Adaptive and Array Signal Processing/Processamento de Sinais Adaptativo

CETUC/PUC-Rio - Prof. Rodrigo de Lamare

Tutorial Questions/Lista de Exercícios - 2

1. In some applications of adaptive signal processing, the data collection process may be flawed so that there are either missing data values or outliers that should be discarded. Suppose that there are either missing data values or outliers that should be discarded. Suppose that we are given N samples of a WSS process $x(n)$ with one value $x(n\_{o})$ missing. Let $x$ be the vector containing the given sample values

$$x=\left[x\left(0\right), x\left(1\right), \cdots ,x\left(n\_{0}-1\right),x\left(n\_{0}+1\right), \cdots ,x(N)\right]^{T}$$

a) Let $R\_{x}$be the autocorrelation matrix for the vector $x$,

$$R\_{x}=E[xx^{H}]$$

Which of the following statements are true?

i)$ R\_{x}$ is Toeplitz.

ii) $R\_{x}$is Hermitian.

iii) $R\_{x}$is positive semidefinite.

b) Given the autocorrelation matrix for $x$, is it possible to find the autocorrelation matrix for the vector

$$x=\left[x\left(0\right), x\left(1\right),\cdots ,x(N)\right]^{T}$$

that does not have $x(n\_{o})$ missing? If so, how does one find it? If not explain why not.

2. A predictor of discrete-time waveforms can be built by forming an estimate of a sample$ n\_{0}$ samples later by observing p consecutive data samples. Consider the estimate of the predictor given by

$$\hat{x}\left(n+n\_{0}\right)=\sum\_{k=1}^{p}a\_{p}\left(k\right)x(n-k)$$

The predictor coefficients $a\_{p}\left(k\right)$ should be chosen to minimize

$$ε\_{p}=\sum\_{n=0}^{\infty }[x\left(n+n\_{0}\right)-\hat{x}\left(n+n\_{0}\right)]^{2}$$

a) Derive the equations required to compute the optimal set of coefficients.

b) If $n\_{0}=0$, how is the formulation of the problem different. Please explain.

3. A first-order real-valued autoregressive (AR) process $x(n)$ satisfies the difference equation

 $x\left(n\right)+a\_{1}x\left(n-1\right)=w(n)$,

where $a\_{1}$ is a constant and $w(n)$ is a white-noise process with variance $σ\_{w}^{2}$.

a) Show that if $w(n)$ has a nonzero mean, the AR process$ x(n)$ is nonstationary.

b) For the case when $w(n)$ has zero mean and the constant $a\_{1}$ satisfies the condition $\left|a\_{1}\right|<1$, show that the variance of $x(n)$ is given by $var\left[x\left(n\right)\right]=\frac{σ\_{n}^{2}}{1-a\_{1}^{2}}$

c)For the conditions in b), find the autocorrelation function of the AR process $x(n)$ . Sketch the autocorrelation function for the two cases $0<a\_{1}<1$ and $-1<a\_{1}<0$.