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Master Thesis

Advanced Robust Adaptive Beamforming for Wireless Networks

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Abstract

This thesis investigate advanced algorithms for robust adaptive beamforming. A literatur survey is carried out along with a development of a number of innovative algorithms, which include algorithms based on the Constrained Constant Modulus design criterion, low-complexity algorithms using a joint optimization strategy based on the Modified Conjugate Gradient and algorithms based on the Low-Complexity Mismatch Estimation (LOCME). In addition an algorithm for distributed beamforming is developed which is based on the Pseudo-SNR.

The worst-case optimization based constrained constant moduls algorithm has been developed, which exploits the constant modulus property of the desired signal. In addition, a condition has been found which garantiees convexity of the problem. An alternative study has been done for the choice of the worst-case parameter. Simulations show that the performance is significantly improved compared to the existing approach.

The low-complexity methods termed Robust Constrained Constant Modulus based on Modified Conjugate Gradient Algorithms and the Robust Constrained Constant Modulus based on Modified Conjugate Gradient Algorithm are proposed. The new methods use a constraint which is very similar to the worst-case constraint. Instead of convex optimization tools, the algorithms use a recursive joint optimization strategy including a modified conjugate gradient method and the adjustment for the robust constraint in parallel. Unlike the convex optimization implementation, this method exploits previous computations, which reduces the computational complexity by more than an order of magnitude. Simulations show that the proposed methods perform equivalently or outperform the existing methods.

A more advanced method is given by the proposed Low-Complexity Mismatch Estimation (LOCME). This method estimates the imprecisely known array steering vector via a projection into a predefined subspace. The LOCME method is proposed within 4 different algorithms using robust constrained minimum variance and robust constrained constant modulus design criteria. Besides the solutions based on convex optimization also its low-complexity counterparts are presented. The simulations show superior performances of all proposed algorithms close to the optimum.

In addition, an alternative method for distributed beamforming is proposed. The method is based on the Pseudo-SNR and therefore is related to the maximum signal to noise ratio (MSNR) approach. Unlike the MSNR approach the new method allows each relay node to compute its own weight autonomously, requiring a very low

level of network collaboration. As a result, the proposed method benefits from local channel state information (CSI) which has a significantly higher accuracy compared to global CSI used in centralized algorithms. The requirement of network collaboration is significantly lower compared to the minimum mean squared error (MMSE) based consensus algorithm which results in a higher spectral efficiency. Simulations show a comparable performance to the MSNR and an improved performance compared to the MMSE based consensus algorithm.

Contents

1 Introduction

In order to fullfil requirements with limited resources, wireless communications and other applications have found methods to exploit the spatial domain in a more efficient way. A remarkable technique in this area is given by beamforming methods. Beamforming approaches for multisensor configurations can be used to receive, transmit or relay signals of interest in the presence of interference and noise. The introduction is organized to provide the reader with an overview to the whole domain similar to [GSS+10].

The receive beamforming approach has been established many decades ago and is still a continuously developing field. It has found many applications in radar, sonar, communications, microphone array speech/audio processing, biomedicine, radio astronomy, seismology and other areas. In wireless communications, multiantenna techniques have become a key technology to deal with the increasing number of users and their ever increasing requirements in data rates, which renewed the interest in beamforming in the last decade. A significant progress has been done in the field of receive beamforming taking advantage of convex optimization. While the conventional methods are quite sensitive against mismatches in the so-called array steering vector (ASV), the presumed signal signature on the sensor array, recently reported algorithms [VGL03] take that into account due to the worst-case optimization-based method. In this class of algorithms robust design criteria are reformulated into convex optimization problems which are finally solved via interior point methods or other appropriate techniques. Besides the worst-case optimization-based approach probabilistically constrained algorithms [VGR07] have been recently established employing convex optimization. Both the worst-case optimization-based and the probability constrained approach have been developed also for the case of multiuser receivers for space-time coded multiple-input multiple-output (MIMO) systems [RSG05],[RVG06]. Another class of robust beamforming algorithms employs designs to obtain an estimate of the array steering vector with its mismatch [HV08].

Also in the field of transmit beamforming significant progress has been made recently. The classical transmit beamforming design means a large inner product of the weight vector and a single array steering vector of interest while the inner products of the weight vector and other array steering vectors are small. Different from that, robust designs employ a set of array steering vectors around the presumed array steering vector. There are also multiple transmit beamforming approaches [RFLT98],[FN98], whose weight vectors are jointly designed according to a balanced interference between different transmissions, or according to an acceptable quality-of-service to each user and a minimized radiated power. Later these approaches have been extended for their robust worst-case optimization-based designs [BO01] using convex optimization and also for their outage probability-constrained design, which provides more flexibility.

In traditional TV and radio broadcast systems the signals are transmitted whether isotropically or with a fixed beampattern to cover a predefined spatial area, caused by the fact that the receivers do not provide feedback to the transmitter. Different from that, modern digital wireless networks, have often access to some level of channel state information (CSI) at the transmitter. Multicast beamforming can exploit this knowledge to improve network reach, coverage, quality-of-service, spectral efficiency and interference.

Relay network beamforming employs a ”virual array” of relay nodes, which retransmits appropriate weighted signals, exploiting different levels of network cooperation [JJ07]. One approach is based on the amplify and forward protocol, where the relay network uses adaptive complex-valued relay weights. There are also advanced schemes where the decode-and-forward protocol can be used. Caused by the fact that the relays can hardly exchange information, these beamforming methods are performed in a distributed way. Besides some other methods and concepts the original approach has been extended for the multiuser scheme, bidirectionality and the filter-and-forward. Often these difficult designs can be reformulated in a way which allows the exploitation of convex optimization with an acceptable computational burden.

1.1 Receive Beamforming

The output of the narrowband beamformer is defined as

$$y = \boldsymbol{w}^H \boldsymbol{x}(i), \tag{1.1}$$

where $\boldsymbol{w} \in \mathbb{C}^{M\times 1}$ is the complex-valued beamforming weight vector and $\boldsymbol{x}(i) \in \mathbb{C}^{M\times 1}$ is the array observation vector at the time index i. The array observation vector can be modeled as

$$\boldsymbol{x}(i) = s_1(i)\boldsymbol{a}_1 + \sum_{m=2}^{D} s_m(i)\boldsymbol{a}_m + \boldsymbol{n}(i), \tag{1.2}$$

where s_1 is the desired signal with its array steering vector $\boldsymbol{a}_1 \in \mathbb{C}^{M\times 1}$, s_m and $\boldsymbol{a}_m$ correspond to the interference and $\boldsymbol{n}(i)$ denotes the additive noise, assumed to be zero-mean, complex Gaussian. The signal model can also be written in a more compact form as

$$\boldsymbol{x}(i) = \boldsymbol{A}\boldsymbol{s}(i) + \boldsymbol{n}(i), \tag{1.3}$$

where $\boldsymbol{A} \in \mathbb{C}^{M\times D}$ contains the array steering vectors and $\boldsymbol{s} \in \mathbb{C}^{D\times 1}$ contains the signals.

In general the beamforming weights are designed to improve the signal-to-interference-plus-noise ratio (SINR) at the beamforming output. The SINR is defined as

$$\text{SINR} = \frac{\boldsymbol{w}^H \boldsymbol{R}_{\text{s}} \boldsymbol{w}}{\boldsymbol{w}^H \boldsymbol{R}_{\text{i+n}} \boldsymbol{w}}, \tag{1.4}$$

where $\boldsymbol{R}_{\text{s}}$ is the covariance matrix due to the desired signal and $\boldsymbol{R}_{\text{i+n}}$ is the interference-plus-noise-covariance matrix. While the perfect covariance matrices are available the weight vector according to the optimal SINR can be obtained as [LS06]

$$\boldsymbol{w}_{\text{opt}} = \mathcal{P}\left\{\boldsymbol{R}_{\text{i+n}}^{-1}\boldsymbol{R}_{\text{s}}\right\}, \tag{1.5}$$

where $\mathcal{P}\{.\}$ is the operator which returns the principal eigenvector. In applications, the true covariance matrices are not available and therefore estimates according to training data are used. The sample estimate of the signal covariance matrix is defined as $\hat{R}_{xx,\text{sample}} = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{x}(i)\boldsymbol{x}^H(i)$, while in adaptive algorithms often the exponentially decayed data window estimate is used which is computed as $\hat{R}_{xx}(i) = \mu\hat{R}_{xx}(i-1) + \boldsymbol{x}(i)\boldsymbol{x}^H(i)$. Unlike the sample estimate, the exponentially decayed data window estimate is asymptotically biased. When $\boldsymbol{R}_{\text{s}}$ can be assmued to have rank one and the corresponding array steering vector is perfectly known as $\boldsymbol{a}$ the optimal weight vector can be approximately obtained as the solution to the optimization problem

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \hat{\boldsymbol{R}}_{xx} \boldsymbol{w} \;\; \text{s. t. } \boldsymbol{w}^H \boldsymbol{a} = 1. \tag{1.6}$$

Its closed form solution is $\boldsymbol{w} = [\hat{\boldsymbol{R}}_{xx}^{-1}\boldsymbol{a}]/[\boldsymbol{a}^H\hat{\boldsymbol{R}}_{xx}^{-1}\boldsymbol{a}]$, while its scaled expression $\boldsymbol{w} = \hat{\boldsymbol{R}}_{xx}^{-1}\boldsymbol{a}$ provides the same SINR. In practical scenarios, circumstances like imperfectly calibrated arrays, local scattering and imprecisely known wavefield propagation cause a mismatch of the presumed array steering vector $\boldsymbol{a}$ of the desired signal. To avoid dramatical performance degradations some robust approaches have been developed in the last decades and the author of this disertation also did which is reported in chapter 2 and chapter 3. The most common way to improve robustness is to add a diagonal loading to the signal covariance matrix. However, there is no easy and reliable way to choose the diagonal loading factor which is scenario dependent. Solutions are given by the worst-case performance based methods, which enjoy a higher grade of theoretical reliability compared to the traditional diagonal loading approach. Besides this class of robust beamfoming algorithms numerous others approaches have been developed and some of them are introduced in the beginning of the second chapter.

Whereas the most critical issue corresponds to the array steering vector mismatch which can cause the so-called signal self nulling effect, there are also approaches providing robustness to other kinds of imprecision. The signal covariance tapering technique gives a solution to widen the nulls in the beampattern of the beamforming weight vector, which is advantageous in the presence of moving interferers. To achieve this effect the signal covariance matrix is elementwise multiplied with the taper matrix which is denoted as $\boldsymbol{T}$

$$\boldsymbol{R}_{\mathrm{T}} = \boldsymbol{R}_{xx} \odot \boldsymbol{T}, \tag{1.7}$$

where the most commonly used taper matrix has been reported in [Mai95] as $\boldsymbol{T} = \frac{\sin(\pi(l-m)\gamma)}{\pi(l-m)\gamma}$, $l, m = 1...M$ and γ determines the width of the nulls in the beampattern. Later it has been shown in [Gue99] that the following method has the equivalent effect

$$\boldsymbol{R}_{\mathrm{T}} = \mathrm{E}\{[\boldsymbol{x}(i) \odot \boldsymbol{e}(\omega)][\boldsymbol{x}(i) \odot \boldsymbol{e}(\omega)]^H\}, \tag{1.8}$$

where $\boldsymbol{e}(\omega) = [1e^{j\omega}...e^{j(M-1)\omega}]^T$ and ω is a dithering variable with the property $\omega \sim \mathcal{U}(-\gamma\pi, \gamma\pi)$. Another technique providing robustness against moving interferers is given by the well established generalized sidelobe canceller.

