algorithms. In the following simulations, unless otherwise stated, we consider that the proposed algorithms and all their counterparts operate with a channel with independent and identically-distributed (i.i.d) block fading model. The channel model is of Rayleigh random fading and the coefficients are taken from complex Gaussian random variables with zero mean and unit variance. Other parameters are also assumed: QPSK is used; The transmitted vectors $\boldsymbol{s}[i]$ are grouped into frames consisting of 500 vectors where the first $\boldsymbol{s}[1], \ldots, \boldsymbol{s}[10]$ vectors are training vectors. In each frame, the channel between a transmit and receive antenna pair is fixed and a single path is assumed.

Fig. 5 demonstrates the MSE for the symbol estimation across all 8 user streams in terms of RLS iterations with 8 receiver antennas configuration. $E_b/N_0 = 20$ dB, and the normalized Doppler frequency f_dT equals to 10^{-3}. The proposed P-DFCC scheme shows the improvement in terms of MSE. From the figure we can see that the P-DFCC has the ability to track the fading channel with $f_dT = 10^{-3}$.

Algorithm 1 Algorithm soft-output log-Max-DFCC Detection

Require: $\boldsymbol{r} \in \mathcal{C}^{N_R \times 1}$, $\boldsymbol{H} \in \mathcal{C}^{N_R \times K}$, constellation set $\mathcal{A}$, σ_v^2, $n \leftarrow 0$, $\boldsymbol{L}(b_{k,j}^{p1})$, TI.

```
Find the set of symbol vectors X^1_{k,j} ∩ L^IDD_k and X^0_{k,j} ∩ L^IDD_k
for lo ← TI {Turbo Iteration} do
  for k ← 1, ..., K do
    for j ← 1, ..., J do
      for s ∈ X^1_{k,j} ∩ L^IDD_k do
        b ← demap(s), b_{k,j} ← 0
        P(x) ← 1/2 (2b_{[k,j]} − 1) L(b^{(p1)}_{k,j})   {Symbol probability}
        λ^1_n ← ln P(x) − ||r − Hs||^2 / σ^2_v
      end for
      for s ∈ X^0_{k,j} ∩ L^IDD_k do
        b ← demap(s), b_{k,j} ← 0
        P(x) ← 1/2 (2b_{[k,j]} − 1) L(b^{(p1)}_{k,j})   {Symbol probability}
        λ^0_n ← ln P(x) − ||r − Hs||^2 / σ^2_v
      end for
      L(b^{(e1)}_{k,j}) ← max{λ^1_n, n = 1, ..., |X^1_{k,j}|} − max{λ^0_n, n = 1, ..., |X^0_{k,j}|}
    end for   {Antenna stream}
  end for   {Bit Label}
  Deinterleave extrinsic L(b^{(e1)}_{k,j})
  Perform BCJR decoding and compute L(b^{(e2)}_{k,j})
  Interleaving extrinsic L(b^{(e2)}_{k,j}) and feedback to detector.
end for   {Turbo Iteration}
Decision of systematic bit is obtained via sign{L(m_k)}
```

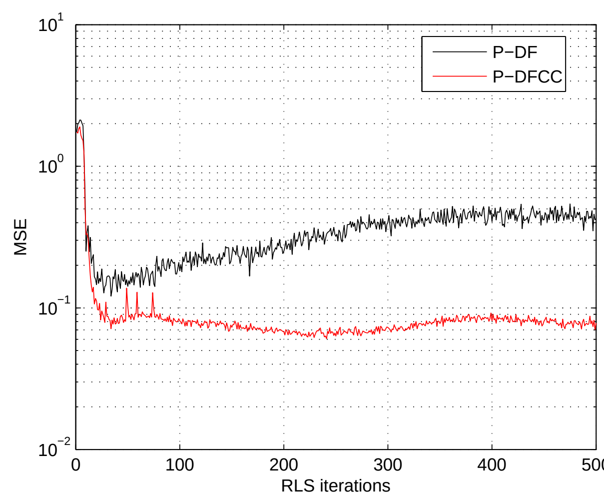


Fig. 5. MSE of the estimated symbols in terms of RLS iterations, with 8 users. After 10 training vectors transmitted, the decision-directed mode is switched on.

