Robust Adaptive Beamforming Algorithms using the Constrained Constant Modulus Criterion

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Abstract

We present a robust adaptive beamforming algorithm based on the worst-case criterion and the constrained constant modulus approach, which exploits the constant modulus property of the desired signal. Similarly to the existing worst-case beamformer with the minimum variance design, the problem can be reformulated as a second-order cone (SOC) program and solved with interior point methods. An analysis of the optimization problem is carried out and conditions are obtained for enforcing its convexity and for adjusting its parameters. Furthermore, low-complexity robust adaptive beamforming algorithms based on the modified conjugate gradient (MCG) and an alternating optimization strategy are proposed. The proposed low-complexity algorithms can compute the existing worst-case constrained minimum variance (WC-CMV) and the proposed worst-case constrained constant modulus (WC-CCM) designs with a quadratic cost in the number of parameters. Simulations show that the proposed WC-CCM algorithm performs better than existing robust beamforming algorithms. Moreover, the numerical results also show that the performances of the proposed low-complexity algorithms are equivalent or better than that of existing robust algorithms, whereas the complexity is more than an order of magnitude lower.

I. Introduction

Beamforming has many applications in wireless communications, radar, sonar, medical imaging, radio astronomy and other areas. One of the most fundamental problems with adaptive beamforming algorithms is the occurrence of mismatches between the presumed and actual signal steering vector [1]. Practical circumstances like local scattering, imperfectly calibrated arrays and imprecisely known wavefield propagation conditions are the typical sources of these mismatches and can lead to a performance degradation of the conventional beamforming algorithms [2]. In the last decades a number of robust approaches have been reported that address this problem [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13]. These robust methods can be broadly categorized into two main groups: techniques based on previous mismatch assumptions [3], [4], [5], [7], [8], [9], [10] and methods that estimate the mismatch or equivalently the actual steering vector [6], [8], [11], [13]. A number of robust designs can be often cast as optimization problems which end up in the so-called second-order cone (SOC) program, which can

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be easily solved with interior point methods. While those designs for beamformers are based on the minimum variance criterion, it is possible to design them using a constant modulus criterion [14], [15], which can exploit prior knowledge about the desired signal and provide a better performance.

The problem we are interested in solving in this paper is the design of cost-effective adaptive robust beamforming algorithms. In particular, we focus on the design of beamforming algorithms which can exploit prior knowledge about the constant modulus property of a desired signal and that can be implemented in an efficient way with an appropriate modification of adaptive signal processing algorithms. In the first part of this work the worst-case optimization-based beamforming algorithm with the constant modulus criterion (CCM) is developed. In order to solve the robust constrained constant modulus we apply an iterative reformulation of the constant modulus cost function introduced in [16], which is a local second-order approximation. Its derivation is based on the assumption that previous computed weight vectors are close to the solution, which is enforced by the additional constraint.

We reformulate the problem as a SOC program in a similar fashion to the approach adopted in [5] and devise an adaptive algorithm to adjust the parameters of the beamformer in time-varying scenarios and that can exploit prior knowledge about the constant modulus of the desired signal. An analysis of the optimization problem is conducted and a condition which ensures convexity is established. In addition, a study about the choice of the parameter ϵ associated with the WC-CCM criterion is carried out. We investigate the performance of the proposed WC-CCM algorithm via simulations. The results show that the proposed WC-CCM algorithm outperforms previously reported methods.

In the second part of this paper low-complexity robust adaptive beamforming algorithms are developed. While the robust constraint is similar to that which is known from the worst-case criterion, the algorithms are based on the modified conjugate gradient (MCG) [17], [18]and an alternating optimization strategy that performs joint adjustment of the constraint and the parameters of the beamformer. The joint optimization strategy exploits previous computations and therefore the computational complexity is reduced by more than an order of magnitude from more than cubic $\mathcal{O}(M^{3.5})$ to quadratic $\mathcal{O}(M^2)$ with the number of sensor elements M as compared to the worst-case optimization-based approach. A low-complexity approach is also developed for the minimum variance design which is termed the robust constrained minimum variance modified conjugate gradient (Robust-CMV-MCG) algorithm. The proposed low-complexity algorithm for the constrained constant modulus design is termed robust constrained constant modulus modified conjugate gradient (Robust-CCM-MCG). While the Robust-CMV-MCG algorithm has a performance equivalent to the worst-case optimization based approach, the Robust-CCM-MCG algorithm which exploits the constant modulus property of the desired signal performs better than existing algorithms. We conduct a simulation study to investigate the performance of the proposed low-complexity algorithms in a number of situations of practical relevance.

This paper is organized as follows. The system model is described in Section II. Section III reviews existing robust adaptive beamforming algorithms. The proposed WC-CCM design is formulated in Section IV. In Section V the SOC implementation and the adaptive algorithm are described. An analysis of the optimization problem is given in Section VI, where a condition is found which ensures convexity and relationships between the parameter ϵ and the signal-to-noise ratio (SNR) are established. In Section VII the corresponding low-complexity solutions are presented. The simulation results are presented and discussed in Section VIII. Section IX gives the conclusion of this work.

II. System Model

Let us consider a linear array of M sensors that receives signals from D narrowband sources. The vector of array observations $\boldsymbol{x}(i) \in \mathbb{C}^{M\times 1}$ at time instant i can be modeled as

$$\boldsymbol{x}(i) = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s}(i) + \boldsymbol{n}(i), \tag{1}$$

where $\boldsymbol{\theta} = [\theta_1, ..., \theta_D]^T \in \mathbb{R}^{D\times 1}$ is the vector with the directions of arrival (DoA) and $(.)^T$ stands for transpose, $\boldsymbol{A}(\theta) = [\boldsymbol{a}_1(\theta_1), ..., \boldsymbol{a}_D(\theta_D)] \in \mathbb{C}^{M\times D}$ is the matrix containing the array steering vectors $\boldsymbol{a}_m(\theta_m) \in \mathbb{C}^{M\times 1}$, for $m = 1, ..., D$. In the following θ_1 is the direction related to the desired user which is roughly known by the system. The vector $\boldsymbol{s}(i) \in \mathbb{C}^{D\times 1}$ represents the uncorrelated sources. The vector $\boldsymbol{n}(i) \in \mathbb{C}^{M\times 1}$ is the sensor noise, which is assumed as zero-mean complex Gaussian. The true array steering vector is assumed as $\boldsymbol{a}_1(\theta_1) = \boldsymbol{a}(\theta_1) + \boldsymbol{e}$, where $\boldsymbol{e}$ is the mismatch vector and $\boldsymbol{a}(\theta_1)$ is the presumed array steering vector which is known by the system. In what follows, we will use $\boldsymbol{a} = \boldsymbol{a}(\theta_1)$. The output of the beamformer is defined as

$$y(i) = \boldsymbol{w}^H\boldsymbol{x}(i) \ , \tag{2}$$

where $\boldsymbol{w} \in \mathbb{C}^{M\times 1}$ is the complex vector of beamforming weights. The notation $(.)^H$ stands for Hermitian transpose. The signal-to-interference-plus-noise ratio (SINR) is defined as

$$\text{SINR} = \frac{\boldsymbol{w}^H\boldsymbol{R}_s\boldsymbol{w}}{\boldsymbol{w}^H\boldsymbol{R}_{i+n}\boldsymbol{w}}, \tag{3}$$

where $\boldsymbol{R}_s$ is the signal covariance matrix corresponding to the desired user and $\boldsymbol{R}_{i+n}$ is the interference-plus-noise covariance matrix.

III. Robust Adaptive Beamforming: A Review

We review a few notable approaches to the design of robust adaptive beamforming algorithms. The most common robust approach is the so-called loaded sample matrix inversion (loaded-SMI) beamformer [3], which includes an additional diagonal loading to the signal covariance matrix. The main problem is how to obtain the optimal diagonal loading factor. Typically it is chosen as 10 σ_n^2, where σ_n^2 is the noise power [3]. Another robust approach is given by the eigen-based beamformer [19]. Here the presumed array steering vector is replaced by its projection onto the signal-plus-interferer subspace. The approach implies that the noise subspace can be identified, which leads to a limitation in high SNR. A similar method is given by the reduced-rank beamforming approach [20], which avoids an eigen-decomposition and exploits the low rank of the signal-plus-interferer subspace. A different robust beamforming strategy is considered by techniques based on diagonal loading [5], [7], [9], [10], which are more advanced compared to [3]. In these techniques, the algorithms determine a diagonal loading parameter which aims to compensate for the mismatch by adding a suitable factor to the diagonal of the covariance matrix of the input signal.

One of these methods is given by the popular worst-case performance optimization-based beamformer [5] which is based on the constraint that the absolute value of the array response is always greater than or equal to a constant for all vectors that belong to a predefined set of vectors in the neighborhood of the presumed vector. In [5] the set of vectors is a sphere $\mathcal{A} = \{\boldsymbol{a} + \boldsymbol{e}, \|\boldsymbol{e}\|_2 \leq \epsilon\}$, where the norm of $\boldsymbol{e}$ is upper-bounded by ϵ. The corresponding optimization problem is given by

$$\begin{aligned} &\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{R}_{xx} \boldsymbol{w} \\ &\text{s. t. } \left|\boldsymbol{w}^H (\boldsymbol{a} + \boldsymbol{e})\right| \geq \delta \quad \text{for all} \, (\boldsymbol{a} + \boldsymbol{e}) \in \mathcal{A}(\epsilon) \end{aligned} \tag{4}$$

where $\boldsymbol{R}_{xx} = \mathrm{E}\left\{\boldsymbol{x}\boldsymbol{x}^H\right\}$ is the covariance matrix of the input signal. The problem can be transformed into the following convex SOC problem:

$$\begin{aligned} \min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{R}_{xx} \boldsymbol{w} \quad \text{s. t. } \mathrm{Re}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} - \delta \geq \epsilon \|\boldsymbol{w}\|_2 \\ \mathrm{Im}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} = 0 \end{aligned} \tag{5}$$

where the operator $\mathrm{Re}\{\cdot\}$ retains the real part of the argument and the operator$\mathrm{Im}\{\cdot\}$ retains the imaginary part of the argument. It has been shown that this kind of beamforming technique is related to the class of diagonal loading. In [9] the set of vectors in the neighborhood can be ellipsoidal as well.

Another notable idea is the probability-constrained approach [21]. Here the constraint satisfies operational conditions that are more likely to occur.

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{R}_{xx} \boldsymbol{w} \quad \text{s. t. } \Pr\left\{\left|\boldsymbol{w}^H (\boldsymbol{a} + \boldsymbol{e})\right| \geq \delta\right\} \geq p, \tag{6}$$

where $\Pr$ denotes the probability operator and p is the desired probability threshold. Here different assumptions on the statistical characteristics of the mismatch-vector $\boldsymbol{e}$ lead to different problem formulations. The solutions for the Gaussian probability density function (pdf) case and the general unknown pdf case have been developed in [21].

Another class of robust methods includes those that estimate the mismatch which have been reported in [6], [8], [11], [13]. The main idea behind these approaches is to compute an estimate of the mismatch and then subsequently use this information to obtain an estimate of the actual steering vector. Recently developed approaches estimate the mismatch vector based on sequential quadratic programming [11] or based on semidefinite relaxation [13].