Another remarkable uncertainty is given by scenarios with just a small number of snapshots as a training data. Recently a solution is reported in [DLS10] where the author computes a certain level of diagonal loading based on minimizing the mean squared error (MSE) of a modified estimate of the signal covariance matrix. The

Table 1.1: Robust Receive Beamforming Principles

Issue:	Moving Interferers	Small Sample Size	Array-Steering-Vector Mismatch
Impact:	Interferer do not match to nulls	Imprecise noise characteristics	Signal-Self-Nulling
Popular Methods:	Matrix-Tapering, Gen. Sidelobe Canceller	Automatically Computed Diagonal Loading Level	Worst-Case Optimization

resulting optimization problem can be cast as

$$\min_{\alpha_0,\beta_0} \; \mathrm{E}\{||\tilde{\boldsymbol{R}}_{xx} - \boldsymbol{R}_{xx}||^2\}, \tag{1.9}$$

where $\boldsymbol{R}_{xx}$ is the true signal covariance matrix and $\tilde{\boldsymbol{R}}_{xx} = \alpha_0 \boldsymbol{I} + \beta_0 \hat{\boldsymbol{R}}_{xx}$ with $\hat{\boldsymbol{R}}_{xx}$ as the sample matrix estimate. Due to the fact that the true signal covariance matrix is not available the solution can be achieved with the following approximation

$$\begin{aligned} \alpha_0 &= \hat{\nu}\frac{\hat{\rho}}{||\hat{\boldsymbol{R}}_{xx} - \hat{\nu}\boldsymbol{I}||^2} \\ \beta_0 &= 1 - \frac{\hat{\rho}}{||\hat{\boldsymbol{R}}_{xx} - \hat{\nu}\boldsymbol{I}||^2}, \end{aligned} \tag{1.10}$$

where $\hat{\rho} = \frac{1}{N^2}\sum_{i=1}^{N} ||\boldsymbol{x}(i)||^4 - \frac{1}{N}\hat{\boldsymbol{R}}_{xx}$ and $\hat{\nu} = \mathrm{tr}(\hat{\boldsymbol{R}}_{xx})/M$. Even though the approach corresponds to the class of diagonal loading, it is not comparable with the commonly used techniques against array steering vector mismatch due to the fact that the loading level is decreasing with the number of training data samples. To give a brief overview, Table 1.1 summarizes different robust beamfoming schemes with its main properties.

1.2 Transmit Beamforming

Downlink Beamforming

Considering a base station with multiple antennas, transmitting different data streams to a set of D users, where each is equiped with a single antenna. The transmitted signal is denoted as

$$\boldsymbol{x}(i) = \sum_{m=1}^{D} s_m(i)\boldsymbol{w}_m, \tag{1.11}$$

where s_m is the data stream for the user m and $\boldsymbol{w}_m$ is the corresponding beamforming weight vector. Introducing the channel $\boldsymbol{h}_m$, the received signal at user m can be modeled as

$$y(i) = \boldsymbol{h}_m^H \boldsymbol{x}(i) + \boldsymbol{n}_m(i), \tag{1.12}$$

where $\boldsymbol{n}$ is the additive noise. To obtain appropiate beamforming weight vectors, the design criterion according to the so-called SINR balancing method can be used. In that formulation the SINR values corresponding to the users are constrained, while the overall transmitted power is to be minimized. While the channel vectors are known at the base station the optimization problem can be cast as

$$\begin{aligned} &\min_{\boldsymbol{w}_{m=1\ldots D}} \sum_{m=1}^{D} \|\boldsymbol{w}_m\|_2^2 \\ &\text{s. t.} \ \frac{\left|\boldsymbol{w}_k^H \boldsymbol{h}_k\right|^2}{\sum_{l\neq k}^{D} \left|\boldsymbol{w}_l^H \boldsymbol{h}_k\right|^2 + \sigma_k^2} \geq \gamma_k \quad \forall k = 1, ..., D, \end{aligned} \tag{1.13}$$

where σ_k^2 is the noise power user k is receiving and γ_k is the desired minimum SINR for user k. The weight vector can be phase rotated in such a way that $\boldsymbol{w}_k^H \boldsymbol{h}_k$ is real valued and positive. As a result the problem can be reformulated as a second order cone program (SOCP). Since instantaneous CSI is often not available, a more practical formulation is based on the second order statistics of the channel $\boldsymbol{R}_m = \mathrm{E}\left\{\boldsymbol{h}_m \boldsymbol{h}_m^H\right\}$. That leads to the following problem

$$\begin{aligned} &\min_{\boldsymbol{w}_{m=1\ldots D}} \sum_{m=1}^{D} \|\boldsymbol{w}_m\|_2^2 \\ &\text{s. t.} \ \frac{\boldsymbol{w}_k^H \boldsymbol{R}_k \boldsymbol{w}_k}{\sum_{l\neq k}^{D} \left|\boldsymbol{w}_l^H \boldsymbol{R}_k \boldsymbol{w}_l\right|^2 + \sigma_k^2} \geq \gamma_k \quad \forall k = 1, ..., D. \end{aligned} \tag{1.14}$$

However, the problem is nonconvex in that formulation. There are different strategies which allows it to be solved. One of them is based on semidefinite relaxation, where the problem can be reformulated as a semidefinte programm (SDP).

Multicast Beamforming

In this scheme the base station is broadcasting a single data stream to a set of D users.

The corresponding SNR-balancing problem can be cast as

$$\min_{\boldsymbol{w}} \|\boldsymbol{w}\|_2 \quad \text{s. t.} \quad \left|\boldsymbol{w}^H \boldsymbol{h}_m\right|^2 \geq \sigma_m^2 \gamma_m \quad \forall m = 1, ..., D \tag{1.15}$$

where γ_m is the desired minimum SNR for user m and σ_m^2 is the variance of the additive noise. While absorbing the SNR consumption and the noise power level in the channel expression $\boldsymbol{h}_m$, there is an alternatively max-min-fair formulation:

$$\max_{\boldsymbol{w}} \min_{m \in \{1,...,D\}} \left|\boldsymbol{w}^H \boldsymbol{h}_m\right|^2 \quad \text{s. t.} \quad \|\boldsymbol{w}\|_2^2 = P, \tag{1.16}$$

where P is the transmission power. This problem can be NP-hard, especially for the case of $D \geq M$. One strategy is based on semidefinite relaxation. Here $\left|\boldsymbol{w}^H \boldsymbol{h}_m\right|^2$ is replaced by $\mathrm{Tr}\{\boldsymbol{W}\boldsymbol{R}_m\}$ and $\|\boldsymbol{w}\|_2^2$ is replaced by $\mathrm{Tr}\{\boldsymbol{W}\}$. The corresponding SDP can be cast as

$$\max_{\boldsymbol{W}} \min_{m \in \{1,...,D\}} \mathrm{Tr}\{\boldsymbol{W}\boldsymbol{R}_m\} \quad \text{s. t.} \ \mathrm{Tr}\{\boldsymbol{W}\} = P, \ \boldsymbol{W} \geq \boldsymbol{0}, \tag{1.17}$$

where $\boldsymbol{W} \geq \boldsymbol{0}$ means $\boldsymbol{W}$ is positive semidefinite. The formulation is not an approximation, if $\mathrm{rank}\{\boldsymbol{W}\} = 1$ holds. In the rank one case the desired weight vector is simply the principal eigenvector of $\boldsymbol{W}$. In other cases a rank one approximation due to $\boldsymbol{W}$ has to be found. However in that case it has been shown, that there are better approximation strategies compared to the choice of the principal eigenvector.

1.3 Relay Network Beamforming

In this scheme the source and the destination have data transmission using a relay network of D nodes in between. In this approach the complex channel path gains between the source and the network nodes are denoted as $f_1...f_D$ and the weights between the network and the destination are termed as $g_1...g_D$. In the early approaches, CSI is assumed to be known perfectly at the destination or the relays. Later approaches are based on second-order statistics of the channels. The signals are weighted at the relay nodes, which can be described as receive beamforming and transmit beamforming at the same time.

Following the amplify-and-forward protocol, the signals received at the relay nodes can be modeled as

$$x_m = \sqrt{P_0} f_m s + n_m, \tag{1.18}$$

where P_0 is the transmitting power, s is the transmitted symbol and n_m is the received noise at the corresponding relay node. While the relay nodes forward the weighted signals, finally the received signal at the destination node can be cast as

$$y = \sum_{m=1}^{D} g_m w_m (\sqrt{P_0} f_m s + n_m) + n_0, \tag{1.19}$$

where w_m is the complex relay weight and n_0 is the received noise at the destination node. A reasonable design criterion to design the weight vector is given by the SNR maximization due to predefined power constraints. Introducing $\boldsymbol{R} = P_0 \, \mathrm{E}\left\{[\boldsymbol{f} \odot \boldsymbol{g}][\boldsymbol{f} \odot \boldsymbol{g}]^H\right\}$ with $\boldsymbol{g} = [g_1, ..., g_D]^T$ and $\boldsymbol{f} = [f_1, ..., f_D]^T$ and $\boldsymbol{Q} = \sigma_{n1}^2 \mathrm{E}\left\{\boldsymbol{g}\boldsymbol{g}^H\right\}$, the beamforming design can be cast as

$$\max_{\boldsymbol{w}} \frac{\boldsymbol{w}^H \boldsymbol{R} \boldsymbol{w}}{\sigma_0^2 + \boldsymbol{w}^H \boldsymbol{Q} \boldsymbol{w}} \quad \text{s. t. } D_{mm} \left|w_m\right|^2 \leq P_m \forall\; m = 1, ..., D, \tag{1.20}$$

where D_{mm} corresponds to $\boldsymbol{D} = P_0 \,\mathrm{diag}\left\{\mathrm{E}\left\{\left|f_1\right|^2\right\}, ..., \mathrm{E}\left\{\left|f_D\right|^2\right\}\right\} + \,\mathrm{diag}\left\{\sigma_1^2, ..., \sigma_D^2\right\}$. The problem can be simplified using the semidefinite relaxation strategy, while replacing $\boldsymbol{X} = \boldsymbol{w}\boldsymbol{w}^H$ and skipping the constraint $\mathrm{rank}\left\{\boldsymbol{X}\right\} = 1$. Finally the optimization problem can be reformulated in a reduced form as

$$\begin{aligned} &\max_{t, \boldsymbol{X}} t \\ &\text{s. t. } \mathrm{Tr}\left\{\boldsymbol{R}\boldsymbol{X} - t\boldsymbol{Q}\boldsymbol{X}\right\} \geq t\sigma_{n2}^2 \\ &X_{mm} \leq P_m / D_{mm} \forall\; m = 1, ..., D, \boldsymbol{X} \geq \boldsymbol{0}, \end{aligned} \tag{1.21}$$

which is feasible for a fixed value of t, while $t_{\mathrm{opt}} \geq t$ holds. Using this relation, t_{opt} can be adjusted and as a result $\boldsymbol{X}$. According to numerical results, $\mathrm{rank}\left\{\boldsymbol{X}\right\} = 1$ holds, which means the semidefinite relaxation is not an approximation, but there is still no proof for that observation. However, the method according to equation (1.20) implies synchronized relay nodes and flat flading channel. In the presence of frequency selective channels there are two different approaches to take that into account. One is to employ the filter-and-forward protocol, which includes an FIR filter in the relay nodes, for compensating the distortions introduced by the channel. Here the burden is put on the relay nodes. There are also other approaches which treat the relay network like an artificial multipath channel with conventional methods known from orthogonal frequency division multiplex (OFDM) schemes. This strategy implies that the burden is given to the destination and source node. There are also approaches developed for the multiuser case. For less or equal than three users, it is proven that involving semidefinite relaxation leads to the exact solution.