The performance is also measured in terms of bit error rate (BER), obtained by 10^4 Monte Carlo runs. In our simulations, the SNR per transmitted information bit is defined as

$$\frac{E_b}{N_0}\bigg|_{\text{dB}} = 10\log_{10}\left(\frac{N_R}{R\log_2 C}\cdot\frac{\sigma_s^2}{\sigma_v^2}\right). \quad (42)$$

The total transmitted power $E_s = K\cdot\sigma_s^2$ which is evenly distributed across K active users. The N_R receive antennas collect a total power of $N_R E_s$ which carries $K\log_2 C$ coded bits or $RK\log_2 C$ information bits. $R\leq 1$ is the channel coding rate which introduces information redundancy. The coding rate $R=1$ is assumed for the simulations without channel coding.

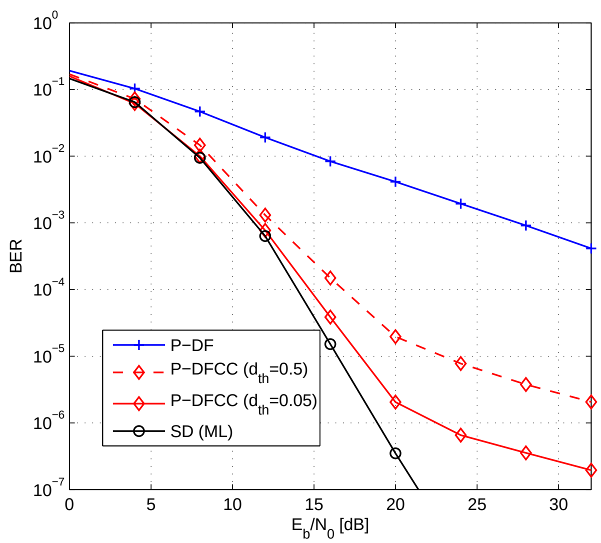


Fig. 6. BER vs. E_b/N_0, the proposed P-DFCC detection achieves a near optimal performance in a 4 user system configuration. The constellation threshold d_{th} introduces a trade off between the performance and the complexity.

Fig.6 shows the BER against E_b/N_0. The channel is estimated by LS algorithms, the P-DF-RLS detector ($\lambda = 0.998$) proposed in [18] exhibits about 7dB performance loss when the target BER equals 10^{-3} compared with the performance of SD. As for the SD, with a sufficiently large sphere radius selected, the SD can always produce an ML solution. With the constellation constraint threshold $d_{\text{th}} = 0.05$, the proposed P-DFCC-RLS ($\lambda = 0.998$) algorithm shows a near-optimal BER performance at the target BER equal to 10^{-3}. From Fig.6, we can verify that the optimal ML detector (or sphere decoder) is able to attain full diversity.

It is also worth to mention that, a MMSE-based successive decision feedback (S-DF) detector is able to obtain a diversity order of $N_R - K + k$, and the BER performance is bounded by the user with the worst performance. The diversity order of the traditional P-DF algorithm is usually lower than the channel power sorted S-DF [9], this is due to the problem of error propagation. In P-DF, an erroneous symbol would propagate through all other user's data stream. However, if all the detected symbols are highly reliable, P-DF may provide a higher diversity order than S-DF, this can be verified by assuming a perfect cancellation scenario, where P-DF achieves full receive diversity order while S-DF has only $N_R - K + k$.

By introducing a reliability checking procedure, the diversity order of the proposed P-DFCC can be adjusted. The control of the diversity order is twofold: (1) the selection of d_{th}. From Fig.6 we can see that the diversity order is directly related to the threshold d_{th}. Namely, decrease the value of d_{th} could change the shape of the constellation constraint and increase the diversity order. (2) The received SNR region. In the low SNR region, the scheme is likely to list a higher number of candidates than those generated in a high SNR region and the performance approaches the ML detector. On the other hand, for the high SNR region, all the symbol estimates are considered reliable and the diversity order tends to be the same of a conventional P-DF. Therefore, for the proposed P-DFCC scheme the gain is higher for a small to medium region of SNR.