All these beamformers are based on the minimum variance criterion. We assume that a number of these beamformers can benefit from using the CCM design criterion instead of the minimum variance one. Prior work with the CCM design criterion includes the design of adaptive beamformers [22] and receivers for spread spectrum systems [14], [15], [23]. The results in the literature indicate that the CCM design has a superior performance to those designs based on the minimum variance. In the following, we develop a worst-case performance optimization-based beamforming algorithm with the CCM design criterion. In addition, we propose low-complexity robust beamforming algorithms.

IV. Proposed Worst-Case Optimization based Constant Modulus Design

The proposed robust beamformer is based on the worst-case approach. In case of the minimum variance design it can be derived from the following optimization problem

$$\begin{aligned} \min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{R}_{xx} \boldsymbol{w} \quad \text{s. t. } \operatorname{Re}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} - \delta \geq \epsilon \left\|\boldsymbol{w}\right\|_2 \\ \operatorname{Im}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} = 0 \ , \end{aligned} \tag{7}$$

where ϵ is the level of steering vector mismatch, which is assumed as known a priori. The proposed beamformer uses the constant modulus criterion, which exploits the constant modulus property of the desired signal instead of the minimum variance design criterion. To this end, we will assume that the signals processed have a constant modulus property during the observation time and the proposed algorithms are designed to exploit this property. The constant modulus cost function is defined by

$$\begin{aligned} J =& \mathrm{E}\left\{\left(\left|y(i)\right|^2 - \gamma\right)^2\right\} \\ =& \mathrm{E}\left\{\left(\boldsymbol{w}^H(i)\boldsymbol{x}(i)\boldsymbol{x}^H(i)\boldsymbol{w}(i) - \gamma\right)^2\right\}, \end{aligned} \tag{8}$$

where $\gamma \geq 0$ which is a parameter related to and should be chosen according to the energy of the signal. If the parameter gamma is different then we need to choose the parameter delta of the constraint to satisfy (30). . By considering the approximation strategy in [16], that is, replacing in (8) $\boldsymbol{w}^H(i)\boldsymbol{x}(i)$ by $\boldsymbol{w}^H(i-1)\boldsymbol{x}(i)$, we obtain a modified cost function which is a second-order local approximation

$$\tilde{J} = \mathrm{E}\left\{(\boldsymbol{w}^H(i)\boldsymbol{x}(i)\boldsymbol{x}(i)^H\boldsymbol{w}(i-1) - \gamma)(\boldsymbol{w}^H(i-1)\boldsymbol{x}(i)\boldsymbol{x}(i)^H\boldsymbol{w}(i) - \gamma)\right\} \tag{9}$$

This is a special case of the established general constant modulus reformulation suggested in [16] whose validity has been confirmed via computer experiments. Furthermore, it should also be mentioned that the underlying assumption that the previous weight vector is close to the solution is additionally enforced by the direction constraint. Besides this strategy, there are similar second-order approximation strategies in the literature that are based on Taylor series expansion [24] approaches. By discarding the constant term, the objective function is given by

$$\hat{J} = \boldsymbol{w}^H \mathrm{E}\left\{\left|y(i)\right|^2 \boldsymbol{x}(i)\boldsymbol{x}^H(i)\right\}\boldsymbol{w} - 2\gamma \operatorname{Re}\left\{\boldsymbol{w}^H \mathrm{E}\left\{y^*(i)\boldsymbol{x}(i)\right\}\right\}, \tag{10}$$

where $y(i) = \boldsymbol{w}^H(i-1)\boldsymbol{x}(i)$ denotes the output which assumes small variations of the beamformer that allows the approximation $\boldsymbol{w}^H(i)\boldsymbol{x}(i) \approx \boldsymbol{w}^H(i-1)\boldsymbol{x}(i)$. In combination with the worst-case constraint, the proposed WC-CCM design can be cast as the following optimization problem

$$\begin{aligned} &\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{R}_a \boldsymbol{w} - 2\gamma \ \operatorname{Re}\left\{\boldsymbol{d}^H \boldsymbol{w}\right\} \\ &\text{s. t. } \boldsymbol{w}^H \boldsymbol{a} - \delta \geq \epsilon \left\|\boldsymbol{w}\right\|_2 \ \text{ and } \ \operatorname{Im}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} = 0 \ , \end{aligned} \tag{11}$$

where $\boldsymbol{R}_a = \mathrm{E}\left\{\left|y(i)\right|^2 \boldsymbol{x}(i)\boldsymbol{x}^H(i)\right\}$ and $\boldsymbol{d} = \mathrm{E}\left\{y^*(i)\boldsymbol{x}(i)\right\}$, are estimated from the previous snapshots which will be explained in the next section.

V. Proposed SOC Implementation and Adaptive Algorithm

In the first part of this section we show how to implement the SOC program and in the second part we devise an adaptive algorithm to adjust the weights of the beamformer according to the WC-CCM design.

A. SOC Implementation

In this subsection, inspired by the approach in [5], we present a SOC implementation of the proposed WC-CCM design. Introducing a scalar variable τ, an equivalent problem to (10) can be formulated

$$\begin{aligned} \min_{\tau,\boldsymbol{w}} \quad \tau \quad \text{s. t.} \quad & \boldsymbol{w}^H \boldsymbol{R}_{\text{ac}}^H \boldsymbol{R}_{\text{ac}} \boldsymbol{w} - 2\gamma \ \operatorname{Re}\left\{\boldsymbol{d}^H \boldsymbol{w}\right\} \leq \tau \\ & \operatorname{Re}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} - \delta \geq \epsilon \|\boldsymbol{w}\|_2 \\ & \operatorname{Im}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} = 0, \end{aligned} \tag{12}$$

where $\boldsymbol{R}_{\text{ac}}^H \boldsymbol{R}_{\text{ac}} = \boldsymbol{R}_a$ is the Cholesky factorization. Introducing the real-valued matrix and the real-valued vectors given by $\breve{\boldsymbol{R}}_{ac} = \begin{bmatrix} \operatorname{Re}\{\boldsymbol{R}_{ac}\} & -\operatorname{Im}\{\boldsymbol{R}_{ac}\} \\ \operatorname{Im}\{\boldsymbol{R}_{ac}\} & \operatorname{Re}\{\boldsymbol{R}_{ac}\} \end{bmatrix} \in \mathbb{R}^{(2M)\times(2M)}$, $\breve{\boldsymbol{d}} = [\operatorname{Re}\{\boldsymbol{d}\}^T, \operatorname{Im}\{\boldsymbol{d}\}^T]^T \in \mathbb{R}^{(2M)\times 1}$, $\breve{\boldsymbol{a}} = [\operatorname{Re}\{\boldsymbol{a}\}^T, \operatorname{Im}\{\boldsymbol{a}\}^T]^T \in \mathbb{R}^{(2M)\times 1}$, $\bar{\boldsymbol{a}} = [\operatorname{Im}\{\boldsymbol{a}\}^T, -\operatorname{Re}\{\boldsymbol{a}\}^T]^T \in \mathbb{R}^{(2M)\times 1}$, $\breve{\boldsymbol{w}} = [\operatorname{Re}\{\boldsymbol{w}\}^T, \operatorname{Im}\{\boldsymbol{w}\}^T]^T \in \mathbb{R}^{(2M)\times 1}$. The problem can be rewritten as

$$\begin{aligned} \min_{\tau,\breve{\boldsymbol{w}}} \quad \tau \quad \text{s. t.} \quad & \breve{\boldsymbol{w}}^T \breve{\boldsymbol{R}}_{ac}^T \breve{\boldsymbol{R}}_{ac} \breve{\boldsymbol{w}} - 2\gamma \ \breve{\boldsymbol{d}}^T \breve{\boldsymbol{w}} \leq \tau \\ & \breve{\boldsymbol{w}}^T \breve{\boldsymbol{a}} - \delta \geq \epsilon \|\breve{\boldsymbol{w}}\|_2 \\ & \breve{\boldsymbol{w}}^T \bar{\boldsymbol{a}} = 0. \end{aligned} \tag{13}$$

The quadratic constraint can be converted into an equivalent SOC constraint because the convexity of the optimization problem can be enforced as will be shown in the next section. This leads to the following optimization problem

$$\begin{aligned} \min_{\tau,\breve{\boldsymbol{w}}} \quad \tau \quad \text{s. t.} \quad & \frac{1}{2} + \gamma \breve{\boldsymbol{d}}^T \breve{\boldsymbol{w}} + \frac{\tau}{2} \geq \left\| \begin{bmatrix} \frac{1}{2} - \gamma \breve{\boldsymbol{d}}^T \breve{\boldsymbol{w}} - \frac{\tau}{2} \\ \breve{\boldsymbol{R}}_{\text{ac}} \breve{\boldsymbol{w}} \end{bmatrix} \right\|_2 \\ & \breve{\boldsymbol{w}}^T \breve{\boldsymbol{a}} - \delta \geq \epsilon \|\breve{\boldsymbol{w}}\|_2 \\ & \breve{\boldsymbol{w}}^T \bar{\boldsymbol{a}} = 0. \end{aligned} \tag{14}$$

Let us define

$$\begin{aligned}
\boldsymbol{p} &= [1, \boldsymbol{0}^T]^T \in \mathbb{R}^{(2M+1)\times 1} \\
\boldsymbol{u} &= [\tau, \breve{\boldsymbol{w}}^T]^T \in \mathbb{R}^{(2M+1)\times 1} \\
\boldsymbol{f} &= [1/2, 1/2, \boldsymbol{0}^T, -\delta, \boldsymbol{0}^T, 0]^T \in \mathbb{R}^{(4M+4)\times 1} \\
\boldsymbol{F}^T &= \begin{bmatrix} \frac{1}{2} & \gamma\breve{\boldsymbol{d}}^T \\ -\frac{1}{2} & -\gamma\breve{\boldsymbol{d}}^T \\ \boldsymbol{0} & \breve{\boldsymbol{R}}_{\text{ac}} \\ 0 & \breve{\boldsymbol{a}} \\ \boldsymbol{0} & \epsilon\boldsymbol{I} \\ 0 & \bar{\boldsymbol{a}} \end{bmatrix} \in \mathbb{R}^{(4M+4)\times(2M+1)},
\end{aligned}$$

where $\boldsymbol{I}$ is a $2M \times 2M$ identity matrix and $\boldsymbol{0}$ is a vector of zeros of compatible dimensions. Finally, the problem can be formulated as the dual form of the SOC problem (equivalent to (8) in [25])

$$\begin{aligned}
\min_{\boldsymbol{u}} \quad & \boldsymbol{p}^T\boldsymbol{u} \ \text{ s. t.} \\
& \boldsymbol{f} + \boldsymbol{F}^T\boldsymbol{u} \ \in \text{SOC}_1^{2M+2} \times \text{SOC}_2^{2M+1} \times \{0\},
\end{aligned} \tag{15}$$

where $\boldsymbol{f}+\boldsymbol{F}^T\boldsymbol{u}$ describes a SOC with a dimension $2M+2$, a SOC with a dimension $2M+1$ and $\{0\}$ is the so-called zero cone that determines the hyperplane due to the equality constraint $\breve{\boldsymbol{w}}^T\bar{\boldsymbol{a}} = 0$. Finally, the weight vector of the beamformer $\boldsymbol{w}$ can be retrieved in the form $\boldsymbol{w} = [\boldsymbol{u}_2, ..., \boldsymbol{u}_{M+1}]^T + j\,[\boldsymbol{u}_{M+2}, ..., \boldsymbol{u}_{2M+1}]^T$. Alternatively (10) can be solved by using [26], which transforms it automatically into an appropriate form.