2 Robust Adaptive Beamforming Algorithms

Besides a review on existing algorithms on robust receive beamforming this chapter presents the proposed adaptive beamforming algorithm termed worst-case optimization-based approach using the constrained constant modulus design which exploits the constant modulus property of the desired signal and the proposed low-complexity robust adaptive beamforming algorithms based on joint-optimization and Modified Conjugate Gradient (MCG) methods.

2.1 Review

This is a review on a few notable approaches to the design of robust adaptive beamforming.

Loaded Sample Matrix Inversion

The most common robust approach is the so-called loaded sample matrix inversion (loaded-SMI) beamformer [Car88],[CZO87], which includes an additional diagonal loading to the signal covariance matrix. The main problem is how to obtain the optimal diagonal loading factor. Typically it is chosen as 10 σ_n^2, where σ_n^2 is the noise power.

Eigenvector-Based Approach

Another robust approach is given by the eigenvector-based beamformer [FG94]. Here the presumed array steering vector is replaced by its projection onto the signal-plus-interferer subspace. While the signal-plus-interferer subspace has a low rank, the method provides an array steering vector with improved accuracy. However, the approach implies that the noise subspace can be identified exactly, which leads to a limitation in high SNR.

Reduced-Rank Approach

A similar method to the Eigenvector-based approach is given by the reduced-rank beamforming approach [dLWF10], which avoids an eigen-decomposition and exploits the low rank, denoted as r, of the signal-plus-interference subspace. The method is based on the following optimization problem

$$\begin{aligned} &\min_{\bar{\boldsymbol{w}},\boldsymbol{S}} \bar{\boldsymbol{w}}^H \boldsymbol{S}^H \boldsymbol{R}_{xx} \boldsymbol{S} \bar{\boldsymbol{w}} \\ &\text{s. t. } \bar{\boldsymbol{w}}^H \boldsymbol{S}^H \boldsymbol{a} = 1, \end{aligned} \tag{2.1}$$

where $\bar{\boldsymbol{w}} \in \mathbb{C}^{r\times 1}$ is the reduced rank weigth vector with reduced dimension and the columns of $\boldsymbol{S} \in \mathbb{C}^{M\times r}$ span the signal-plus-interference subspace. This optimization problem can be solved using a joint optimization strategy where the $\bar{\boldsymbol{w}}$ and $\boldsymbol{S}$ are adjusted alternately while exchanging information.

Worst-Case Optimization-Based Approach

The popular worst-case performance optimization-based beamformer is based on a constraint that the absolute value of the array response is always greater or equal to a constant for all vectors that belong to a predefined set of vectors in the neighborhood of the presumed vector.

$$\left|\boldsymbol{w}^H(\boldsymbol{a}+\boldsymbol{e})\right| = \left|\boldsymbol{w}^H\boldsymbol{a}+\boldsymbol{w}^H\boldsymbol{e}\right| \geq \delta, \quad \forall \quad (\boldsymbol{e}+\boldsymbol{a}) \in \mathcal{A} \tag{2.2}$$

In [VGL03] the set of vectors is a sphere $\mathcal{A} = \{\boldsymbol{a}+\boldsymbol{e}, \|\boldsymbol{e}\|_2 \leq \epsilon\}$, where the norm of $\boldsymbol{e}$ is upper-bounded by ϵ. Therefore the corresponding optimization problem can be cast as

$$\begin{aligned} &\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{R}_{xx} \boldsymbol{w} \\ &\text{s. t. } \left|\boldsymbol{w}^H\left(\boldsymbol{a}+\boldsymbol{e}\right)\right| \geq \delta, \quad \forall \left(\boldsymbol{a}+\boldsymbol{e}\right) \in \mathcal{A}(\epsilon), \end{aligned} \tag{2.3}$$

where $\boldsymbol{R}_{xx} = \mathrm{E}\left\{\boldsymbol{x}\boldsymbol{x}^H\right\}$ is the covariance matrix of the input signal. While applying the triangle and the Cauchy-Schwarz inequalities

$$\left|\boldsymbol{w}^H\boldsymbol{a}+\boldsymbol{w}^H\boldsymbol{e}\right| \geq \left|\boldsymbol{w}^H\boldsymbol{a}\right| - \left|\boldsymbol{w}^H\boldsymbol{e}\right| \geq \left|\boldsymbol{w}^H\boldsymbol{a}\right| - \epsilon \left\|\boldsymbol{w}\right\|_2, \tag{2.4}$$

a lower bound can be introduced. The overall problem does not change if $\boldsymbol{w}^H\boldsymbol{a}$ undergoes an arbitary phase rotation and so it is allowed to set the imaginary part to zero which results in the corresponding convex SOC problem:

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{R}_{xx} \boldsymbol{w} \quad \text{s. t. } \operatorname{Re}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} - \delta \geq \epsilon \left\|\boldsymbol{w}\right\|_2 \qquad \operatorname{Im}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} = 0 \ . \tag{2.5}$$

It has been shown that this kind of beamformer is related to the class of diagonal loading. In [LB05] the set of vectors in the neighborhood can be ellipsoidal as well.

Probability-Constrained Approach

Another notable idea is the probability-constrained approach [VGR07]. Here the constraint satisfies operational conditions that are more likely to occur.

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{R}_{xx} \boldsymbol{w} \quad \text{s. t. } \Pr\left\{\left|\boldsymbol{w}^H \left(\boldsymbol{a} + \boldsymbol{e}\right)\right| \geq \delta\right\} \geq p, \tag{2.6}$$

where Pr denotes the probability operator and p is the desired probability threshold. Here different assumptions on the statistical characteristics of the mismatch-vector $\boldsymbol{e}$ lead to different problem formulations. The solutions for the Gaussian probability density function (pdf) case and the general unknown pdf case are developed in [VGR07].

Robust-Capon-Approach

The robust Capon approach [LS03] is based on an estimation of the array steering vector, which is assumed as a point in a predefined set. The estimate can be obtained by solving

$$\max_{\sigma^2, \boldsymbol{a}} \sigma^2 \quad \text{s. t. } \boldsymbol{R}_{xx} - \sigma^2 \boldsymbol{a}\boldsymbol{a}^H \geq 0, \quad \boldsymbol{a} \in \epsilon_M(\bar{\boldsymbol{a}}, \boldsymbol{E}), \tag{2.7}$$

where $\sigma^2 \boldsymbol{a}\boldsymbol{a}^H$ represents the estimate of the true covariance matrix corresponding to the signal of interest and the ellipsoidal set ϵ_M is described with its center the presumed array steering vector $\bar{\boldsymbol{a}}$ and the positive semidefinite matrix $\boldsymbol{E}$.

$$\epsilon_M(\bar{\boldsymbol{a}}, \boldsymbol{E}) = \left\{\boldsymbol{a} \in \mathbb{C}^M \,\middle|\, [\boldsymbol{a} - \bar{\boldsymbol{a}}]^H \boldsymbol{E} [\boldsymbol{a} - \bar{\boldsymbol{a}}] \leq 1\right\} \tag{2.8}$$

The problem can be reduced to

$$\min_{\boldsymbol{a}} = \boldsymbol{a}^H \boldsymbol{R}_{xx}^{-1} \boldsymbol{a} \quad \text{s. t. } \boldsymbol{a} \in \epsilon_M(\bar{\boldsymbol{a}}, \boldsymbol{E}) \tag{2.9}$$

Sequential Quadratic Programming

Another recently developed approach [HV08] identifies the mismatch vector $\boldsymbol{e}$ in the imprecisely known array steering vector given by $\boldsymbol{a} = \boldsymbol{a}(\theta_\mathrm{d})$ which is a function of the presumed direction. The approach uses sequential quadratic programming (SQP) to obtain an estimate with a higher accuracy. In order to do so, the authors compute $\boldsymbol{C} = \int_\Theta \boldsymbol{a}(\theta)\boldsymbol{a}^H(\theta)d\theta$, where $\Theta = [\theta_1, \theta_2]$ represents the range of the angular location of the desired signal. They also compute its counterpart, defined as $\bar{\boldsymbol{C}} = \int_{\bar{\Theta}} \boldsymbol{a}(\theta)\boldsymbol{a}^H(\theta)d\theta$, where $\bar{\Theta}$ reprensents the range outside of the angular location of the desired signal. In addition, $\boldsymbol{P} = \boldsymbol{I} - \boldsymbol{U}\boldsymbol{U}^H$, while $\boldsymbol{U}$ contains the K normalized prinicipal eigenvectors of $\boldsymbol{C}$.

With that development the constrained energy maximization problem can be cast as

$$\begin{aligned} \min_{\boldsymbol{e}} \;\; & (\boldsymbol{a}+\boldsymbol{e})^H \boldsymbol{R}_{xx}^{-1} (\boldsymbol{a}+\boldsymbol{e}) \\ \text{s. t. } & \boldsymbol{P}(\boldsymbol{a}+\boldsymbol{e}) = 0, \;\; \|\boldsymbol{a}+\boldsymbol{e}\|_2 \leq \sqrt{M} + \delta \\ & \boldsymbol{a}^H\boldsymbol{e} = 0, \;\; (\boldsymbol{a}+\boldsymbol{e})^H \bar{\boldsymbol{C}} (\boldsymbol{a}+\boldsymbol{e}) \leq \zeta, \end{aligned} \tag{2.10}$$

where $\zeta = \boldsymbol{a}^H\bar{\boldsymbol{C}}\boldsymbol{a}$ and δ determine the maximum stepsize. The solution of the optimization problem provides an estimate of the array steering vector mismatch. While the energy has improved, which means $(\boldsymbol{a}+\boldsymbol{e})^H\boldsymbol{R}_{xx}^{-1}(\boldsymbol{a}+\boldsymbol{e}) \leq \boldsymbol{a}^H\boldsymbol{R}_{xx}^{-1}\boldsymbol{a}$, the array steering vector can be updated as follows $\boldsymbol{a} \Rightarrow \sqrt{M}\frac{\boldsymbol{a}+\boldsymbol{e}}{\|\boldsymbol{a}+\boldsymbol{e}\|_2}$ and the optimization problem can be solved again.

Even if the method provides an array steering vector estimate with high accuracy, it implies a matrix inversion and most notably a multiple optimization problem which is not desirable for real time applications.

2.2 Worst-Case Criterion using the CCM Design

Unlike the worst-case optimzation-based approach [VGL03] combined with the constrained minimum variance design criterion this section describes the proposed worst-case optimization-based approach using the constrained constant modulus [dLSN05] design criterion (WC-CCM), which provides a better performance exploiting the constant modulus property of the desired signal. Besides the design development and its implementation this section includes an additional analysis about the adjustment of the ϵ parameter in the robust constraint function and the analysis of the optimization problem. As a result a condition has been found which garanties convexity of the problem. The content of this section is published by the author in [LdLH11b].

