Information theory and error control coding/Teoria da informação e códigos corretores de erros

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Tutorial Questions/Lista de Exercícios - 3

1. Determine the channel capacity for the channel shown in the figure below.



1. Consider the following channel known as the Binary Erasure Channel



The probabilities of the input symols are P(X=0) = a and P (X=1) = 1 – a.

1. Determine the average mutual information I(X;Y) in bits
2. Determine the value of a that maximizes I(X;Y), i. e., the channel capacity C in bits/channel use and plot C as a function of p for the optimum value of a.
3. For the value of a found in b), determine the mutual information I(X,Y) = I (0;0), I(1;1), I(0;e) and I(1;e).

3. Prove that the mutual information of a channel is related to the joint entropy of the channel input $X$ and the channel output$ Y$ by

$I\left(X;Y\right)=H\left(X\right)+H\left(Y\right)-H(X;Y)$,

where the joint entropy $H(X;Y)$, is defined by

$$H\left(X;Y\right)=\sum\_{j=0}^{J-1}\sum\_{k=0}^{K-1}p\left(x\_{j}.y\_{k}\right)log\_{2}\left(\frac{1}{p\left(x\_{j}.y\_{k}\right)}\right)$$