B. Adaptive Algorithm

It has already been mentioned that the optimization problem corresponding to the WC-CCM algorithm design is solved iteratively. As a result, the underlying optimization problem is to be solved periodically. In this case the proposed adaptive algorithm solves it at each time instant. For the adaptive implementation we use an exponentially decayed data window for the estimation of $\boldsymbol{R}_a$ and $\boldsymbol{d}$ given by

$$\hat{\boldsymbol{R}}_a(i) = \mu\hat{\boldsymbol{R}}_a(i-1) + |y(i)|^2\,\boldsymbol{x}(i)\boldsymbol{x}^H(i) \tag{16}$$

$$\hat{\boldsymbol{d}}(i) = \mu\hat{\boldsymbol{d}}(i-1) + \boldsymbol{x}(i)y^*(i), \tag{17}$$

where $0 < \mu < 1$ is the forgetting factor. Each iteration includes a Cholesky factorization and also a transformation into a real valued problem. Finally, the problem is formulated in the dual form of the SOC problem and solved with SeDuMi [25]. The structure of the adaptive algorithm is summarized in Table I. Compared to the algorithm based on the minimum variance constraint, the proposed algorithm increases the dimension of the first SOC from $2M+1$ to $2M+2$.

VI. Analysis of the Optimization Problem

In this section we analyze the optimization problem associated with the design of the proposed robust WC-CCM beamformer. For the purpose of analysis, we rely on the equality of the robust constraint described in (11). In particular, we derive a sufficient condition for enforcing the convexity of the proposed WC-CCM beamformer design as a function of the power of the desired signal. We also provide design guidelines for the adjustment of the parameter ϵ in the optimization problem.

A. Convexity of the Optimization Problem

The objective function for the constant modulus design criterion is

$$J_{\text{cm}} = \mathrm{E}\left\{ \left(|y(i)|^2 - \gamma \right)^2 \right\}. \tag{18}$$

To ensure that the constraint $\boldsymbol{w}^H \boldsymbol{a} = \delta + \epsilon \ \|\boldsymbol{w}\|_2$ is fulfilled, the beamformer $\boldsymbol{w}$ is replaced by

$$\boldsymbol{w} = \frac{\boldsymbol{a}}{M} \ \left(\delta + \epsilon \, \|\boldsymbol{w}\|_2\right) + \boldsymbol{B}\boldsymbol{z}, \tag{19}$$

where the columns of $\boldsymbol{B}$ are unitary and span the null space of $\boldsymbol{a}^H$, $\boldsymbol{z} \in \mathbb{C}^{M-1\times 1}$ and $\boldsymbol{a}^H\boldsymbol{a} = M$. To obtain a function which does not depend on $\|\boldsymbol{w}\|_2$, we compute the squared norm of (19) and obtain the following quadratic equation to be solved:

$$\|\boldsymbol{w}\|_2 = \tau = \sqrt{\frac{1}{M} \left(\delta + \epsilon \ \tau\right)^2 + \boldsymbol{z}^H \boldsymbol{z}} \tag{20}$$

Since the norm is greater than zero the following holds

$$\tau = \frac{\epsilon \ \delta}{M - \epsilon^2} + \sqrt{\frac{M\boldsymbol{z}^H\boldsymbol{z} + \delta^2}{M - \epsilon^2} + \left(\frac{\epsilon \ \delta}{M - \epsilon^2}\right)^2}. \tag{21}$$

Therefore, by inserting (20) and (21) in (19) , the resulting weight vector $\boldsymbol{w}$ is a function of $\boldsymbol{z}$ as described by

$$\boldsymbol{w}\left(\boldsymbol{z}\right) = \frac{\boldsymbol{a}}{M} \ \left(\delta + \frac{\epsilon^2 \ \delta}{M - \epsilon^2} + \epsilon \sqrt{\frac{M\boldsymbol{z}^H\boldsymbol{z} + \delta^2}{M - \epsilon^2} + \left(\frac{\epsilon \ \delta}{M - \epsilon^2}\right)^2}\right) + \boldsymbol{B}\boldsymbol{z} \tag{22}$$

Replacing the $\boldsymbol{w}$ in the objective function leads to an equivalent problem to the original:

$$J = \mathrm{E}\left\{ |y(i)|^2 - \gamma \right\} = \mathrm{E}\left\{ \left[\boldsymbol{w}^H\left(\boldsymbol{z}\right) \boldsymbol{x}\boldsymbol{x}^H \boldsymbol{w}\left(\boldsymbol{z}\right) - \gamma \right]^2 \right\} \tag{23}$$

The function above is convex if the Hessian $\boldsymbol{H} = \frac{\partial}{\partial \boldsymbol{z}^H}\left(\frac{\partial J}{\partial \boldsymbol{z}}\right)$ is positive semi-definite. The Hessian corresponding to the objective function is given by

$$\begin{aligned} \boldsymbol{H} = {} & 2\frac{\partial}{\partial \boldsymbol{z}^H}\left(\mathrm{E}\left\{|y|^2 - \gamma\right\}\right)\frac{\partial}{\partial \boldsymbol{z}}\left(\mathrm{E}\left\{|y|^2 - \gamma\right\}\right) \\ & + 2 \ \mathrm{E}\left\{|y|^2 - \gamma\right\}\frac{\partial}{\partial \boldsymbol{z}^H}\frac{\partial}{\partial \boldsymbol{z}}\left(\mathrm{E}\left\{|y|^2 - \gamma\right\}\right) \end{aligned} \tag{24}$$

Since it is the product of a vector multiplied with its Hermitian transposed the first term in (24), is positive semi-definite. While it is assumed that $\mathrm{E}\left\{|y(i)|^2 - \gamma\right\} \geq 0$ the positive semi-definiteness of $\boldsymbol{H}_2 = \frac{\partial}{\partial \boldsymbol{z}^H}\frac{\partial}{\partial \boldsymbol{z}}\left(\mathrm{E}\left\{|y|^2 - \gamma\right\}\right)$

still needs to be shown. It can be expressed as a sum $\boldsymbol{H}_2 = \sum_{k=1}^{4} \boldsymbol{H}_{2,k}$ and is given by

$$\begin{aligned}
&\boldsymbol{H}_2 = \\
&\mathrm{E}\Bigg\{ \left(-\frac{\epsilon}{2}\left(\frac{1}{\sqrt{\alpha}}\right)\right)^3 \left(\frac{M}{M-\epsilon^2}\right)^2 \mathrm{Re}\left\{\xi\right\} \boldsymbol{z}\boldsymbol{z}^H \\
&+ \epsilon\frac{1}{\sqrt{\alpha}}\left(\frac{M}{M-\epsilon^2}\right) \mathrm{Re}\left\{\xi\right\} \boldsymbol{I}_{M-1} \\
&+ \frac{\epsilon}{2}\frac{1}{\sqrt{\alpha}}\left(\frac{M}{M-\epsilon^2}\right)\frac{\boldsymbol{a}^H}{M}\boldsymbol{x}\boldsymbol{x}^H\frac{\boldsymbol{a}}{M}\frac{\epsilon}{2}\frac{1}{\sqrt{\alpha}}\left(\frac{M}{M-\epsilon^2}\right)\boldsymbol{z}\boldsymbol{z}^H \\
&+ \left(\frac{\epsilon}{2}\frac{1}{\sqrt{\alpha}}\left(\frac{M}{M-\epsilon^2}\right)\boldsymbol{z}\frac{\boldsymbol{a}^H}{M} + \boldsymbol{B}^H\right)\boldsymbol{x}\boldsymbol{x}^H\left(\frac{\boldsymbol{a}}{M}\frac{\epsilon}{2}\frac{1}{\sqrt{\alpha}}\left(\frac{M}{M-\epsilon^2}\right)\boldsymbol{z}^H + \boldsymbol{B}\right)\Bigg\},
\end{aligned} \tag{25}$$

where $\alpha = \left(\frac{M\boldsymbol{z}^H\boldsymbol{z}+\delta^2}{M-\epsilon^2} + \left(\frac{\epsilon\ \delta}{M-\epsilon^2}\right)^2\right)$, $\beta = \left(\delta + \frac{\epsilon^2\ \delta}{M-\epsilon^2} + \epsilon\ \sqrt{\alpha}\right)$ and $\xi = \frac{\boldsymbol{a}^H}{M}\boldsymbol{x}\boldsymbol{x}^H\left(\beta\frac{\boldsymbol{a}}{M} + \boldsymbol{B}\boldsymbol{z}\right)$. To show that $\boldsymbol{H}_2$ is positive semidefinite the following steps are made. Here it is assumed that

$$\mathrm{Re}\left\{\xi\right\} = \mathrm{Re}\left\{\frac{\boldsymbol{a}^H}{M}\boldsymbol{x}\boldsymbol{x}^H\left(\beta\frac{\boldsymbol{a}}{M} + \boldsymbol{B}\boldsymbol{z}\right)\right\} \geq 0. \tag{26}$$

This assumption is reasonable as far as the term $\boldsymbol{x}^H\boldsymbol{B}\boldsymbol{z}$ is basically the compensating term of the unwanted contribution of $\boldsymbol{x}^H\left(\beta\frac{\boldsymbol{a}}{M}\right)$. Under this condition all terms in the sum of $\boldsymbol{H}_2$ are positive semi-definite except the first term $\boldsymbol{H}_{2,1}$. The inequality $\boldsymbol{v}^H(\boldsymbol{H}_{2,2})\boldsymbol{v} \geq \boldsymbol{v}^H(-\boldsymbol{H}_{2,1})\boldsymbol{v}\ \ \forall\boldsymbol{v}$ is a sufficient condition to ensure positive semi-definiteness which is described as

$$\begin{aligned}
&\boldsymbol{v}^H\epsilon\frac{1}{\sqrt{\alpha}}\left(\frac{M}{M-\epsilon^2}\right)\mathrm{Re}\left\{\xi\right\}\boldsymbol{I}_{M-1}\boldsymbol{v} \\
&\geq \boldsymbol{z}^H\left(\frac{\epsilon}{2}\left(\frac{1}{\sqrt{\alpha}}\right)\right)^3\left(\frac{M}{M-\epsilon^2}\right)^2\mathrm{Re}\left\{\xi\right\}\boldsymbol{z}\boldsymbol{z}^H\boldsymbol{z} \\
&\geq \boldsymbol{v}^H\left(\frac{\epsilon}{2}\left(\frac{1}{\sqrt{\alpha}}\right)\right)^3\left(\frac{M}{M-\epsilon^2}\right)^2\mathrm{Re}\left\{\xi\right\}\boldsymbol{z}\boldsymbol{z}^H\boldsymbol{v},
\end{aligned} \tag{27}$$

where $\boldsymbol{v}$ is any vector with the same norm of $\boldsymbol{z}$ and $\boldsymbol{z}^H(-\boldsymbol{H}_{2,1})\boldsymbol{z}$ is intruduced as the upper bound for $\boldsymbol{v}^H(-\boldsymbol{H}_{2,1})\boldsymbol{v}$. Since $\boldsymbol{z}^H\boldsymbol{z} = \boldsymbol{v}^H\boldsymbol{v}$, the inequality can be reduced to

$$2\alpha \geq \left(\frac{M\boldsymbol{z}^H\boldsymbol{z}}{M-\epsilon^2}\right). \tag{28}$$

Replacing α gives the proof for positive semi-definiteness

$$2\left(\frac{M\boldsymbol{z}^H\boldsymbol{z}+\delta}{M-\epsilon^2} + \left(\frac{\epsilon\ \delta}{M-\epsilon^2}\right)^2\right) \geq \left(\frac{M\boldsymbol{z}^H\boldsymbol{z}}{M-\epsilon^2}\right), \tag{29}$$

which is always true. To ensure that $\mathrm{E}\left\{|y|^2 - \gamma\right\} \geq 0$ it can be assumed that $\boldsymbol{w}^H\left(\boldsymbol{a}+\boldsymbol{e}\right) \geq \delta$, where $\boldsymbol{e}$ is the array steering vector mismatch. Therefore,

$$\gamma \leq \delta\ \mathrm{E}\left\{|s_1|^2\right\} \tag{30}$$

is a sufficient condition to enforce convexity, where $|s_1|^2$ is the power of the desired user. Therefore, the parameter gamma should be chosen such that the convexity condition given in (30) is satisfied.

