AdaptiveandArraySignalProcessing/Processamento de Sinais Adaptativo

CETUC/PUC-Rio - Prof. Rodrigo de Lamare

Tutorial Questions/Lista de Exercícios - 3

1. Consider a Wiener filtering problem characterised as follows: The correlation matrix of the input data vector is

The cross-correlation vector between and the desired signal is

1. Compute the Wiener filter using both analytical and numerical values.
2. What is the minimum mean-squared error produced by this Wiener filter?
3. Write down a representation of the Wiener filter in terms of eigenvalues of and associated eigenvectors.

2. The minimum mean-square error (MMSE) of a linear filter is defined by

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where is the variance of the desired response , is the correlation matrix of the input , and is the cross-correlation vector between and . By applying the unitary similarity transformation to the inverse of the correlation matrix, , show that

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whereis the kth eigenvalue of the correlation matrix , and is the corresponding eigenvector.

3. Consider the design of a three-step predictor using a first order filter in which the observed signal is the input to the filter , which generates the following minimum mean-square estimate of :

1. What are the Wiener-Hopf equations for the three-step predictor?
2. If the values of for lags and are find the optimum three-step predictor.
3. Compute the Wiener-Hopf equations and the optimum three-step predictor for the same values above using a second-order filter. Compare the mean-square error of the first-order and the second-order filters.

4. Consider a multi-antenna wireless communications system using N transmit and receive antennas as described below

**Tx**

**Rx**

This system can be modelled by a simple linear equation given by

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where is the N x N channel matrix, is the N x 1 vector of data symbols and is the N x 1 noise vector. The elements of the matrix are randomly generated using a complex Gaussian variable with zero mean and unit variance. The vector contains binary-phase shift-keying (BPSK) symbols that are randomly generated from a discrete random variable with zero mean, variance and its entries are independent and identically distributed (i.i.d.). The vector is a complex Gaussian random vector with zero mean and variance . The vectors andare assumed to be statistically independent.

1. Design a Wiener filter to perform linear equalisation on the received data,and write down the Wiener-Hopf equations.
2. Compute the minimum mean-square error of the Wiener filter.
3. Write a down a simulation code of a multi-antenna system that transmits packets of data between the Tx and the Rx and measure the bit error rate (BER) performance against the signal-to-noise ratio (SNR) defined by . Plot a curve showing the BER against the SNR for values of SNR between 0 and 20 dB.