2.2.1 Proposed Design

The proposed beamformer is based on the worst-case approach. In case of the minimum variance design it can be derived from the following optimization problem

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{R}_{xx} \boldsymbol{w} \quad \text{s. t. } \operatorname{Re}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} - \delta \geq \epsilon \left\|\boldsymbol{w}\right\|_2 \tag{2.11}$$

$$\operatorname{Im}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} = 0 \; , \tag{2.12}$$

where ϵ is the level of steering vector mismatch, which is assumed as known a priori. The proposed beamformer uses the constant modulus criterion, which takes advantage of the knowledge of the signal amplitude of the desired user, instead of the minimum variance design criterion. Its objective function is defined by

$$J = \operatorname{E}\left\{\left(|y|^2 - \gamma\right)^2\right\} , \tag{2.13}$$

where $\gamma \geq 0$ which is a parameter related to the energy of the signal. A closed-form solution which will minimize the constant modulus cost function appears to be impossible because it is a fourth-order function with a more complicated structure [LS06]. However, the optimization problem can be solved iteratively. Therefore the constant modulus cost function can be written as follows

$$J = \operatorname{E}\left\{|y|^2 \boldsymbol{w}^H \boldsymbol{x}\boldsymbol{x}^H \boldsymbol{w} - \gamma y^* \boldsymbol{w}^H \boldsymbol{x} - \gamma \boldsymbol{x}^H \boldsymbol{w} y + \gamma^2\right\} . \tag{2.14}$$

Reformulating (2.14) leads to the equivalent expression

$$\hat{J} = \boldsymbol{w}^H \mathrm{E}\left\{ |y|^2 \boldsymbol{x}\boldsymbol{x}^H \right\} \boldsymbol{w} - 2\gamma \,\mathrm{Re}\left\{ \boldsymbol{w}^H \mathrm{E}\left\{ y^* \boldsymbol{x} \right\} \right\}, \tag{2.15}$$

where y denotes the output which is computed with the previously computed weight vector. In combination with the worst-case constraint, the proposed WC-CCM design can be cast as the following optimization problem

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{R}_a \boldsymbol{w} - 2\gamma \;\mathrm{Re}\left\{ \boldsymbol{d}^H \boldsymbol{w} \right\} \tag{2.16}$$

$$\text{s. t. } \boldsymbol{w}^H \boldsymbol{a} - \delta \geq \epsilon \left\| \boldsymbol{w} \right\|_2 \text{ and } \mathrm{Im}\left\{ \boldsymbol{w}^H \boldsymbol{a} \right\} = 0, \tag{2.17}$$

where $\boldsymbol{R}_a = \mathrm{E}\left\{ |y|^2 \boldsymbol{x}\boldsymbol{x}^H \right\}$ and $\boldsymbol{d} = \mathrm{E}\left\{ y^* \boldsymbol{x} \right\}$ are estimated from the previous snapshots, which will be explained in the next section.

2.2.2 Proposed SOC Implementation and Adaptive Algorithm

The first part in this subsection describes how to implement the SOC program and in the second part the adaptive algorithm to adjust the weights of the beamformer according to the WC-CCM design is presented.

2.2.2.1 SOC Implementation

In this subsection we follow the approach in [VGL03] and present the SOC impementation of the proposed WC-CCM design. Introducing a scalar variable τ, an equivalent problem to equation (2.17) can be formulated

$$\begin{aligned} \min_{\tau, \boldsymbol{w}} \quad \tau \quad \text{s. t.} \quad & \boldsymbol{w}^H \boldsymbol{R}_{\mathrm{ac}}^H \boldsymbol{R}_{\mathrm{ac}} \boldsymbol{w} - 2\gamma \;\mathrm{Re}\left\{ \boldsymbol{d}^H \boldsymbol{w} \right\} \leq \tau \\ & \mathrm{Re}\left\{ \boldsymbol{w}^H \boldsymbol{a} \right\} - \delta \geq \epsilon \left\| \boldsymbol{w} \right\|_2 \\ & \mathrm{Im}\left\{ \boldsymbol{w}^H \boldsymbol{a} \right\} = 0, \end{aligned} \tag{2.18}$$

where $\boldsymbol{R}_{\text{ac}}^{H}\boldsymbol{R}_{\text{ac}} = \boldsymbol{R}_a$ is the Cholesky factorization. Introducing the real-valued matrix and the real-valued vectors given by

$$\begin{aligned}
\breve{\boldsymbol{R}}_{ac} &= \begin{bmatrix} \mathrm{Re}\left\{\boldsymbol{R}_{ac}\right\} & -\mathrm{Im}\left\{\boldsymbol{R}_{ac}\right\} \\ \mathrm{Im}\left\{\boldsymbol{R}_{ac}\right\} & \mathrm{Re}\left\{\boldsymbol{R}_{ac}\right\} \end{bmatrix} \\
\breve{\boldsymbol{d}} &= [\mathrm{Re}\left\{\boldsymbol{d}\right\}^T, \mathrm{Im}\left\{\boldsymbol{d}\right\}^T]^T \\
\breve{\boldsymbol{a}} &= [\mathrm{Re}\left\{\boldsymbol{a}\right\}^T, \mathrm{Im}\left\{\boldsymbol{a}\right\}^T]^T \\
\bar{\boldsymbol{a}} &= [\mathrm{Im}\left\{\boldsymbol{a}\right\}^T, -\mathrm{Re}\left\{\boldsymbol{a}\right\}^T]^T \\
\breve{\boldsymbol{w}} &= [\mathrm{Re}\left\{\boldsymbol{w}\right\}^T, \mathrm{Im}\left\{\boldsymbol{w}\right\}^T]^T,
\end{aligned}$$

the problem can be rewriten as

$$\begin{aligned}
\min_{\tau,\breve{\boldsymbol{w}}} \quad \tau \quad \text{s. t.} \quad & \breve{\boldsymbol{w}}^T \breve{\boldsymbol{R}}_{ac}^T \breve{\boldsymbol{R}}_{ac} \breve{\boldsymbol{w}} - 2\gamma\, \breve{\boldsymbol{d}}^T \breve{\boldsymbol{w}} \leq \tau \\
& \breve{\boldsymbol{w}}^T \breve{\boldsymbol{a}} - \delta \geq \epsilon \left\| \breve{\boldsymbol{w}} \right\|_2 \\
& \breve{\boldsymbol{w}}^T \bar{\boldsymbol{a}} = 0.
\end{aligned} \tag{2.19}$$

The quadratic constraint can be converted in an equivalent SOC constraint, which leads to the following optimization problem

$$\begin{aligned}
\min_{\tau,\breve{\boldsymbol{w}}} \quad \tau \quad \text{s. t.} \quad & \frac{1}{2} + \gamma \breve{\boldsymbol{d}}^T \breve{\boldsymbol{w}} + \frac{\tau}{2} \geq \left\| \begin{bmatrix} \frac{1}{2} - \gamma \breve{\boldsymbol{d}}^T \breve{\boldsymbol{w}} - \frac{\tau}{2} \\ \breve{\boldsymbol{R}}_{\text{ac}} \breve{\boldsymbol{w}} \end{bmatrix} \right\|_2 \\
& \breve{\boldsymbol{w}}^T \breve{\boldsymbol{a}} - \delta \geq \epsilon \left\| \breve{\boldsymbol{w}} \right\|_2 \\
& \breve{\boldsymbol{w}}^T \bar{\boldsymbol{a}} = 0.
\end{aligned} \tag{2.20}$$

For the implementation, let us define

$$\begin{aligned}
\boldsymbol{p} &= [1, \boldsymbol{0}^T]^T \in \mathbb{R}^{(2M+1)\times 1} \\
\boldsymbol{u} &= [\tau, \breve{\boldsymbol{w}}^T]^T \in \mathbb{R}^{(2M+1)\times 1} \\
\boldsymbol{f} &= [1/2, 1/2, \boldsymbol{0}^T, -\delta, \boldsymbol{0}^T, 0]^T \in \mathbb{R}^{(4M+4)\times 1} \\
\boldsymbol{F}^T &= \begin{bmatrix} \frac{1}{2} & \gamma \breve{\boldsymbol{d}}^T \\ -\frac{1}{2} & -\gamma \breve{\boldsymbol{d}}^T \\ \boldsymbol{0} & \breve{\boldsymbol{R}}_{\text{ac}} \\ 0 & \breve{\boldsymbol{a}} \\ \boldsymbol{0} & \epsilon \boldsymbol{I} \\ 0 & \bar{\boldsymbol{a}} \end{bmatrix} \in \mathbb{R}^{(4M+4)\times(2M+1)},
\end{aligned}$$

where $\boldsymbol{I}$ is the identity matrix and $\mathbf{0}$ is a vector of zeros of compatible dimensions. Finally, the problem can be formulated as the dual form of the SOC problem (equivalent to (8) in [Stu98])

$$\begin{aligned} \min_{\boldsymbol{u}} \quad & \boldsymbol{p}^T \boldsymbol{u} \quad \text{s. t.} \\ & \boldsymbol{f} + \boldsymbol{F}^T \boldsymbol{u} \quad \in \mathrm{SOC}_1^{2M+2} \times \mathrm{SOC}_2^{2M+1} \times \{0\}, \end{aligned} \tag{2.21}$$

where $\boldsymbol{f} + \boldsymbol{F}^T \boldsymbol{u}$ describes a SOC with a dimension $2M+2$, a SOC with a dimension $2M+1$ and a zero cone in that order. Finally, the weight vector of the beamformer $\boldsymbol{w}$ can be put together in the following form

$$\boldsymbol{w} = \left[\boldsymbol{u}_2, ..., \boldsymbol{u}_{M+1}\right]^T + j\left[\boldsymbol{u}_{M+2}, ..., \boldsymbol{u}_{2M+1}\right]^T \tag{2.22}$$

Alternatively (2.17) can be solved by using [GB11], which transforms it automatically in an appropriate form.

2.2.2.2 Adaptive Algorithm

It has already been mentioned that the optimization problem corresponding to the WC-CCM algorithm design is solved iteratively. As a result, the underlying optimization problem is to be solved periodically. In this case the proposed adaptive algorithm solves it at each time instant. For the adaptive implementation we use an exponentially decayed data window for the estimation of $\boldsymbol{R}_a$ and $\boldsymbol{d}$ given by

$$\hat{\boldsymbol{R}}_a(i) = \mu \hat{\boldsymbol{R}}_a(i-1) + |y(i)|^2 \boldsymbol{x}(i)\boldsymbol{x}^H(i) \tag{2.23}$$

$$\hat{\boldsymbol{d}}(i) = \mu \hat{\boldsymbol{d}}(i-1) + \boldsymbol{x}(i)y^*(i), \tag{2.24}$$

where μ is the forgetting factor. Each iteration includes a Cholesky factorization and also a transformation in a real valued problem. Finally, the problem is formulated in the dual form of the SOC problem and solved with SeDuMi [Stu98]. The adaptive algorithm structure is detailed in Table 2.1.

Compared to the algorithm based on the minimum variance constraint, the proposed algorithm increases the dimension of the first SOC from $2M+1$ to $2M+2$.