B. Adjustment of the Design Parameter ϵ

Let us define the beamforming weight vector as

$$\boldsymbol{w} = c\ \boldsymbol{a}/M + \boldsymbol{b}, \tag{31}$$

where c is a scalar, and $\boldsymbol{b}$ is orthogonal to $\boldsymbol{a}$. Using it with the worst-case constraint leads to

$$c - \delta \geq \epsilon\ \sqrt{\frac{c^2}{M} + \boldsymbol{b}^H \boldsymbol{b}}. \tag{32}$$

From the above inequality the following relation holds

$$c - \delta \geq \epsilon\ \sqrt{\frac{c^2}{M} + \boldsymbol{b}^H \boldsymbol{b}} \geq \epsilon\ \sqrt{\frac{c^2}{M}}. \tag{33}$$

Rewriting the relation shows that c tends to infinity when ϵ is close to $\sqrt{M}$

$$c \geq \frac{\delta}{1 - \epsilon/\sqrt{M}}. \tag{34}$$

In addition, it is mentioned in [7] that for $\|\boldsymbol{a}\|_2 \leq \epsilon$ there is no $\boldsymbol{w}$ that satisfies the constraint. By rewriting the inequality in (33), we obtain

$$c \geq \frac{M\ \delta}{M - \epsilon^2} + \sqrt{\frac{M\epsilon^2 \boldsymbol{b}^H \boldsymbol{b} - M\ \delta}{M - \epsilon^2} + \left(\frac{M\ \delta}{M - \epsilon^2}\right)^2} \tag{35}$$

Now by assuming that $\epsilon \approx \sqrt{M}$ and strictly less than $\sqrt{M}$, then we have $c \gg \delta$. In that case, the inequality in (33) can be rewritten as

$$c \geq \epsilon \sqrt{\frac{c^2}{M} + \boldsymbol{b}^H \boldsymbol{b}}, \tag{36}$$

or equivalently as

$$\frac{\boldsymbol{b}^H \boldsymbol{b}}{c^2} \leq \frac{1}{\epsilon^2} - \frac{1}{M}. \tag{37}$$

As a result of (37), the choice of ϵ affects the ratio between the components of the weight vector defined by (31), which can become negligible. This corresponds to $\boldsymbol{w} \approx c\ \boldsymbol{a}/M$ and an equivalent diagonal loading which is above the level of the interference. Hence, the diagonal loading can be chosen by an appropriate procedure if ϵ is chosen in the allowed interval $[0, \sqrt{M}]$, where the constraint can be enforced. Obviously, in the case of ϵ being close to $\sqrt{M}$ the ratio $\frac{\boldsymbol{b}^H \boldsymbol{b}}{c^2}$ tends to a small value, which can lead to a performance degradation. The ratio is small for low SNR values, and this is caused by the assumption that the additional noise appears as a diagonal loading in the signal covariance matrix and this eventually decreases $\|\boldsymbol{b}\|$. This means that the relation in (37) and its penalty has a more significant impact on the performance for higher SNR values. As a consequence our suggestion is to choose ϵ with respect to the SNR as well as with respect to the assumed mismatch level. This will be investigated in the simulations (see Fig.3)

VII. Low-Complexity Algorithms using the Modified Conjugate Gradient

The existing algorithms which use the worst-case optimization-based constraint do not take advantage of previous computations as the conventional SMI beamforming algorithm solved by the modified conjugate method (MCG) algorithm or the recursive-least-squares (RLS) algorithm in the so-called *on-line* mode. For this reason, the existing robust beamforming algorithms are not suitable for low-complexity implementations and are unable to track time-varying signals.

In this section a robust constraint is shown which is just slightly different compared to the worst-case optimization-based approaches. As a result the corresponding optimization problem is a quadratically constrained quadratic program (QCQP) instead of a second order cone (SOC) program. It is shown how to solve the problem with a joint optimization strategy. The method includes a system of equations which is solved efficiently with a modified conjugate gradient algorithm and an alternating optimization strategy [27]. As a result, the computational complexity is reduced from more than cubic $\mathcal{O}(M^{3.5})$ to quadratic $\mathcal{O}(M^2)$ with the number of sensor elements, while the SINR performance is equivalent to the worst-case optimization-based approach. The proposed method is presented in the robust constrained minimum variance design using the modified conjugate gradient method (RCMV-MCG) and in the robust constrained constant modulus design using the modified conjugate gradient method (RCCM-MCG), which exploits the constant modulus property of the desired signal.

A. Proposed Design and Joint Optimization Approach

In this part, we detail the main steps of the proposed design and the low-complexity algorithms as well as the joint optimization approach that is employed to compute the parameters of the adaptive robust beamformer and the diagonal loading. Specifically, the proposed algorithms are based on an alternating optimization strategy [27] that updates the beamformer $\boldsymbol{w}(i)$ while the diagonal loading $\lambda(i)$ is fixed and then updates $\lambda(i)$ while $\boldsymbol{w}(i)$ is held fixed. The algorithm is illustrated in Fig. 1.

Since the joint optimization of the parameters $\boldsymbol{w}(i)$ and $\lambda(i)$ is not a convex optimization problem, the first question that arises is whether the proposed algorithms will converge to their global minima. The proposed algorithms have been widely tested and we have not observed problems with local minima. This is corroborated by the recent results reported in [27] that shows that alternating optimization techniques similar to that proposed here converge to the global optimum provided that typical assumptions used for adaptive algorithms such as step size values, forgetting factors and the statistical independence of the noise and the source data processed hold.

1) Robust Constrained Minimum Variance Design: The proposed low-complexity beamforming algorithms are related to the worst-case approach [5]. In order to obtain a design which can be solved with a low complexity, the robust constraint reported in [5] is modified. According to [9] it is sufficient to use the real part of the constraint. In addition, it is assumed that the use of $\tilde{\epsilon}\left\|\boldsymbol{w}\right\|_2^2$ instead of $\epsilon\left\|\boldsymbol{w}\right\|_2$ from the conventional constraint has a comparable impact. Finally, the proposed design criterion for the minimum variance case is

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{R}_{xx} \boldsymbol{w}, \quad \text{s. t. } \operatorname{Re}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} - \delta \geq \tilde{\epsilon} \left\|\boldsymbol{w}\right\|_2^2. \tag{38}$$

Using the method of Lagrange multipliers gives

$$\mathcal{L}_{\text{CMV}}\left(\boldsymbol{w}, \lambda\right) = \boldsymbol{w}^H \boldsymbol{R}_{xx} \boldsymbol{w} + \lambda \left[\tilde{\epsilon} \ \boldsymbol{w}^H \boldsymbol{w} - \operatorname{Re}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} + \delta\right], \tag{39}$$

where λ is the Lagrange multiplier. Computing the gradient of (39) with respect to $\boldsymbol{w}^*$, and equating it to zero leads to

$$\boldsymbol{w} = \left(\boldsymbol{R}_{xx} + \tilde{\epsilon}\lambda \boldsymbol{I}\right)^{-1} \lambda \boldsymbol{a}/2. \tag{40}$$

Since there is no known method in the literature that can obtain the Lagrange multiplier in a closed form, here it is proposed a strategy to adjust both the beamformer $\boldsymbol{w}$ and the Lagrange multiplier in an alternating fashion. In this joint optimization the Lagrange multiplier is interpreted as a penalty factor and the condition $\lambda > 0$ holds all the time. The adjustment increases the penalty factor when the constraint is not fulfilled and decreases it otherwise. To this end, we devise the following algorithm

$$\lambda(i) = \lambda(i-1) + \mu_\lambda \left(\tilde{\epsilon} \left\|\boldsymbol{w}(i)\right\|_2^2 - \operatorname{Re}\left\{\boldsymbol{w}(i)^H \boldsymbol{a}\right\} + \delta\right), \tag{41}$$

where μ_λ is the step size. In addition, it is reasonable to define boundaries for the update term.

In order to obtain an operation range for the parameter $\tilde{\epsilon}$ the weight vector can be expressed as $\boldsymbol{w} = \frac{\boldsymbol{a}}{M} + \boldsymbol{b}$. Rearranging the constraint function leads to the inequality

$$\tilde{\epsilon} \leq \frac{c - \delta}{\frac{1}{M}c^2 + \boldsymbol{b}^H \boldsymbol{b}} \leq M \frac{c - \delta}{c^2} \leq \frac{M}{2}, \tag{42}$$

which clearly indicates that there is no solution for $\tilde{\epsilon} > \frac{M}{2}$. From our experiments we know that the parameter has to be chosen significantly smaller.

2) Robust Constrained Constant Modulus Design: In case of constant modulus signals it has been shown that the constant modulus design performs better than the minimum variance design [14], [15]. Similarly, the robust approach can be combined with the constrained constant modulus criterion. The corresponding optimization problem for the iteratively solved constant modulus objective function can be cast as

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{R}_{\text{a}} \boldsymbol{w} - 2\gamma \operatorname{Re}\left\{\boldsymbol{w}^H \boldsymbol{d}\right\}, \tag{43}$$

$$\text{s. t. } \operatorname{Re}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} - \delta \geq \tilde{\epsilon} \left\|\boldsymbol{w}\right\|_2^2 \tag{44}$$

Using the method of Lagrange multipliers gives

$$\begin{aligned} \mathcal{L}_{\text{CCM}}\left(\boldsymbol{w}, \lambda\right) = & \boldsymbol{w}^H \boldsymbol{R}_{\text{a}} \boldsymbol{w} - 2\ \gamma \operatorname{Re}\left\{\boldsymbol{w}^H \boldsymbol{d}\right\} \\ & + \lambda \left[\tilde{\epsilon} \ \boldsymbol{w}^H \boldsymbol{w} - \operatorname{Re}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} + \delta\right]. \end{aligned} \tag{45}$$

Computing the gradient of (45) with respect to $\boldsymbol{w}^*$, and equating it to zero leads to

$$\boldsymbol{w} = [\boldsymbol{R}_{\mathrm{a}} + \tilde{\epsilon}\lambda \boldsymbol{I}]^{-1} [\gamma \boldsymbol{d} + \lambda \boldsymbol{a}/2] \ , \tag{46}$$

where $\boldsymbol{I}$ is an M-dimensional identity matrix. The adjustment of the Lagrange multiplier λ can be performed in the same way as in the minimum variance case.

B. Adaptive Algorithms

To take advantage of the proposed joint optimization approach an *on-line* modified conjugate gradient method, with one iteration per snapshot is used to solve the resulting problem. Its derivation is based on [28] and it can be interpreted as an extension of the idea in [17].