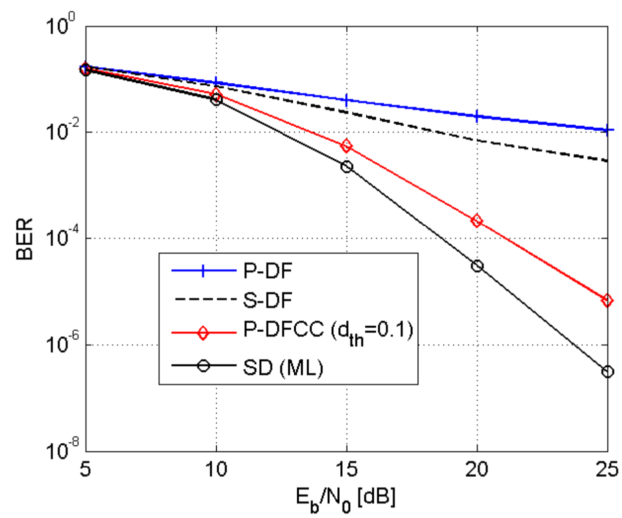


Fig. 7. BER vs. E_b/N_0, the proposed P-DFCC detection achieves a near optimal performance in a 4-user system configuration with 16-QAM symbols

Another simulation is carried out with 16-QAM symbols. The SNR against BER curves are plotted in Fig.7. The threshold is set to $d_{\text{th}} = 0.1$. With QPSK modulation the proposed P-DFCC detection algorithm is able to achieve a better performance compared with traditional P-DF as well as S-DF algorithms.

Fig.8 presents the comparison of BER performance for various normalized Doppler frequency $f_d T$ (in the time-varying channels) when $E_b/N_0 = 14$ dB. In this simulation, each channel between a transmit and receive antenna pair varies accodrding to the Jakes' model [20]. LS channel estimation is applied to the unknown channel. The length of the training sequence is $I = 20$. The simulation results show that the proposed P-DFCC significantly improves the traditional P-DF detector and approaches the SD performance in time-varying channels.

In Fig. 9, the complexity is given by counting the required complex multiplications as the number of users increases. P-DFCC has a complexity slightly above the P-DF while it achieves a significant performance improvement. The threshold d_{th} is introduced to reduce the complexity and improve the performance. We use fixed complexity sphere decoders (FSD)

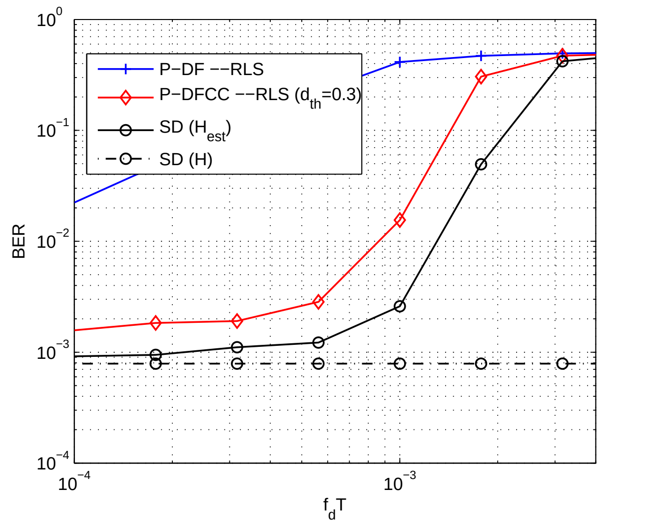


Fig. 8. Comparison of BER performance for various normalized Doppler frequency f_dT when $K = 4$ and E_b/N_0 = 14dB.

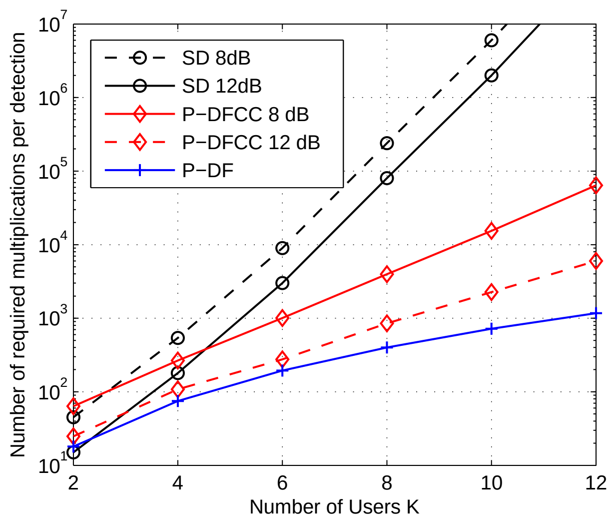


Fig. 9. Complexity in terms of arithmetic operations against the transmit antennas, the P-DFCC has a comparable complexity with P-DF algorithm. $d_{\text{th}} = 0.3$.

[29] to compare the complexity. It should be noted that FSD is one of the lowest complexity SD algorithms that are known.