Table 2.1: Proposed WC-CCM Algorithm

initialization: $\hat{\boldsymbol{R}}_a(0) = \sigma_n^2 \boldsymbol{I}; \hat{\boldsymbol{d}}(0) = \boldsymbol{0}; \boldsymbol{w}(0) = \frac{\boldsymbol{a}}{M}$
Update for each time instant i = 1,...,N
$y(i) = \boldsymbol{w}^H(i-1)\boldsymbol{x}(i)$
$\hat{\boldsymbol{R}}_a(i) = \mu \hat{\boldsymbol{R}}_a(i-1) + \lvert y(i) \rvert^2 \boldsymbol{x}(i)\boldsymbol{x}^H(i)$
$\boldsymbol{R}_{\mathrm{ac}}(i) = \mathrm{chol}\left(\hat{\boldsymbol{R}}_a(i)\right)$
$\hat{\boldsymbol{d}}(i) = \mu \hat{\boldsymbol{d}}(i-1) + \boldsymbol{x}(i)y^*(i)$
$\boldsymbol{R}_{\mathrm{acr}}(i) = \begin{bmatrix} \mathrm{Re}\{\boldsymbol{R}_{\mathrm{ac}}(i)\} & -\mathrm{Im}\{\boldsymbol{R}_{\mathrm{ac}}(i)\} \\ \mathrm{Im}\{\boldsymbol{R}_{\mathrm{ac}}(i)\} & \mathrm{Re}\{\boldsymbol{R}_{\mathrm{ac}}(i)\} \end{bmatrix}$
$\boldsymbol{d}_{\mathrm{r}}(i) = \left[\mathrm{Re}\left\{\hat{\boldsymbol{d}}(i)\right\}^T, \mathrm{Im}\left\{\hat{\boldsymbol{d}}(i)\right\}^T\right]^T$
$\boldsymbol{p} = [1, \boldsymbol{0}^T]^T$
$\boldsymbol{f} = [1/2, 1/2, \boldsymbol{0}^T, -\delta, \boldsymbol{0}^T, 0]^T$
$\boldsymbol{F}^T = \begin{bmatrix} \frac{1}{2} & \gamma \boldsymbol{d}_{\mathrm{r}}^T(i) \\ -\frac{1}{2} & -\gamma \boldsymbol{d}_{\mathrm{r}}^T(i) \\ \boldsymbol{0} & \boldsymbol{R}_{\mathrm{acr}}(i) \\ 0 & \breve{\boldsymbol{a}} \\ \boldsymbol{0} & \epsilon \boldsymbol{I} \\ 0 & \bar{\boldsymbol{a}} \end{bmatrix}$
$\min_{\boldsymbol{u}} \quad \boldsymbol{p}^T \boldsymbol{u} \quad \text{s. t.}$
$\qquad \boldsymbol{f} + \boldsymbol{F}^T \boldsymbol{u} \quad \in \mathrm{SOC}_1^{2M+2} \times \mathrm{SOC}_2^{2M+1} \times \{0\}$
$\boldsymbol{w}(i) = [\boldsymbol{u}_2, ..., \boldsymbol{u}_{M+1}]^T + j\,[\boldsymbol{u}_{M+2}, ..., \boldsymbol{u}_{2M+1}]^T$

2.2.3 Adjustment of the Parameter ϵ

The beamforming weight vector can be defined as

$$\boldsymbol{w} = c\,\boldsymbol{a}/M + \boldsymbol{b}, \tag{2.25}$$

where c is a scalar, and $\boldsymbol{b}$ is orthogonal to $\boldsymbol{a}$. Using it with the worst-case constraint leads to

$$c - \delta \geq \epsilon \sqrt{\frac{c^2}{M} + \boldsymbol{b}^H \boldsymbol{b}}. \tag{2.26}$$

Obviously the following relation holds

$$c - \delta \geq \epsilon \sqrt{\frac{c^2}{M} + \boldsymbol{b}^H \boldsymbol{b}} \geq \epsilon \sqrt{\frac{c^2}{M}}. \tag{2.27}$$

Rewriting the relation shows that there is a pole for $\epsilon = \sqrt{M}$.

$$c \geq \frac{\delta}{1 - \epsilon/\sqrt{M}} \tag{2.28}$$

In addition, it is mentioned in [LS03] that for $\|\boldsymbol{a}\|_2 \leq \epsilon$ and $\|\boldsymbol{a}\|_2 = \sqrt{M}$ there is no $\boldsymbol{w}$ that satisfies the constraint. Retaining $\boldsymbol{b}$ in (2.26) leads to

$$c \geq \frac{M\ \delta}{M - \epsilon^2} + \sqrt{\frac{M\epsilon^2 \boldsymbol{b}^H \boldsymbol{b} - M\ \delta}{M - \epsilon^2} + \left(\frac{M\ \delta}{M - \epsilon^2}\right)^2} \tag{2.29}$$

Since $\boldsymbol{b}^H \boldsymbol{b}$ increases with increasing c, the pole is additionally enforced.

Now it is assumed $\epsilon \lesssim \sqrt{M}$, which leads to $c \gg \delta$, $c - \delta \approx c$. In that case, the constraint (2.26) can be rewritten as

$$c \geq \epsilon \sqrt{\frac{c^2}{M} + \boldsymbol{b}^H \boldsymbol{b}}, \tag{2.30}$$

in what follows

$$\frac{\boldsymbol{b}^H \boldsymbol{b}}{c^2} = \frac{1}{\epsilon^2} - \frac{1}{M}. \tag{2.31}$$

As a result of (2.31), depending on the choice of ϵ the ratio between the components of the weight vector defined by (2.25) can be infinitesimal. That corresponds to $\boldsymbol{w} \approx c\ \boldsymbol{a}/M$ and a diagonal loading which is above the interferer's level. Hence, there is no limitation for the diagonal loading even if ϵ is chosen in the interval $[0, \sqrt{M}]$ for ϵ, where the constraint can be fulfilled theoretically. Obviously, in case of ϵ close to $\sqrt{M}$ the ratio $\frac{\boldsymbol{b}^H \boldsymbol{b}}{c^2}$ is constrained to a small value, which can lead to a performance degradation. Without loss of generality, the ratio is already smaller for lower SNR values, caused by the fact that additional noise reduces the eigenvalue spread of the signal covariance matrix. That means the relation of the quantities in (2.31) and its penalty has more impact for higher SNR values. As a consequence it is advantageous to choose ϵ with respect to the SNR as well as with respect to the assumed mismatch level.

2.2.4 Analysis of the Optimization Problem

The objective function for the constant modulus design criterion is

$$J_{\mathrm{cm}} = \mathrm{E}\left\{ \left(|y|^2 - \gamma \right)^2 \right\}. \tag{2.32}$$

To ensure that the constraint $\boldsymbol{w}^H \boldsymbol{a} = \delta + \epsilon \ \|\boldsymbol{w}\|_2$ is fulfilled $\boldsymbol{w}$ is replaced by

$$\boldsymbol{w} = \frac{\boldsymbol{a}}{M} \ (\delta + \epsilon \, \|\boldsymbol{w}\|_2) + \boldsymbol{B}\boldsymbol{z}, \tag{2.33}$$

where $\boldsymbol{B} = \mathrm{null}\left\{\boldsymbol{a}^H\right\}$ and $\boldsymbol{z} \in \mathbb{C}^{M-1 \times 1}$. To obtain a function which does not depend on $\|\boldsymbol{w}\|_2$, a quadratic equation needs to be solved.

$$\|\boldsymbol{w}\|_2 = \tau = \sqrt{\frac{1}{M} \left(\delta + \epsilon \ \tau\right)^2 + \boldsymbol{z}^H \boldsymbol{z}} \tag{2.34}$$

Because the norm is greater than zero the following holds

$$\tau = \frac{\epsilon \ \delta}{M - \epsilon^2} + \sqrt{\frac{M \boldsymbol{z}^H \boldsymbol{z} + \delta^2}{M - \epsilon^2} + \left(\frac{\epsilon \ \delta}{M - \epsilon^2} \right)^2}. \tag{2.35}$$

Therefore, the resulting weight vector is a function of $\boldsymbol{z}$.

$$\boldsymbol{w} = \frac{\boldsymbol{a}}{M} \left(\delta + \frac{\epsilon^2 \ \delta}{M - \epsilon^2} + \epsilon \ \sqrt{\frac{M \boldsymbol{z}^H \boldsymbol{z} + \delta^2}{M - \epsilon^2} + \left(\frac{\epsilon \ \delta}{M - \epsilon^2} \right)^2} \right) + \boldsymbol{B}\boldsymbol{z} \tag{2.36}$$

Replacing the $\boldsymbol{w}$ in the objective function leads to an equivalent problem to the original:

$$\begin{aligned} J = \mathrm{E}\Bigg\{ \Bigg[& \left(\frac{\boldsymbol{a}^H}{M} \left(\delta + \frac{\epsilon^2 \ \delta}{M - \epsilon^2} + \epsilon \ \sqrt{\frac{M \boldsymbol{z}^H \boldsymbol{z} + \delta^2}{M - \epsilon^2} + \left(\frac{\epsilon \ \delta}{M - \epsilon^2} \right)^2} \right) + \boldsymbol{z}^H \boldsymbol{B}^H \right) \\ & \boldsymbol{x}\boldsymbol{x}^H \left(\frac{\boldsymbol{a}}{M} \left(\delta + \frac{\epsilon^2 \ \delta}{M - \epsilon^2} + \epsilon \ \sqrt{\frac{M \boldsymbol{z}^H \boldsymbol{z} + \delta^2}{M - \epsilon^2} + \left(\frac{\epsilon \ \delta}{M - \epsilon^2} \right)^2} \right) + \boldsymbol{B}\boldsymbol{z} \right) - \gamma \Bigg]^2 \Bigg\} \end{aligned} \tag{2.37}$$

The function is convex, when the Hessian $\boldsymbol{H} = \frac{\partial}{\partial \boldsymbol{z}^H}\left(\frac{\partial J}{\partial \boldsymbol{z}}\right)$ is positive semidefinite. The Hessian corresponding to the objective function is given by

$$\begin{aligned}\boldsymbol{H} = & 2\frac{\partial}{\partial \boldsymbol{z}^H}\left(\mathrm{E}\left\{|y|^2-\gamma\right\}\right)\frac{\partial}{\partial \boldsymbol{z}}\left(\mathrm{E}\left\{|y|^2-\gamma\right\}\right)\\ & +2\ \mathrm{E}\left\{|y|^2-\gamma\right\}\frac{\partial}{\partial \boldsymbol{z}^H}\frac{\partial}{\partial \boldsymbol{z}}\left(\mathrm{E}\left\{|y|^2-\gamma\right\}\right)\end{aligned} \tag{2.38}$$