1) Robust-CMV-MCG: In the proposed algorithm an exponentially decayed data window is used to estimate the matrix $\boldsymbol{R}_{xx}$ as described by

$$\hat{\boldsymbol{R}}_{xx}(i) = \mu \hat{\boldsymbol{R}}_{xx}(i-1) + \boldsymbol{x}(i)\boldsymbol{x}^H(i) \ , \tag{47}$$

where μ is the forgetting factor. According to [1]

$$\boldsymbol{R}_{xx} \simeq [1-\mu] \, \hat{\boldsymbol{R}}_{xx}(i) \tag{48}$$

can be assumed for large i. Replacing $\boldsymbol{R}_{xx}$ in (40), introducing $\hat{\lambda}(i) = \frac{\lambda(i)}{1-\mu}$, leads to $\boldsymbol{w}(i) = [\hat{\boldsymbol{R}}_{xx}(i) + \tilde{\epsilon}\hat{\lambda}(i)\boldsymbol{I}]^{-1}\hat{\lambda}(i)\boldsymbol{a}/2$. Let us introduce the CG weight vector $\boldsymbol{v}(i)$ as follows $\boldsymbol{w}(i) = \boldsymbol{v}(i)\frac{\hat{\lambda}(i)}{2}$. The conjugate gradient algorithm solves the problem by iteratively updating the CG weight vector

$$\boldsymbol{v}(i) = \boldsymbol{v}(i-1) + \alpha(i)\boldsymbol{p}(i), \tag{49}$$

where $\boldsymbol{p}(i)$ is the direction vector and $\alpha(i)$ is the adaptive step size. One way to realize the conjugate gradient method performing one iteration per snapshot is the application of the degenerated scheme [28]. Under this condition the adaptive step size $\alpha(i)$ has to fulfill the convergence bound given by

$$0 \leq \mathrm{E}\left\{\boldsymbol{p}^H(i)\boldsymbol{g}(i)\right\} \leq 0.5 \ \mathrm{E}\left\{\boldsymbol{p}^H(i)\boldsymbol{g}(i-1)\right\}, \tag{50}$$

where $\mathrm{E}\left\{\mathrm{Im}\left\{\boldsymbol{p}^H(i)\boldsymbol{g}(i-1)\right\}\right\} \approx 0$ and $\mathrm{E}\left\{\mathrm{Im}\left\{\boldsymbol{p}^H(i)\boldsymbol{g}(i)\right\}\right\} \approx 0$ can be neglected. The negative gradient vector and its recursive expression are considered in a similar fashion to [28],[17] as described by

$$\begin{aligned}
\boldsymbol{g}(i) =& \boldsymbol{a} - [\hat{\boldsymbol{R}}_{xx}(i) + \tilde{\epsilon}\hat{\lambda}(i)\boldsymbol{I}]\boldsymbol{v}(i) \\
=& \boldsymbol{a}[1-\mu] + \mu \boldsymbol{g}(i-1) \\
& - [\boldsymbol{x}\boldsymbol{x}^H + \tilde{\epsilon}(\hat{\lambda}(i) - \mu\hat{\lambda}(i-1))\boldsymbol{I}]\boldsymbol{v}(i-1) \\
& - \alpha(i)[\hat{\boldsymbol{R}}_{xx}(i) + \tilde{\epsilon}\hat{\lambda}(i)\boldsymbol{I}]\boldsymbol{p}(i)
\end{aligned} \tag{51}$$

Pre-multiplying with $\boldsymbol{p}^H(i)$, taking expectations on both sides and considering $\boldsymbol{p}(i)$ uncorrelated with $\boldsymbol{a}$, $\boldsymbol{x}(i)$ and $\boldsymbol{v}(i-1)$ leads to

$$\begin{aligned}\mathrm{E}\left\{\boldsymbol{p}^H(i)\boldsymbol{g}(i)\right\} \approx&\mu\mathrm{E}\left\{\boldsymbol{p}^H(i)\boldsymbol{g}(i-1)\right\}\\ &-\mathrm{E}\left\{\alpha(i)\right\}\mathrm{E}\left\{\boldsymbol{p}^H(i)[\hat{\boldsymbol{R}}_{xx}(i)+\tilde{\epsilon}\hat{\lambda}(i)\boldsymbol{I}]\boldsymbol{p}(i)\right\}.\end{aligned} \tag{52}$$

Here it is assumed that the algorithm has converged, which implies $\boldsymbol{a}[1-\mu]-[\mathrm{E}\left\{\boldsymbol{x}\boldsymbol{x}^H\right\}+\tilde{\epsilon}\hat{\lambda}(i)[1-\mu]\boldsymbol{I}]\boldsymbol{v}(i-1)=\boldsymbol{0}$, where equation (48) is taken into account and $\hat{\lambda}(i)\approx\hat{\lambda}(i-1)$. Introducing $\boldsymbol{p}_R=[\hat{\boldsymbol{R}}_{xx}(i)+\hat{\lambda(i)}\tilde{\epsilon}\boldsymbol{I}]\boldsymbol{p}(i)$, rearranging (52) and inserting it into (50) determines the step size within its boundaries as follows

$$\alpha(i)=\left[\boldsymbol{p}^H(i)\boldsymbol{p}_R\right]^{-1}(\mu-\eta)\,\boldsymbol{p}^H(i)\boldsymbol{g}(i-1), \tag{53}$$

where $0\leq\eta\leq 0.5$. The direction vector is a linear combination of the previous direction vector and the negative gradient given by

$$\boldsymbol{p}(i+1)=\boldsymbol{p}(i)+\beta(i)\boldsymbol{g}(i), \tag{54}$$

where $\beta(i)$ is computed to avoid the reset procedure by employing the Polak-Ribiere approach [18].

$$\beta=[\boldsymbol{g}^H(i-1)\boldsymbol{g}(i-1)]^{-1}[\boldsymbol{g}(i)-\boldsymbol{g}(i-1)]^H\boldsymbol{g}(i) \tag{55}$$

The proposed algorithm, which is termed Robust-CMV-MCG, is described in Table II.

Note that for the alternating algorithm to adjust the Lagrange multiplier, we divide the update-term by 2, if the Lagrange multiplier is outside a predefined range, as it is described in Table I. The application of the proposed algorithm corresponds to a computational effort which is quadratic with the number of sensor elements M.

2) Robust-CCM-MCG: The adaptive algorithm in the case of the constrained constant modulus criterion is developed analogously to the minimum variance case. The estimates of $\boldsymbol{R}_a$ and $\boldsymbol{d}$ are based on an exponentially decayed data window and are given by

$$\hat{\boldsymbol{R}}_{\mathrm{a}}(i) = \mu\hat{\boldsymbol{R}}_{\mathrm{a}}(i-1)+|y(i)|^2\boldsymbol{x}(i)\boldsymbol{x}^H(i) \tag{56}$$

$$\hat{\boldsymbol{d}}(i) = \mu\hat{\boldsymbol{d}}(i-1)+\boldsymbol{x}(i)y^*(i) \tag{57}$$

Following the steps of the derivation of the MCG algorithm and taking into account that

$$\boldsymbol{R}_{\mathrm{a}}\simeq[1-\mu]\hat{\boldsymbol{R}}_{\mathrm{a}}(i) \tag{58}$$

$$\boldsymbol{d}\simeq[1-\mu]\hat{\boldsymbol{d}}(i) \tag{59}$$

leads to the adaptive algorithm. Note that, in contrast to the CMV case, here the beamforming weight vector is the same as the conjugate gradient weight vector, which means $\boldsymbol{w}=[\hat{\boldsymbol{R}}_{\mathrm{a}}+\tilde{\epsilon}\hat{\lambda}\boldsymbol{I}]^{-1}[\gamma\hat{\boldsymbol{d}}+\hat{\lambda}\boldsymbol{a}/2]$. The negative gradient vector and its recursive expression are defined as

$$\begin{aligned}\boldsymbol{g}(i)=&[\gamma\hat{\boldsymbol{d}}+\hat{\lambda}\boldsymbol{a}/2]-[\hat{\boldsymbol{R}}_{\mathrm{a}}+\tilde{\epsilon}\hat{\lambda}\boldsymbol{I}]\boldsymbol{w}(i)\\ =&\mu\boldsymbol{g}(i-1)-\alpha(i)\boldsymbol{p}_R-\left(|y(i)|^2\boldsymbol{x}(i)\boldsymbol{x}^H(i)\right)\boldsymbol{w}(i-1)\\ &+\gamma\boldsymbol{x}(i)y^*(i)+\nu\left[\boldsymbol{a}/\left(2\tilde{\epsilon}\right)-\boldsymbol{w}(i-1)\right],\end{aligned} \tag{60}$$

where $\nu = \left[\hat{\lambda}(i) - \mu\hat{\lambda}(i-1)\right]\tilde{\epsilon}$. The proposed algorithm, which is termed Robust-CCM-MCG, is described in Table III.

VIII. Simulations

In this section, we present a number of simulation examples that illustrate the performance of the proposed robust beamforming algorithms and compare them with existing robust techniques that are representative of the prior work in this area. A uniform linear sensor array is used with $M = 10$ sensors. Specifically, we consider comparisons of the proposed algorithms with the loaded-SMI [3], the optimal SINR [2] (page 54) and the WC-CMV in [5]. We examine scenarios in which the SINR is measured against the parameter ϵ that arises from the worst-case optimization, the number of snapshots and the SNR. We also consider a specific situation in which the array steering vector is corrupted by local coherent scattering, and scenarios in which there are changes in the environment and the tracking performance of the beamformers is evaluated. These experiments are important to assess the performance of the proposed algorithms and to illustrate how they perform against existing methods.

A. Proposed WC-CCM Algorithm

In this part of the simulations, the WC-CCM design algorithm of Table I that uses a SOC program is compared to the loaded-SMI [3], the optimal SINR [2] and the worst-case optimization-based constrained minimum variance algorithm [5]. In the next simulations, it is considered that $|s_1| = 1$, $\delta = 1$, $\gamma = 1$, $\epsilon = 2.1$ and $\mu = 0.995$ unless otherwise specified. In addition to user 1, the desired signal, there are 4 interferers, the powers (P) relative to user 1 and directions of arrival (DoA) in degrees of which are detailed in Table IV.

The array steering vector is corrupted by local coherent scattering

$$\boldsymbol{a}_1 = \boldsymbol{a} + \sum_{k=1}^{4} \mathrm{e}^{j\Phi_k} \boldsymbol{a}_{\mathrm{sc}}\left(\theta_k\right), \tag{61}$$

where Φ_k is uniformly distributed between zero and 2π and θ_k is uniformly distributed with a standard deviation of 2 degrees with the assumed direction as the mean. The mismatch changes for every realization and is fixed over the snapshots of each simulation trial.

Fig. 2 shows the SINR as a function of the design parameter ϵ for different levels of mismatch, where its level corresponds to the standard deviation of the local scattering. In Fig. 3 no mismatch is considered but different noise levels. Both simulations show performance degradations when ϵ is chosen close to $\sqrt{M}$, especially for high SNR values. The simulations corroborate the analysis and show that the optimal value for ϵ depends on the SNR.

Fig. 4 presents the SINR performance over the snapshots in the presence of local coherent scattering. At time index $i = 1000$ the interference scenario changes according to Table IV and interferers assume different power levels and DoAs. With this change, the beamformers must adapt to the new environment and their tracking performance is assessed by a plot showing the SINR performance against the snapshots. The proposed WC-CCM algorithm shows in Fig. 4 a significantly better SINR performance than the WC-CMV [5] and the loaded-SMI algorithm.