The curves in Fig.10 are given for convolutionally coded BER performance on a Rayleigh block fading channel. The proposed P-DFCC with $d_{th} = 0.3$ improves the conventional P-DF detection performance about 3 dB at the target coded BER equals to 10^{-4}. The P-DFCC detector approaches the optimal MAP detection performance with only 1.5 dB performance loss when coded BER = 10^{-4}.

VI. Conclusion

In this paper, we have derived an adaptive decision feedback based detector for MIMO transmission systems with varying channels. In this context, we have presented a novel way to improve the BER performance by using the parallel decision

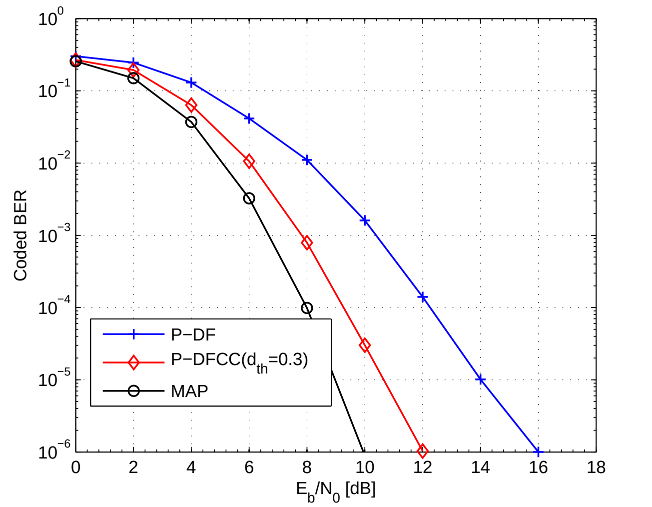


Fig. 10. Coded BER curves of QPSK over 4×4 MIMO channels; block size 1000 message bits, code rate $R = 1/2$, memory 2 convolutional code.

feedback with constellation constraints approach, a threshold is introduced to reduce the complexity and improve the performance. This approach has the ability to reduce the MSE of traditional parallel decision feedback detection, effectively improve the BER performance of parallel interference cancellation schemes and obtain a close to optimal performance with a low additional detection complexity.

References

[1] S. Verdú, *Multiuser Detection*. Cambridge, U.K.: Cambridge Univ. Press, 1998.

[2] N. Jindal, A. Goldsmith, “Dirty-paper coding versus TDMA for MIMO Broadcast channels,” *IEEE Trans. on Info. Theory*, vol. 51, no. 5, pp. 1783-1794, May 2005.

[3] D. J. Love, R. W. Heath, “Limited feedback unitary precoding for spatial multiplexing systems,” *IEEE Trans. on Info. Theory*, vol.51, no.8, pp. 2967-2976, Aug. 2005.

[4] R. C. de Lamare and R. Sampaio-Neto, ”Minimum Mean Squared Error Iterative Successive Parallel Arbitrated Decision Feedback Detectors for DS-CDMA Systems,”*IEEE Trans. on Commun.*, May, 2008, pp. 778 - 789.

[5] Y. Cai and R. C. de Lamare, “Adaptive Space-Time Decision Feedback Detectors with Multiple Feedback Cancellation”, IEEE Transactions on Vehicular Technology, vol. 58, no. 8, pp. 4129-4140, Oct. 2009.

[6] E. Viterbo, J. Boutros, ”A universal lattice code decoder for fading channels”, *IEEE Trans. on Info. Theory*, vol.45, pp. 1639-1642, July. 1999.

[7] B. Hochwald and S. T. Brink, Achieving near-capacity on a mutliple-antenna channel,” *IEEE Trans. Commun.*, vol. 51, pp. 389-399, Mar. 2003.

[8] H. Vikalo, B. Hassibi, ”The Expected Complexity of Sphere Decoding, Part I: Theory, Part II: Applications,” *IEEE Trans. on Signal Proc.,2003*

[9] R. Fa, R. C. de Lamare, “Multi-Branch Successive Interference Cancellation for MIMO Spatial Multiplexing Systems”, *IET Communications*, vol. 5, no. 4, pp. 484 - 494, March 2011.

[10] P. Li, R. C. de Lamare and R. Fa, “Multiple Feedback Successive Interference Cancellation Detection for Multiuser MIMO Systems,” *IEEE Transactions on Wireless Communications*, vol. 10, no. 8, pp. 2434 - 2439, August 2011.