Since it is the product of a vector multiplied with its Hermitian transposed the first term in (2.38), the Hessian is positive semidefinite. While it is assumed that $\mathrm{E}\left\{|y|^2-\gamma\right\} \geq 0$ the positive semidefiniteness of $\boldsymbol{H}_2 = \frac{\partial}{\partial \boldsymbol{z}^H}\frac{\partial}{\partial \boldsymbol{z}}\left(\mathrm{E}\left\{|y|^2-\gamma\right\}\right)$ still needs to be shown. It can be expressed as a sum $\boldsymbol{H}_2 = \sum_{k=1}^{6}\boldsymbol{H}_{2k}$ and is given by

$$\begin{aligned}\boldsymbol{H}_2 =\mathrm{E}\Bigg\{\Bigg[& \left(-\frac{\epsilon}{4}\left(\frac{1}{\sqrt{\alpha}}\right)\right)^3\left(\frac{M}{M-\epsilon^2}\right)^2\frac{\boldsymbol{a}^H}{M}\boldsymbol{x}\boldsymbol{x}^H\left(\beta\frac{\boldsymbol{a}}{M}+\boldsymbol{B}\boldsymbol{z}\right)\boldsymbol{z}\boldsymbol{z}^H\\ & +\frac{\epsilon}{2}\frac{1}{\sqrt{\alpha}}\left(\frac{M}{M-\epsilon^2}\right)\frac{\boldsymbol{a}^H}{M}\boldsymbol{x}\boldsymbol{x}^H\left(\beta\frac{\boldsymbol{a}}{M}+\boldsymbol{B}\boldsymbol{z}\right)\boldsymbol{I}_{M-1}\\ & +\left(\beta\frac{\boldsymbol{a}^H}{M}+\boldsymbol{z}^H\boldsymbol{B}^H\right)\boldsymbol{x}\boldsymbol{x}^H\frac{\boldsymbol{a}}{M}\left(-\frac{\epsilon}{4}\left(\frac{1}{\sqrt{\alpha}}\right)\right)^3\left(\frac{M}{M-\epsilon^2}\right)^2\boldsymbol{z}\boldsymbol{z}^H\\ & +\left(\beta\frac{\boldsymbol{a}^H}{M}+\boldsymbol{z}^H\boldsymbol{B}^H\right)\boldsymbol{x}\boldsymbol{x}^H\frac{\boldsymbol{a}}{M}\frac{\epsilon}{2}\frac{1}{\sqrt{\alpha}}\left(\frac{M}{M-\epsilon^2}\right)\boldsymbol{I}_{M-1}\\ & +\frac{\epsilon}{2}\frac{1}{\sqrt{\alpha}}\left(\frac{M}{M-\epsilon^2}\right)\frac{\boldsymbol{a}^H}{M}\boldsymbol{x}\boldsymbol{x}^H\frac{\boldsymbol{a}}{M}\frac{\epsilon}{2}\frac{1}{\sqrt{\alpha}}\left(\frac{M}{M-\epsilon^2}\right)\boldsymbol{z}\boldsymbol{z}^H\\ & +\left(\frac{\epsilon}{2}\frac{1}{\sqrt{\alpha}}\left(\frac{M}{M-\epsilon^2}\right)\boldsymbol{z}\frac{\boldsymbol{a}^H}{M}+\boldsymbol{B}^H\right)\boldsymbol{x}\boldsymbol{x}^H\left(\frac{\boldsymbol{a}}{M}\frac{\epsilon}{2}\frac{1}{\sqrt{\alpha}}\left(\frac{M}{M-\epsilon^2}\right)\boldsymbol{z}^H+\boldsymbol{B}\right)\Bigg],\end{aligned} \tag{2.39}$$

where $\alpha = \left(\frac{M\boldsymbol{z}^H\boldsymbol{z}+\delta^2}{M-\epsilon^2}+\left(\frac{\epsilon\ \delta}{M-\epsilon^2}\right)^2\right)$ and $\beta = \left(\delta+\frac{\epsilon^2\ \delta}{M-\epsilon^2}+\epsilon\ \sqrt{\alpha}\right)$. To show that $\boldsymbol{H}_2$ is positive semidefinite the following steps are made. Here it is assumed that

$$\frac{\boldsymbol{a}^H}{M}\boldsymbol{x}\boldsymbol{x}^H\left(\beta\frac{\boldsymbol{a}}{M}+\boldsymbol{B}\boldsymbol{z}\right) \geq 0. \tag{2.40}$$

This assumption is reasonable as far as the term $\boldsymbol{x}^H\boldsymbol{B}\boldsymbol{z}$ is basically the compensating term of the unwanted contribution of $\boldsymbol{x}^H\left(\beta\frac{\boldsymbol{a}}{M}\right)$. Under this condition all terms in the sum of $\boldsymbol{H}_2$ are positive semidefinite except the first term $\boldsymbol{H}_{21}$ and the third term $\boldsymbol{H}_{23}$, where $\boldsymbol{H}_{23} = \boldsymbol{H}_{21}^H$. But it can be shown that $\boldsymbol{H}_{21}+\boldsymbol{H}_{22}$ and $\boldsymbol{H}_{23}+\boldsymbol{H}_{24}$ are always positive semidefinite. To prove that, the inequality $\boldsymbol{v}^H(\boldsymbol{H}_{22})\boldsymbol{v} \geq \boldsymbol{v}^H(-\boldsymbol{H}_{21})\boldsymbol{v}$

is described as

$$\begin{aligned}
&\boldsymbol{v}^H \frac{\epsilon}{2} \frac{1}{\sqrt{\alpha}} \left(\frac{M}{M-\epsilon^2}\right) \frac{\boldsymbol{a}^H}{M} \boldsymbol{x}\boldsymbol{x}^H \left(\beta \frac{\boldsymbol{a}}{M} + \boldsymbol{B}\boldsymbol{z}\right) \boldsymbol{I}_{M-1} \boldsymbol{v} \\
&\geq \boldsymbol{z}^H \left(\frac{\epsilon}{4} \left(\frac{1}{\sqrt{\alpha}}\right)\right)^3 \left(\frac{M}{M-\epsilon^2}\right)^2 \frac{\boldsymbol{a}^H}{M} \boldsymbol{x}\boldsymbol{x}^H \left(\beta \frac{\boldsymbol{a}}{M} + \boldsymbol{B}\boldsymbol{z}\right) \boldsymbol{z}\boldsymbol{z}^H \boldsymbol{z} \\
&\geq \boldsymbol{v}^H \left(\frac{\epsilon}{4} \left(\frac{1}{\sqrt{\alpha}}\right)\right)^3 \left(\frac{M}{M-\epsilon^2}\right)^2 \frac{\boldsymbol{a}^H}{M} \boldsymbol{x}\boldsymbol{x}^H \left(\beta \frac{\boldsymbol{a}}{M} + \boldsymbol{B}\boldsymbol{z}\right) \boldsymbol{z}\boldsymbol{z}^H \boldsymbol{v},
\end{aligned} \tag{2.41}$$

where $\boldsymbol{v}$ is any vector with the length of $\boldsymbol{z}$ and there is an upper bound for $\boldsymbol{v} = \boldsymbol{z}$. Since $\boldsymbol{z}^H\boldsymbol{z} = \boldsymbol{v}^H\boldsymbol{v}$, the inequality can be reduced to

$$2\alpha \geq \left(\frac{M\boldsymbol{z}^H\boldsymbol{z}}{M-\epsilon^2}\right). \tag{2.42}$$

Replacing α gives the proof for positive semidefiniteness

$$2\left(\frac{M\boldsymbol{z}^H\boldsymbol{z}+\delta}{M-\epsilon^2} + \left(\frac{\epsilon\,\delta}{M-\epsilon^2}\right)^2\right) \geq \left(\frac{M\boldsymbol{z}^H\boldsymbol{z}}{M-\epsilon^2}\right), \tag{2.43}$$

which always holds. The same can be done in an analogous way for the third term ($\boldsymbol{H}_{23}$) and the fourth term ($\boldsymbol{H}_{24}$). To ensure that $\mathrm{E}\left\{|y|^2 - \gamma\right\} \geq 0$ it can be assumed that $\boldsymbol{w}^H(\boldsymbol{a}+\boldsymbol{e}) \geq \delta$, where $\boldsymbol{e}$ is the array steering vector mismatch. Therefore,

$$\gamma \leq \delta\ \mathrm{E}\left\{|s_1|^2\right\} \tag{2.44}$$

is a sufficient condition to enforce convexity, where $|s_1|^2$ is the power of the desired user.

2.2.5 Simulations

In the simulations the WC-CCM design is compared to the loaded-SMI [CZO87] the optimal SINR [LS06] and the worst-case optimization-based constrained minimum variance algorithm [VGL03]. A uniform linear sensor array is used with $M = 10$ sensors. In all simulations is considered $|s_1| = 1$, $\delta = 1$, $\gamma = 1$ and $\mu = 0.995$. Besides user 1, the desired signal, there are 4 interferers, the powers (P) relative to user 1 and directions of arrival (DoA) in degrees of which are detailed in Table 2.2. For time-index $i = 1000$ an environmental event is considered.

In the first examples, the SINR performance is shown as a function of ϵ to illustrate

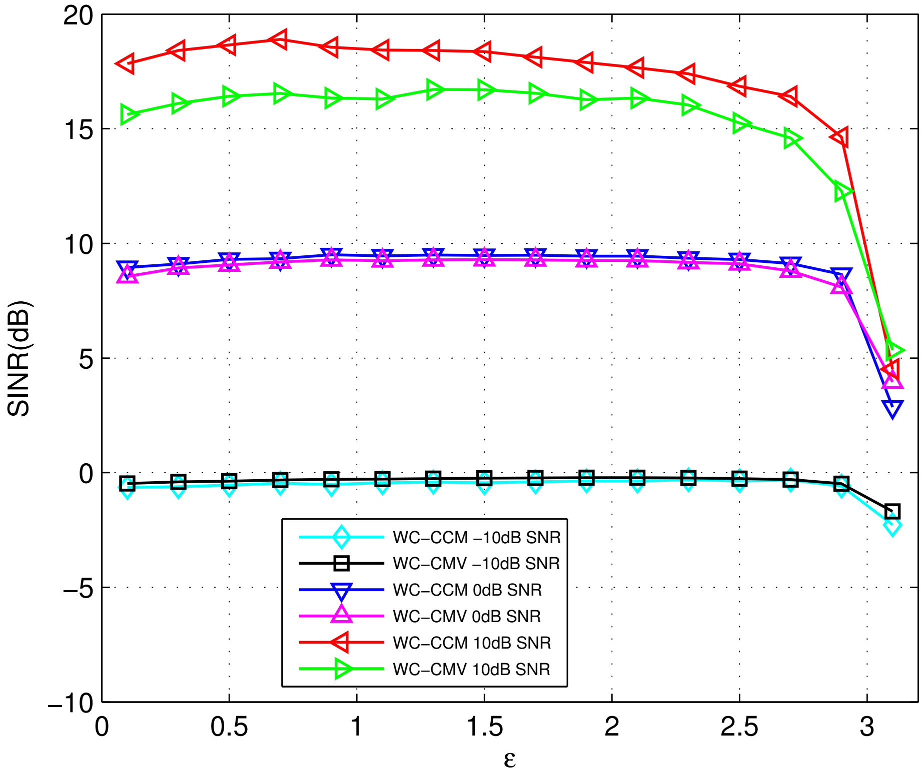

Figure 2.1: SINR versus ϵ, for different SNRs, perfect ASV, $M = 10$

Table 2.2: Interference scenario
P(dB) relative to user1 / DoA

snapshot	user 1	user 2	user 3	user 4	user 5
1-1000	$0/93°$	$13/120°$	$1/140°$	$22/67°$	$10/157°$
1001-2000	$0/93°$	$30/120°$	$25/170°$	$4/104°$	$9/68°$

the discussion on the choice of ϵ and to verify the assumption that the constraint of the worst-case optimization approach has the same behavior when combined with the CCM design. Each simulation of the worst-case constraint is done with the minimum variance and with the constant modulus design criteria. Fig. 2.1 shows the SINR performance when the algorithms work with perfectly known array steering vectors. As expected, the performance drops down when ϵ is close to $\sqrt{M}$. The simulations corroborate the analysis and show that the optimal value for ϵ depends on the SNR.