In terms of tracking performance, the proposed WC-CCM algorithm of Table I is able to effectively adjust to the new environment. Fig. 5 shows the SINR performance against the SNR for $i = 500$ snapshots. The curves show that the proposed WC-CCM algorithm is more robust against mismatch problems than the existing WC-CMV and loaded-SMI agorithms.

B. Low-Complexity Robust Adaptive Beamforming

In this subsection, we assess the SINR performance of the proposed low-complexity robust beamforming algorithms in Tables II and III that are devised for an online operation. In the simulations, the same parameters of the previous subsection are used and, in addition, the step sizes are $\mu_\lambda(\text{CMV}) = 800$ and $\mu_\lambda(\text{CCM}) = 100$. The limitation on the update is set to $\delta_{\lambda max} = 200$. For the robust constraints, we employ $\epsilon = \tilde{\epsilon} = 2.1$ and the parameters $|s_1| = 1$, $\delta = 1$, $\gamma = 1$,. According to the different constraint functions, the equality is a special case for $M = 10$ and cannot be generalized. In addition to the desired user (user 1), there are 4 interferers whose relative powers (P) with respect to the desired user and directions of arrival (DoA) in degrees are detailed in Table V. At time index $i = 1000$ the adaptive beamforming algorithms are confronted with a change of scenario given in Table IV and the interferers assume different power levels and DoAs. In this situation, the adaptive beamforming algorithms must adapt to the new conditions and their tracking performance is evaluated.

Fig. 6 shows the SINR performance as a function of the number of snapshots in the presence of local coherent scattering. The results of Fig. 6 show that the proposed Robust CCM-MCG algorithm has a superior SINR performance to the existing WC-CMV [5] algorithm, the proposed Robust CMV-MCG algorithm and the loaded-SMI algorithm. The Robust CMV-MCG algorithm has a comparable performance to the WC-CMV [5] algorithm but the latter has a significantly higher computational cost. The SINR performance versus the SNR is presented in Fig. 7. While the proposed Robust CMV-MCG algorithm shows an equivalent performance to the WC-CMV [5], the proposed Robust CCM-MCG algorithm exploits the constant modulus property and performs better than existing approaches. Fig. 8 shows the SINR performance against the number of snapshots for the same scenario as in Fig. 6 with different values of γ whilst keeping delta fixed. The results show that for certain values the convexity constraint is satisfied and the algorithm converges to a higher SINR value, whereas for smaller values of gamma the algorithm converges to lower values of SINR, suggesting that a local minimum of the constant modulus cost function might have been reached. Therefore, the values of γ should be set appropriately in order to ensure an optimized performance. This adjustment could be performed with either some prior knowledge about the energy of the signal or with the help of a procedure that computes the energy of the signal online.

IX. Conclusion

We have proposed a robust beamforming algorithm based on the worst case constraint and the constrained constant modulus (CCM) design criterion which is called worst-case constant modulus criterion (WC-CCM). The proposed approach exploits the constant modulus property of the desired signal. The problem can be solved iteratively, where each iteration is effectively solved by a SOC program. Compared to the conventional worst-case optimization based approach using the minimum variance design, the proposed algorithm shows better results especially in the high SNR regime.

In addition to the WC-CCM algorithm, we have also developed two low-complexity robust adaptive beamforming algorithms, namely, the Robust-CMV-MCG and the Robust-CCM-MCG. The proposed algorithms use a constraint similar to the worst-case optimization based approach. It has been shown that the joint optimization approach allows the exploitation of highly efficient *on-line* algorithms like the modified conjugate gradient method which performs just one iteration per snapshot taking advantage of previous computations. As a result the complexity is reduced by more than an order of magnitude compared to the worst-case optimization based beamformer which is solved with a second-order cone program. While the proposed Robust-CMV-MCG performs equivalently, the proposed Robust-CCM-MCG algorithm based on the CCM design criterion, shows a better performance which takes advantage of the constant modulus property of the signal amplitude of the desired user.

REFERENCES

[1] H. V. Trees, *Optimum Array Processing*. John Wiley, 2002.

[2] J. Li and P. Stoica, *Robust Adaptive Beamforming*. NJ: Wiley: Hoboken, 2006.

[3] H. Cox, R. M. Zeskind, and M. H. Owen, “Robust adaptive beamforming,” *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-35, pp. 1365–1376, Oct. 1987.

[4] A. B. Gershman, “Robust adaptive beamforming in sensor arrays,” *Int. J. Electron. Commun.*, vol. 53, p. 305314, Dec. 1999.

[5] S. A. Vorobyov, A. B. Gershman, and Z.-Q. Luo, “Robust adaptive beamforming using worst-case performance optimization: A solution to the signal mismatch problem,” *IEEE Trans. Signal Processing*, vol. 51, pp. 313–324, Feb. 2003.

[6] P. Stoica, Z. Wang, and J. Li, “Robust capon beamforming,” *IEEE Signal Processing Letters*, vol. 10, no. 6, pp. 172 –175, june 2003.

[7] J. Li and P. Stoica, “On robust capon beamforming and diagonal loading,” *IEEE Trans. Signal Processing*, vol. 51, no. 7, pp. 1702–1715, Jul. 2003.

[8] J. Li, P. Stoica, and Z. Wang, “Doubly constrained robust capon beamformer,” *IEEE Transactions on Signal Processing*, vol. 52, no. 9, pp. 2407 – 2423, sept. 2004.

[9] R. Lorenz and S. Boyd, “Robust minimum variance beamforming,” *IEEE Trans. Signal Processing*, vol. 53, pp. 1684–1696, May 2005.

[10] C.-Y. Chen and P. P. Vaidyanathan, “Quadratically constrained beamforming robust against direction-of-arrival mismatch,” *IEEE Trans. Signal Processing*, vol. 55, no. 8, 2007.

[11] A. Hassanien and S. A. Vorobyov, “Robust adaptive beamforming using sequential quadratic programming: An iterative solution to the mismatch problem,” *IEEE Signal Processing Lett.*, vol. 15, pp. 733–736, 2008.

[12] A. B. Gershman, N. D. Sidiropoulos, S. Shahbazpanahi, M. Bengtsson, and B. Ottersten, “Convex optimization-based beamforming,” *IEEE Signal Processing Magazin*, pp. 62–75, May 2010.

[13] A. Khabbazibasmenj, S. Vorobyov, and A. Hassanien, “Robust adaptive beamforming based on steering vector estimation with as little as possible prior information,” *IEEE Transactions on Signal Processing*, vol. 60, no. 6, pp. 2974 –2987, june 2012.

[14] R. C. de Lamare and R. Sampaio-Neto, “Blind adaptive code-constrained constant modulus algorithms for CDMA interference suppression in multipath channels,” *IEEE Signal Processing Lett.*, vol. 9, no. 4, pp. 334–336, Apr. 2005.

[15] R. C. de Lamare, M. Haardt, and R. Sampaio-Neto, “Blind adaptive constrained reduced-rank parameter estimation based on constant modulus design for CDMA interference suppression,” *IEEE Trans. Signal Processing*, vol. 56, no. 2, pp. 2470–2482, Jun. 2008.

[16] Y. Chen, T. Le-Ngoc, B. Champagne, and C. Xu, “Recursive least squares constant modulus algorithm for blind adaptive array,” *Signal Processing, IEEE Transactions on*, vol. 52, no. 5, pp. 1452 – 1456, may 2004.

[17] L. Wang and R. C. de Lamare, “Constrained adaptive filtering algorithms based on conjugate gradient techniques for beamforming,” *IET Signal Processing*, vol. 4, pp. 686–697, Dec. 2010.

[18] D. G. Luenberger, *Linear and Nonlinear Programming 2nd ed.* Reading, MA: Addison-Wesley, 1984.

[19] D. D. Feldman and L. J. Griffiths, “A projection approach to robust adaptive beamforming,” *IEEE Trans. Signal Processing*, vol. 42, pp. 867–876, Apr. 1994.

[20] R. C. de Lamare, L. Wang, and R. Fa, “Adaptive reduced-rank LCMV beamforming algorithms based on joint iterative optimization of filters: Design and analysis,” *Signal Processing*, vol. 90, no. 2, pp. 640–652, Feb. 2010.

[21] S. A. Vorobyov, A. B. Gershman, and Y. Rong, “On the relationship between the worst-case optimization-based and probability-constrained approaches to robust adaptive beamforming,” in *Proc. IEEE ICASSP*, vol. 2, Honolulu, HI, Apr. 2007, pp. 977–980.

[22] L. Wang and R. C. de Lamare, “Low-complexity adaptive step size constrained constant modulus sg algorithms for blind adaptive beamforming,” *Signal Processing*, vol. 89, no. 12, pp. 2503–2513, 2009.

[23] R. C. de Lamare, R. Sampaio-Neto, and M. Haardt, “Blind adaptive constrained constant-modulus reduced-rank interference suppression algorithms based on interpolation and switched decimation,” *IEEE Transactions on Signal Processing*, vol. 59, no. 2, pp. 681–695, 2011.

[24] Y. Meng, M. You, and J. Liu, “Robust noise suppression least square constant modulus interference cancellation,” in *Personal, Indoor and Mobile Radio Communications, 2009 IEEE 20th International Symposium on*, sept. 2009, pp. 526 –530.

[25] J. F. Sturm, “Using sedumi 1.02, a matlab toolbox for optimization over symmetric cones,” Optim. Meth. Softw., Aug. 1999.

[26] M. Grant and S. Boyd, “CVX: Matlab software for disciplined convex programming, version 1.21,” http://cvxr.com/cvx, Jan. 2011.

[27] U. Niesen, D. Shah, and G. W. Wornell, *Adaptive Alternating Minimization Algorithms*, 2009, vol. 55, no. 3.

[28] P. S. Chang and A. N. Willson, “Analysis of conjugate gradient algorithms for adaptive filtering,” *IEEE Trans. Signal Processing*, vol. 48, pp. 409–418, Feb. 2000.

Figures

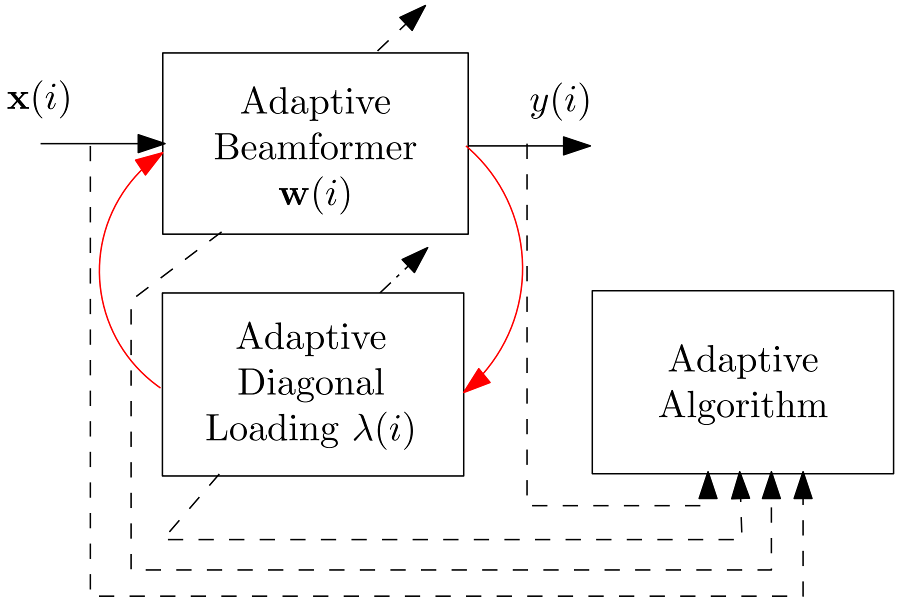

Fig. 1. Proposed adaptive scheme with alternating updates of beamforming weights and diagonal loading.