[11] R. C. de Lamare and R. Sampaio-Neto, “Adaptive reduced-rank equalization algorithms based on alternating optimization design techniques for MIMO systems”, *IEEE Transactions on Vehicular Technology*, vol. 60, no. 6, 2482-2494, 2011.

[12] P. Li and R. C. de Lamare, ”Adaptive Decision-Feedback Detection With Constellation Constraints for MIMO Systems”, *IEEE Transactions on Vehicular Technology*, vol. 61, no. 2, 853-859, 2012.

[13] G. Ginis, J. M. Cioffi, ”On the relation between V-BLAST and the GDFE”, *IEEE Commun. Lett.*, 2001, 5, (9), pp. 364366

[14] M. Chiani, ”Introducing erasures in decision-feedback equalization to reduce error propagation,” *IEEE Trans. Commun.*, vol. 45, no. 7, pp. 757-760, July 1997.

[15] R. C. de Lamare, R. Sampaio-Neto, ”Adaptive MBER decision feedback multiuser receivers in frequency selective fading channels”, *IEEE Commun. Lett.*, 2003, 7, (2), pp. 7375

[16] G. D. Golden, C. J. Foschini, R. A. Valenzuela, and P. W. Wolniansky, ”Detection algorithm and initial laboratory result using V-BLAST space-time communication architecture.” *Electron. Lett.*, 35(1), 1415. 1999

[17] Q. Sun and D. C. Cox, ”Training-based channel estimation for continuous flat fading BLAST,” in *proc. IEEE ICC*, Helsinki, Finland, Jun. 2002, pp. 325-329.

[18] J. H. Choi, H. Y. Yu, Y. H. Lee, ”Adaptive MIMO decision feedback equalization for receivers with time-varying channels”, *IEEE Trans. Signal Proc.*, 2005, 53, (11), pp. 4295-4303.

[19] S. Haykin, Adaptive Filter Theory, 4th edn, PHIPE. 2001.

[20] W. C. Jakes, *Microwave Mobile Communications*. New York: Wiley, 1974.

[21] R. C. de Lamare and R. Sampaio-Neto, “Adaptive Reduced-Rank Processing Based on Joint and Iterative Interpolation, Decimation, and Filtering,” *IEEE Transactions on Signal Processing*, vol. 57, no. 7, July 2009, pp. 2503 - 2514.

[22] T. Wang, R. C. de Lamare, P. D. Mitchell, “Low-Complexity Set-Membership Channel Estimation for Cooperative Wireless Sensor Networks,” IEEE Transactions on Vehicular Technology, vol. 60, no. 6, pp. 2594-2607, July 2011.

[23] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, “Optimal decoding of linear codes for minimizing symbol error rate,” *IEEE Transactions on Information Theory,* 20:284287, March 1974.

[24] J. Hagenauer, P. Hoher, “A viterbi algorithm with soft-decision outputs and its applications,” *IEEE Globecom Conference*, pages 16801686, Dallas, USA, January 1989.

[25] X. Wang and H.V. Poor, “Iterative (Turbo) soft interference cancellation and decoding for coded CDMA,” *IEEE Transactions on Communications*, vol. 47, no. 7, pp. 1046-1061, 1999.

[26] A. G. D. Uchoa; C. Healy; R. C. de Lamare, R. D. Souza, “Design of LDPC Codes Based on Progressive Edge Growth Techniques for Block Fading Channels,” IEEE Communications Letters, vol. 15, no. 11, pp. 1221-1223, November 2011.

[27] J. Liu and R. C. de Lamare,“Low-Latency Reweighted Belief Propagation Decoding for LDPC Codes”, *IEEE Communications Letters*, vol. 16 , no. 10, pp. 1660-1663, 2012.

[28] H. Vikalo, B. Hassibi, T. Kailath, “Iterative decoding for MIMO channels via modified sphere decoding,” *IEEE Transactions on Wireless Communications,* vol. 3 no. 6 pp. 2299-2311, Nov. 2004.

[29] L. G. Barbero, J. S. Thompson, “Fixing the Complexity of the Sphere Decoder for MIMO Detection,” *IEEE Transactions on Wireless Communications,* vol. 7, no. 6, pp. 2131-2142, June 2008.

[30] C. Studer, “Iterative MIMO decoding: algorithms and VLSI implementation aspects,” *PhD thesis* Hartung-Gorre Publisher, 2009.