Fig. 2.2 shows the same experiment with an array steering vector mismatch. The

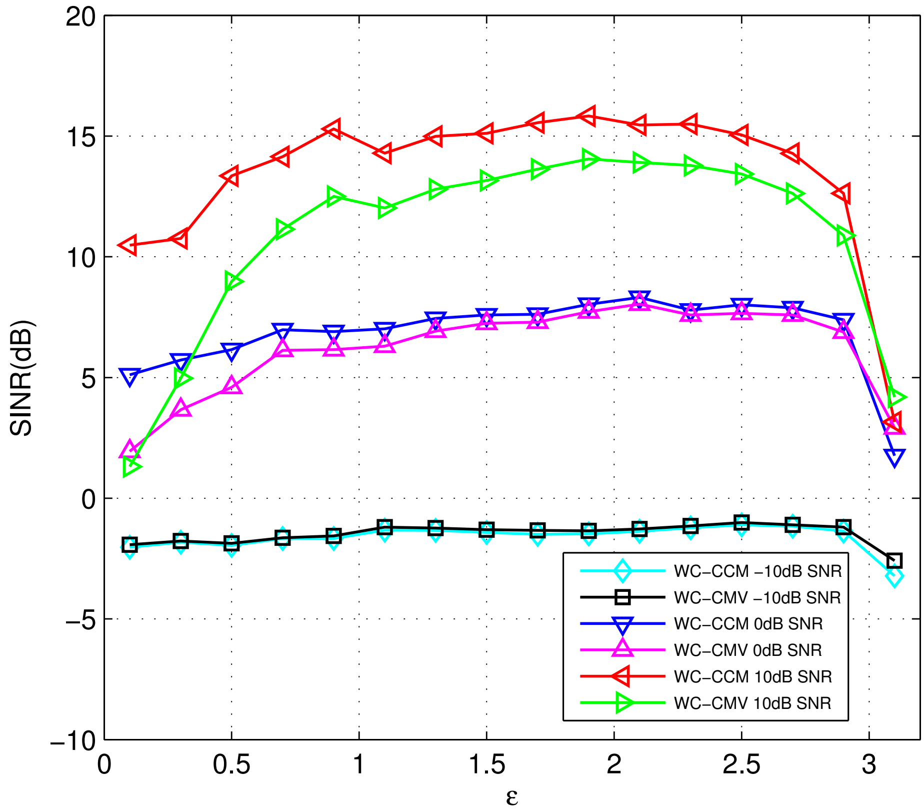

Figure 2.2: SINR versus ϵ, for different SNRs, local coherent scattering, $M = 10$

signal steering vector is corrupted by a local coherent scattering (LCS), given by

$$\breve{\boldsymbol{a}} = \boldsymbol{a} + \sum_{i=1}^{4} \mathrm{e}^{j\Phi_i} \boldsymbol{a}_{\mathrm{sc}}\left(\theta_i\right), \tag{2.45}$$

where Φ_i is uniformly distributed between zero and 2π and θ_i is uniformly distributed with a standard deviation of 2 degrees with the assumed direction as the mean. The mismatch changes for every realization and is fixed over the snapshots of each simulation trial. As expected, the optimal ϵ is shifted to higher values, compared to Fig. 2.1.

Fig. 2.3 shows the SINR performance over ϵ for different mismatch levels. Here the mismatch level refers to the standard deviation, where 100% corresponds to 2 degrees. Here the influence of the noise is reduced by setting the SNR to a high value. It is illustrated how the mismatch impacts the performance of the beamforming algorithms. Note that for the CCM design, the curves are more flat, which means this design approach is less sensitive to different mismatch levels. As a conclusion, it is shown that the optimal ϵ depends on the mismatch as well as the SNR. In what

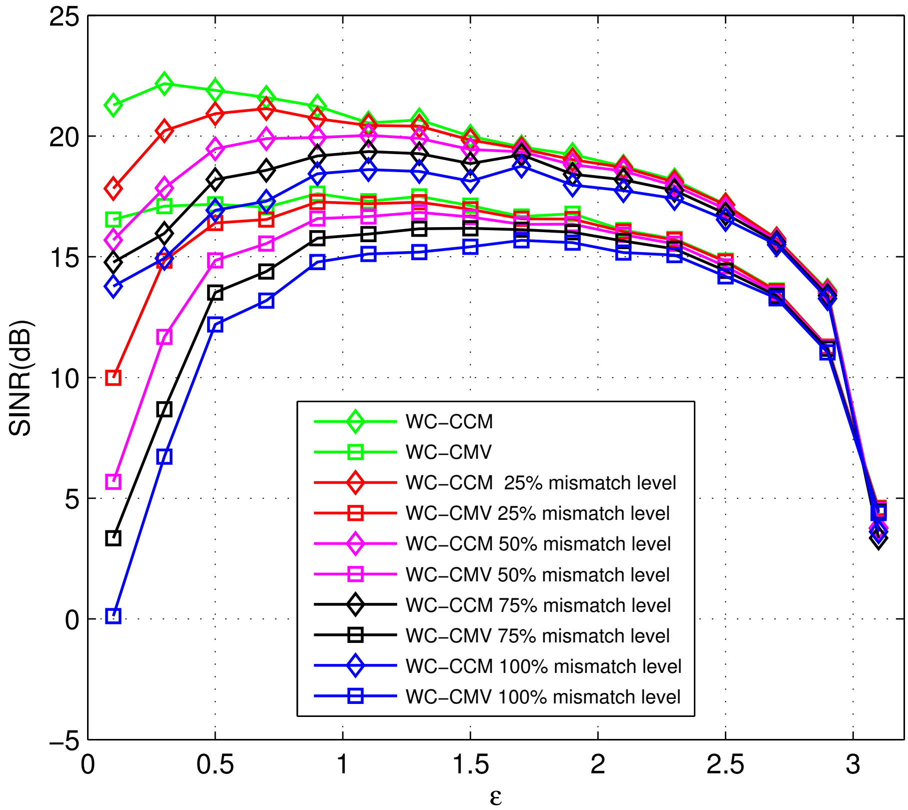

Figure 2.3: SINR versus ϵ, different levels of LCS, SNR = 15dB, i=200, $M = 10$

follows, the mismatch level is simulated with 100% and $\epsilon = 2.1$ for the CMV and for the CCM design criteria.

In the last part the proposed WC-CCM is compared with the WC-CMV [VGL03], the loaded-SMI inversion with a diagonal loading factor determined as 10 σ_n^2 and the optimal SINR [LS06]. As in the previous section, the mismatch is caused by local coherent scattering. Fig. 2.4 presents the SINR performance over the snapshots. For time index $i = 1000$ the interference scenario changes according to Table 2.2. This influences the performance and the beamformers adapt to the environment. The proposed WC-CCM shows a significantly better performance than the WC-CMV and the loaded-SMI algorithm. Fig. 2.5 shows the SINR performance against the SNR for $i = 500$ snapshots. The curves show that the proposed WC-CCM algorithm is more robust against mismatch problems than the existing WC-CMV and loaded-SMI agorithms.

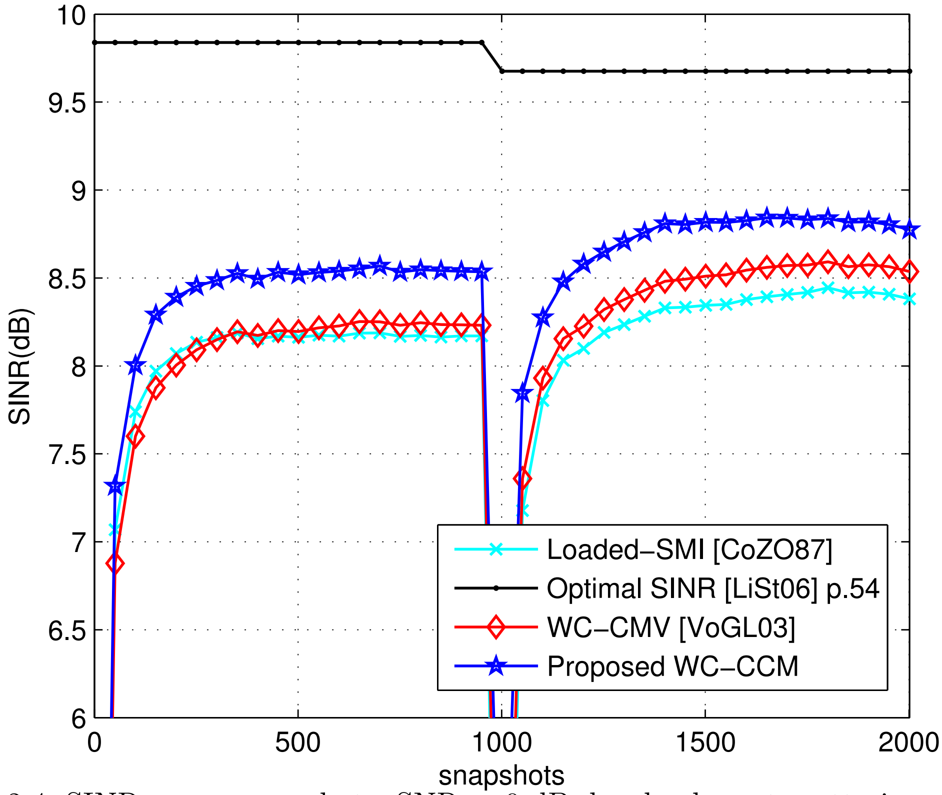

Figure 2.4: SINR versus snapshots, SNR = 0 dB, local coherent scattering, $M = 10$

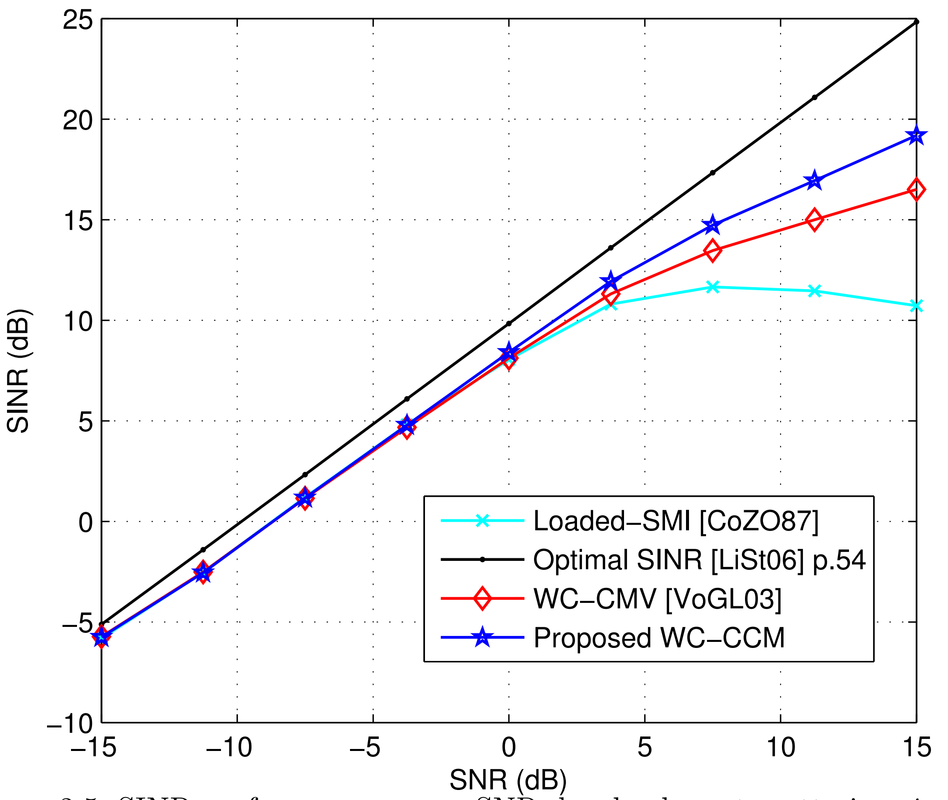

Figure 2.5: SINR performance versus SNR, local coherent scattering, $i = 500$

2.2.6 Conclusions and Futurework

Conclusion

The author has proposed a robust beamforming algorithm based on the worst case constraint and the CCM design criterion. The proposed approach exploits prior knowledge of the desired signal's amplitude. The problem can be solved iteratively, where each iteration is effectively solved by a SOC program. In addition, an alternative study about the worst-case constraint is considered, which can be useful for the choice of the parameter ϵ. As a result, the proposed beamformer outperforms the conventional design especially in high SNR regions. Based on the results it can be expected that the CCM design criterion can improve also other robust beamforming approaches.