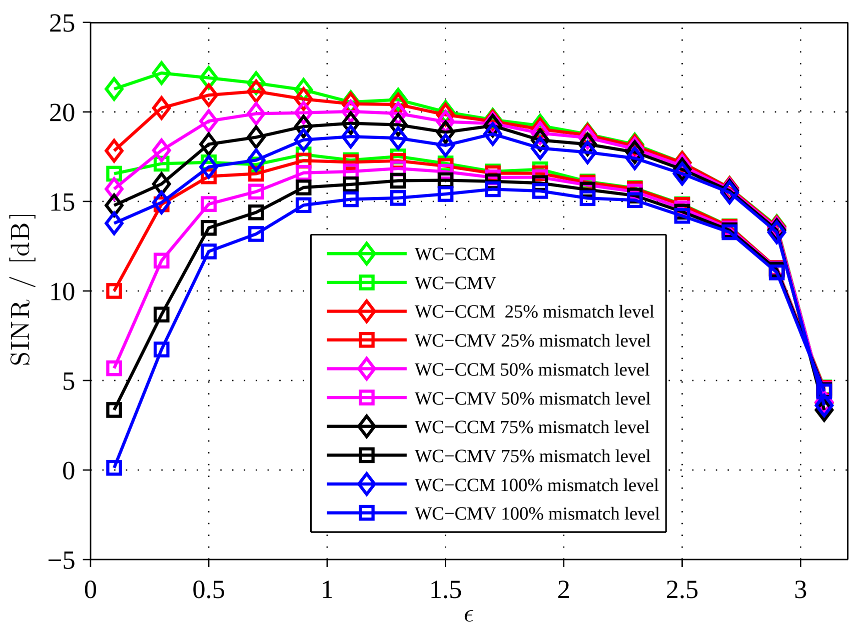

Fig. 2. SINR versus ϵ, SNR = 15dB, $M = 10$, $i = 200$.

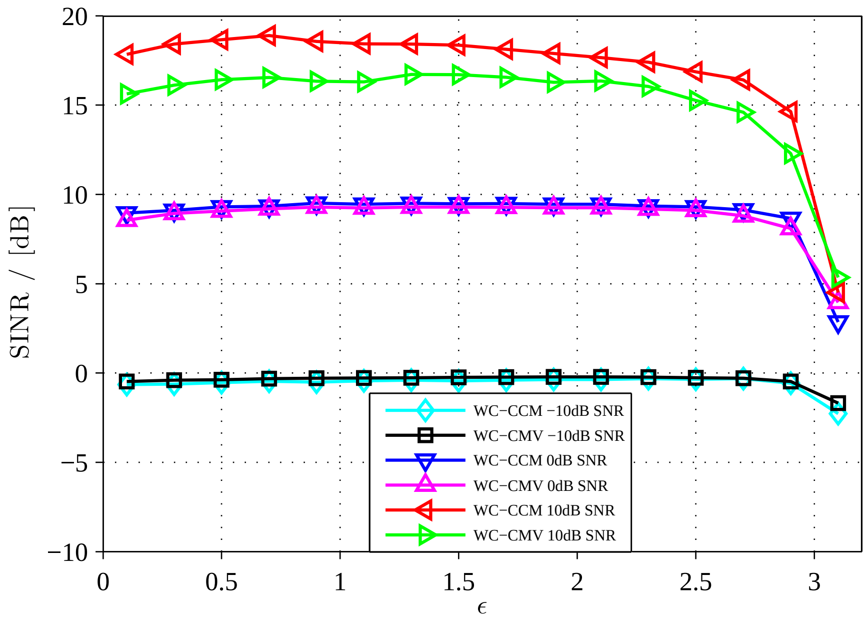

Fig. 3. SINR versus ϵ, perfect ASV, $M = 10$.

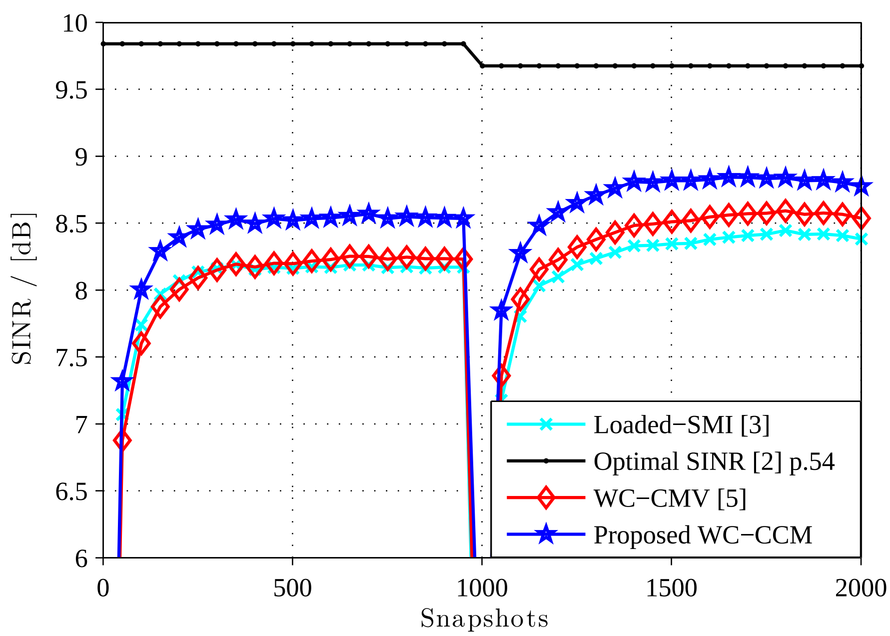

Fig. 4. SINR versus snapshots, SNR = 0 dB, local coherent scattering.

TABLES

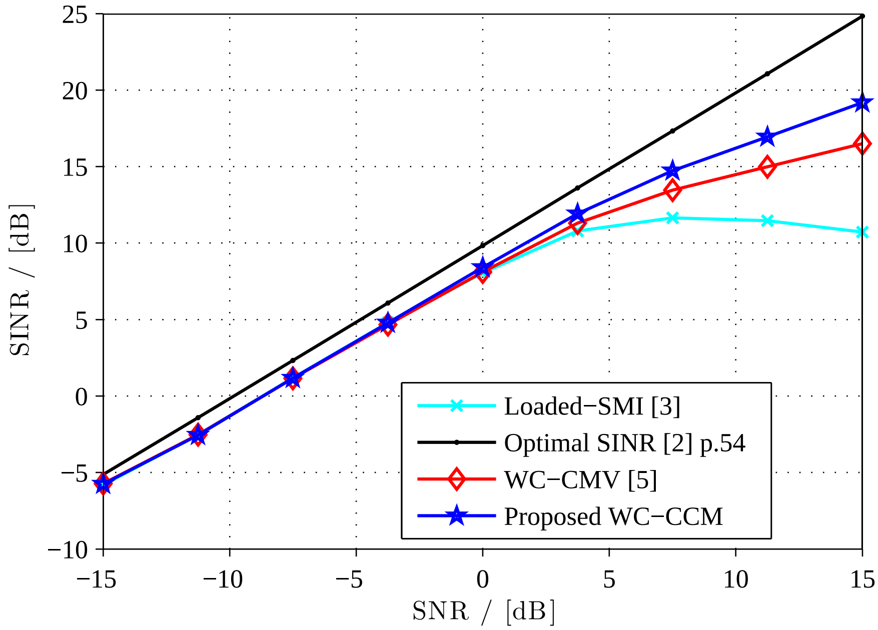

Fig. 5. SINR versus SNR, local coherent scattering, $i = 500$, $M = 10$.

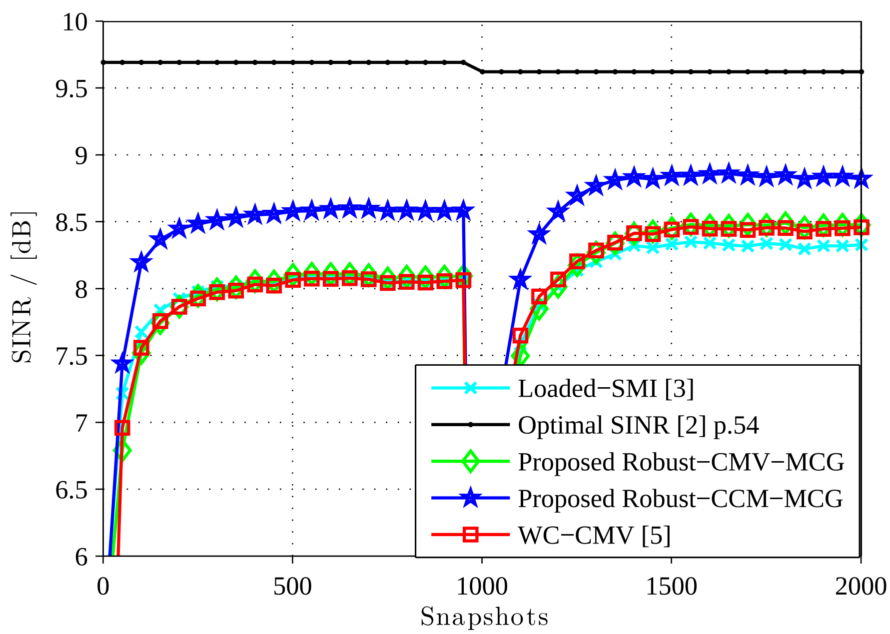

Fig. 6. SINR versus snapshots, local coherent scattering, SNR = 0dB and $|s_1| = 1$ and $\delta = 1$.

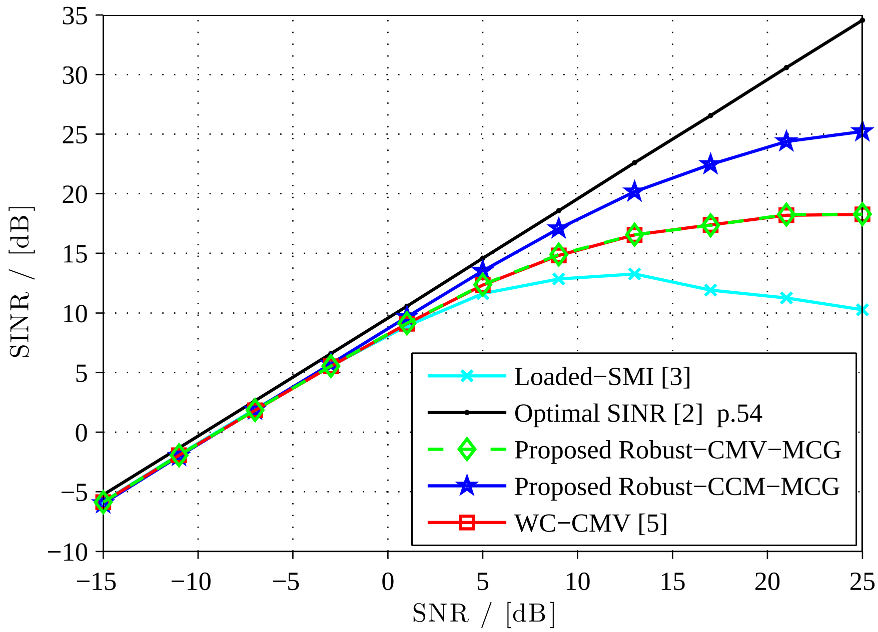

Fig. 7. SINR versus SNR, local coherent scattering, $i = 1500$, $M = 10$.

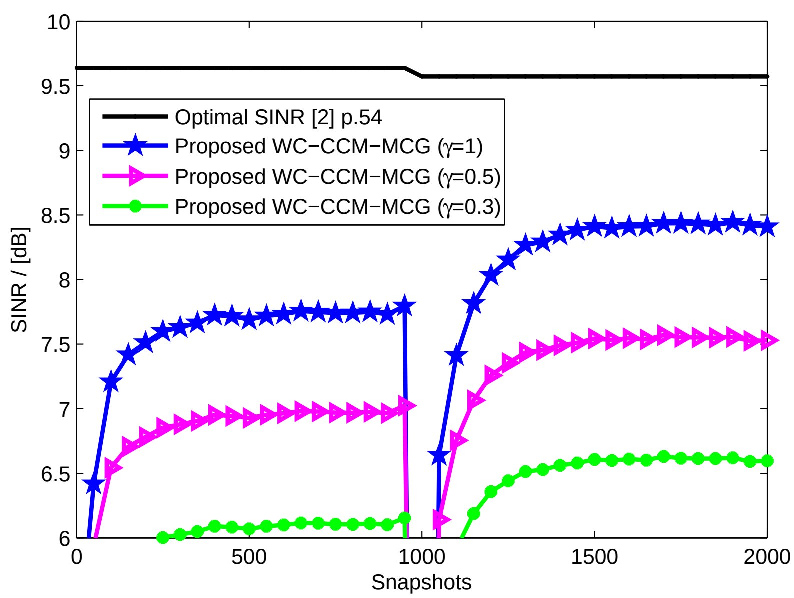

Fig. 8. SINR versus snapshots with different γ, local coherent scattering, SNR = 0dB, .

TABLE I

PROPOSED WC-CCM ALGORITHM

initialization: $\hat{\boldsymbol{R}}_a(0) = \sigma_n^2 \boldsymbol{I}; \hat{\boldsymbol{d}}(0) = \boldsymbol{0}; \boldsymbol{w}(0) = \frac{\boldsymbol{a}}{M}$

Update for each time instant i = 1,...,N

$$y(i) = \boldsymbol{w}^H(i-1)\boldsymbol{x}(i)$$

$$\hat{\boldsymbol{R}}_a(i) = \mu \hat{\boldsymbol{R}}_a(i-1) + |y(i)|^2 \, \boldsymbol{x}(i)\boldsymbol{x}^H(i)$$

$$\boldsymbol{R}_{\text{ac}}(i) = \text{chol}\left(\hat{\boldsymbol{R}}_a(i)\right)$$

$$\hat{\boldsymbol{d}}(i) = \mu \hat{\boldsymbol{d}}(i-1) + \boldsymbol{x}(i)y^*(i)$$

$$\boldsymbol{R}_{\text{acr}}(i) = \begin{bmatrix} \text{Re}\left\{\boldsymbol{R}_{\text{ac}}(i)\right\} & -\text{Im}\left\{\boldsymbol{R}_{\text{ac}}(i)\right\} \\ \text{Im}\left\{\boldsymbol{R}_{\text{ac}}(i)\right\} & \text{Re}\left\{\boldsymbol{R}_{\text{ac}}(i)\right\} \end{bmatrix}$$

$$\boldsymbol{d}_{\text{r}}(i) = \left[\text{Re}\left\{\hat{\boldsymbol{d}}(i)\right\}^T, \text{Im}\left\{\hat{\boldsymbol{d}}(i)\right\}^T\right]^T$$

$$\boldsymbol{p} = [1, \boldsymbol{0}^T]^T$$
$$\boldsymbol{f} = [1/2, 1/2, \boldsymbol{0}^T, -\delta, \boldsymbol{0}^T, 0]^T$$

$$\boldsymbol{F}^T = \begin{bmatrix} \frac{1}{2} & \gamma \boldsymbol{d}_{\text{r}}^T(i) \\ -\frac{1}{2} & -\gamma \boldsymbol{d}_{\text{r}}^T(i) \\ \boldsymbol{0} & \boldsymbol{R}_{\text{acr}}(i) \\ 0 & \breve{\boldsymbol{a}} \\ \boldsymbol{0} & \epsilon \boldsymbol{I} \\ 0 & \bar{\boldsymbol{a}} \end{bmatrix}$$

$$\min_{\boldsymbol{u}} \quad \boldsymbol{p}^T \boldsymbol{u} \quad \text{s. t.}$$

$$\boldsymbol{f} + \boldsymbol{F}^T \boldsymbol{u} \quad \in \text{SOC}_1^{2M+2} \times \text{SOC}_2^{2M+1} \times \{0\}$$

$$\boldsymbol{w}(i) = [\boldsymbol{u}_2, ..., \boldsymbol{u}_{M+1}]^T + j\,[\boldsymbol{u}_{M+2}, ..., \boldsymbol{u}_{2M+1}]^T$$

TABLE II

PROPOSED RCMV-MCG ALGORITHM

$\boldsymbol{v}(0) = \boldsymbol{0};\ \boldsymbol{p}(1) = \boldsymbol{g}(0) = \boldsymbol{a};\ \hat{\boldsymbol{R}}(0) = \delta\boldsymbol{I};\ \hat{\lambda}(0) = \hat{\lambda}(1) = \hat{\lambda}_0$
For each time instant $i = 1, ..., N$
$\hat{\boldsymbol{R}}_{xx}(i) = \mu\hat{\boldsymbol{R}}_{xx}(i-1) + \boldsymbol{x}(i)\boldsymbol{x}^H(i)$
$\boldsymbol{p}_R = [\hat{\boldsymbol{R}}_{xx}(i) + \hat{\lambda}(i)\tilde{\epsilon}\boldsymbol{I}]\boldsymbol{p}(i); \quad \nu = \left[\hat{\lambda}(i) - \mu\hat{\lambda}(i-1)\right]\tilde{\epsilon}$
$\alpha(i) = \left[\boldsymbol{p}^H(i)\boldsymbol{p}_R\right]^{-1}(\mu-\eta)\,\boldsymbol{p}^H(i)\boldsymbol{g}(i-1); \quad (0 \leq \eta \leq 0.5)$
$\boldsymbol{v}(i) = \boldsymbol{v}(i-1) + \alpha(i)\boldsymbol{p}(i)$
$\boldsymbol{g}(i) = [1-\mu]\boldsymbol{a} + \mu\boldsymbol{g}(i-1) - \alpha(i)\boldsymbol{p}_R$
$\quad - \left(\boldsymbol{x}(i)\boldsymbol{x}^H(i) + \nu\boldsymbol{I}\right)\boldsymbol{v}(i-1)$
$\beta(i) = \left[\boldsymbol{g}^H(i-1)\boldsymbol{g}(i-1)\right]^{-1}\left[\boldsymbol{g}(i) - \boldsymbol{g}(i-1)\right]^H \boldsymbol{g}(i)$
$\boldsymbol{p}(i+1) = \boldsymbol{g}(i) + \beta(i)\boldsymbol{p}(i)$
$\boldsymbol{w}(i) = \lambda(i)\boldsymbol{v}(i)/2$
$\delta_\lambda = \mu_\lambda[\tilde{\epsilon}\,\Vert\boldsymbol{w}(i)\Vert_2^2 - \mathrm{Re}\left\{\boldsymbol{w}^H(i)\boldsymbol{a}\right\} + \delta]$
while $\delta_\lambda \leq -\lambda(i)$ or $\delta_\lambda \geq \delta_{\lambda\max}$
$\quad \delta_\lambda \Rightarrow \delta_\lambda/2$
end
$\hat{\lambda}(i+1) = \hat{\lambda}(i) + \delta_\lambda$

TABLE III

PROPOSED RCCM-MCG ALGORITHM

$\boldsymbol{p}(1) = \boldsymbol{g}(0) = \boldsymbol{a};\ \hat{\boldsymbol{R}}_{\text{a}}(0) = \delta \boldsymbol{I};\ \hat{\boldsymbol{d}}(0) = \boldsymbol{0};$

$\hat{\lambda}(0) = \hat{\lambda}(1) = \hat{\lambda}_0;\ \boldsymbol{w} = \boldsymbol{a}/M$

For each time instant $i = 1, ..., N$

$$\hat{\boldsymbol{R}}_{\text{a}}(i) = \mu \hat{\boldsymbol{R}}_{\text{a}}(i-1) + |y(i)|^2\, \boldsymbol{x}(i)\boldsymbol{x}^H(i)$$

$$\boldsymbol{p}_R = [\hat{\boldsymbol{R}}_{\text{a}}(i) + \hat{\lambda}(i)\tilde{\epsilon}\boldsymbol{I}]\boldsymbol{p}(i); \quad \nu = \left[\hat{\lambda}(i) - \mu\hat{\lambda}(i-1)\right]\tilde{\epsilon}$$

$$\alpha(i) = \left[\boldsymbol{p}^H(i)\boldsymbol{p}_R\right]^{-1}(\mu - \eta)\,\boldsymbol{p}^H(i)\boldsymbol{g}(i-1); \quad (0 \le \eta \le 0.5)$$

$$\boldsymbol{w}(i) = \boldsymbol{w}(i-1) + \alpha(i)\boldsymbol{p}(i)$$

$$\boldsymbol{g}(i) = \mu\boldsymbol{g}(i-1) - \alpha(i)\boldsymbol{p}_R - \left(|y(i)|^2\, \boldsymbol{x}(i)\boldsymbol{x}^H(i)\right)\boldsymbol{w}(i-1)$$

$$+\gamma\boldsymbol{x}(i)y^*(i) + \nu\left[\boldsymbol{a}/\left(2\tilde{\epsilon}\right) - \boldsymbol{w}(i-1)\right]$$

$$\beta(i) = \left[\boldsymbol{g}^H(i-1)\boldsymbol{g}(i-1)\right]^{-1}\left[\boldsymbol{g}(i) - \boldsymbol{g}(i-1)\right]^H \boldsymbol{g}(i)$$

$$\boldsymbol{p}(i+1) = \boldsymbol{g}(i) + \beta(i)\boldsymbol{p}(i)$$

$$\delta_{\hat{\lambda}} = \mu_{\hat{\lambda}}[\hat{\epsilon}\,\|\boldsymbol{w}(i)\|_2^2 - \text{Re}\left\{\boldsymbol{w}^H(i)\boldsymbol{a}\right\} + \delta]$$

while $\delta_\lambda \le -\hat{\lambda}(i)$ or $\delta_\lambda \ge \delta_{\lambda\max}$

$\delta_\lambda \Rightarrow \delta_\lambda/2$

end

$$\hat{\lambda}(i+1) = \hat{\lambda}(i) + \delta_\lambda$$

TABLE IV

INTERFERENCE SCENARIO

	Snapshot	user 1 (desired user)	user 2	user 3	user 4	user 5
P(dB) relative to user1 / DoA	1-1000	0/93	13/120	1/140	22/67	10/157
	1001-2000	0/93	30/120	25/170	4/104	9/68

TABLE V

INTERFERENCE SCENARIO

	Snapshot	user 1 (desired user)	user 2	user 3	user 4	user 5
P(dB) relative to user1 / DoA	1-1000	0/93	10/120	5/140	10/150	7/105
	1001-2000	0/93	30/120	34/170	6/104	9/68