Futurework

The proposed constrained constant modulus algorithm is slightly sensitive against channel gain fluctuations. This sensitivity could be studied and an extention could be introduced, which takes that into account.

2.3 Joint Optimization using Modified Conjugate Gradient Method

The existing algorithms which use the worst-case optimization-based constraint do not take advantage of previous computations as the conventioanl SMI beamforming algorithm solved by the modified conjugate method (MCG) algorithm or the recursive-least-squares (RLS) algorithm in the so-called *on-line* mode.
In the next section a robust constraint is shown which is just slightly different compared to the worst-case optimization-based approaches. As a result the corresponding optimization problem is a quadratically constrained quadratic program (QCQP) instead of a second order cone (SOC) program. It is shown how to solve the problem with a joint optimization strategy. The method includes a system of equations which is solved efficently with a modified conjugate gradient algorithm. Finally the complexity is reduced from more than cubic $\mathcal{O}(M^{3.5})$ to squared $\mathcal{O}(M^2)$ with the number of sensor elements, while the SINR performance is equivalent compared to the worst-case optimization-based approach. The new method is presented in the robust constrained minimum variance design using the modified conjugate gradient method (RCMV-MCG) and in the robust constrained constant modulus design using the modified conjugate gradient method (RCCM-MCG), which exploits the knowledge of the signal energy of the desired user. The content of this chapter is published by the author in [LdLWH11].

2.3.1 Proposed Design and Joint-Optimization Approach

Robust Constrained Minimum Variance Design

The proposed beamformers are related to the worst case approach, whose performance is already well established [LS06],[VGL03]. In case of the minimum variance design it can be transformed in the following optimization problem.

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{R}_{xx} \boldsymbol{w}, \quad \text{s. t. } \operatorname{Re}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} - \delta \geq \epsilon \left\|\boldsymbol{w}\right\|_2, \tag{2.46}$$

where $\boldsymbol{R}_{xx} = \mathrm{E}\left\{\boldsymbol{x}(i)\boldsymbol{x}(i)^H\right\}$ is the covariance matrix of the input signal. According to [LB05] it is sufficient to constrain the real part in the constraint. It has been shown that this kind of beamformer belongs to the class of diagonal loading schemes [VGL03]. As already mentioned, this problem can be solved by interior point methods while its complexity is more than cubic with the number of sensor elements. Contrary to that approach the proposed *on-line* algorithms take advantage of the previous calculations, which leads to a complexity reduction of more than an order of magnitude. In order to do so, the constraint is slightly modified. Here it is assumed that the use of $\tilde{\epsilon}\left\|\boldsymbol{w}\right\|_2^2$ instead of $\epsilon\left\|\boldsymbol{w}\right\|_2$ from the conventional constraint has a comparable impact. Finally, the proposed design criterion for the minimum variance case is

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{R}_{xx} \boldsymbol{w}, \quad \text{s. t. } \operatorname{Re}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} - \delta \geq \tilde{\epsilon}\left\|\boldsymbol{w}\right\|_2^2 . \tag{2.47}$$

Using the method of Lagrange multipliers gives

$$\mathcal{L}_{\mathrm{CMV}}\left(\boldsymbol{w}, \lambda\right) = \boldsymbol{w}^H \boldsymbol{R}_{xx} \boldsymbol{w} + \lambda\left[\tilde{\epsilon}\ \boldsymbol{w}^H \boldsymbol{w} - \operatorname{Re}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} + \delta\right], \tag{2.48}$$

where λ is the Lagrange multiplier. Computing the gradient of (2.48) with respect to $\boldsymbol{w}^*$, and equating it to zero leads to

$$\boldsymbol{w} = \left(\boldsymbol{R}_{xx} + \tilde{\epsilon}\lambda \boldsymbol{I}\right)^{-1} \lambda \boldsymbol{a}/2. \tag{2.49}$$

Because it is not clear how to obtain the Lagrange multiplier in a closed form, here it is proposed to adjust it in a parallel algorithm. In this joint optimization the Lagrange multiplier is interpreted as a penalty factor. In this case $\lambda > 0$ needs to hold all the time. The adjustment rises the penalty factor when the constraint is not fullfilled and decreases it otherwise. In case of a too small penalty the minimum variance design leads to a non fulfilled constraint. For that we devise the following algorithm

$$\lambda(i) = \lambda(i-1) + \mu_\lambda\left(\tilde{\epsilon}\left\|\boldsymbol{w}(i)\right\|_2^2 - \operatorname{Re}\left\{\boldsymbol{w}(i)^H \boldsymbol{a}\right\} + \delta\right), \tag{2.50}$$

where μ_λ is the step size. In addition it is reasonable to define boundaries for the update term.

Robust Constrained Constant Modulus Design

In case of constant modulus signals it has been shown that the constant modulus design performs better than the minimum variance design [dLSN05],[dLHSN08]. Similarly, the robust approach can be combined with the constrained constant modulus criterion. The constant modulus cost function is defined by

$$J = \mathrm{E}\left\{\left(|y|^2 - \gamma\right)^2\right\}. \tag{2.51}$$

Since it is a fourth order function with a more complicated structure a closed form solution appears to be not possible. For this reason, the problem is solved iteratively which corresponds to the following underlying objective function

$$\hat{J} = \boldsymbol{w}^H \boldsymbol{R}_{\mathrm{a}} \boldsymbol{w} - 2\gamma \operatorname{Re}\left\{\boldsymbol{d}^H \boldsymbol{w}\right\}, \tag{2.52}$$

where $\boldsymbol{R}_{\mathrm{a}} = \mathrm{E}\left\{|y|^2 \boldsymbol{x}\boldsymbol{x}^H\right\}$ and $\boldsymbol{d} = \mathrm{E}\left\{\boldsymbol{x}y^*\right\}$ are estimated using the previously computed weight vector. Finally the optimization problem can be cast as

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{R}_{\mathrm{a}} \boldsymbol{w} - 2\gamma \operatorname{Re}\left\{\boldsymbol{w}^H \boldsymbol{d}\right\}, \tag{2.53}$$

$$\text{s. t. } \operatorname{Re}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} - \delta \geq \tilde{\epsilon} \left\|\boldsymbol{w}\right\|_2^2 \tag{2.54}$$

Using the method of Lagrange multipliers gives

$$\begin{aligned}\mathcal{L}_{\mathrm{CCM}}\left(\boldsymbol{w}, \lambda\right) =& \boldsymbol{w}^H \boldsymbol{R}_{\mathrm{a}} \boldsymbol{w} - 2\,\gamma \operatorname{Re}\left\{\boldsymbol{w}^H \boldsymbol{d}\right\} \\ &+ \lambda\left[\tilde{\epsilon}\; \boldsymbol{w}^H \boldsymbol{w} - \operatorname{Re}\left\{\boldsymbol{w}^H \boldsymbol{a}\right\} + \delta\right].\end{aligned} \tag{2.55}$$

Computing the gradient of (2.55) with respect to $\boldsymbol{w}^*$, and equating it to zero leads to

$$\boldsymbol{w} = \left[\boldsymbol{R}_{\mathrm{a}} + \tilde{\epsilon}\lambda \boldsymbol{I}\right]^{-1} \left[\gamma \boldsymbol{d} + \lambda \boldsymbol{a}/2\right] . \tag{2.56}$$

The adjustment of the Lagrange multiplier λ can be done in the same way as in the minimum variance case.

2.3.2 Adaptive Algorithms

To take advantage of the joint optimization approach an *on-line* modified conjugate gradient method, with one iteration per snapshot is used to solve the resulting problem. Its derivation is based on [CW00] and it can be interpreted as an extension of the idea in [WdL10].

Robust-CMV-MCG

In the algorithm an exponentially decayed data window is used to estimate $\boldsymbol{R}_{xx}$

$$\hat{\boldsymbol{R}}_{xx}(i) = \mu \hat{\boldsymbol{R}}_{xx}(i-1) + \boldsymbol{x}(i)\boldsymbol{x}^H(i) , \tag{2.57}$$

where μ is the forgetting factor. According to [Tre02]

$$\boldsymbol{R}_{xx} \simeq (1-\mu)\, \hat{\boldsymbol{R}}_{xx}(i) \tag{2.58}$$

can be assumed for large i. Replacing $\boldsymbol{R}_{xx}$ in (2.49), introducing $\hat{\lambda}(i) = \frac{\lambda(i)}{1-\mu}$, leads to $\boldsymbol{w}(i) = [\hat{\boldsymbol{R}}_{xx}(i) + \tilde{\epsilon}\hat{\lambda}(i)\boldsymbol{I}]^{-1}\hat{\lambda}(i)\boldsymbol{a}/2$. Let us introduce the CG weight vector $\boldsymbol{v}(i)$ as follows $\boldsymbol{w}(i) = \boldsymbol{v}(i)\frac{\hat{\lambda}(i)}{2}$. The conjugate gradient algorithm solves the problem by iteratively updating the CG weight vector

$$\boldsymbol{v}(i) = \boldsymbol{v}(i-1) + \alpha(i)\boldsymbol{p}(i), \tag{2.59}$$

where $\boldsymbol{p}(i)$ is the direction vector and $\alpha(i)$ is the adaptive step size. One way [CW00] to realize the conjugate gradient method performing one iteration per snapshot is the application of the degenerated scheme. Under this condition the adaptive step size $\alpha(i)$ has to fulfill the convergence bound given by

$$0 \leq \boldsymbol{p}^H(i)\boldsymbol{g}(i) \leq 0.5\ \boldsymbol{p}^H(i)\boldsymbol{g}(i-1). \tag{2.60}$$

To rearrange (2.60) the negative gradient vector and its recursive expression are considered as

$$\begin{aligned}\boldsymbol{g}(i) =& \boldsymbol{a} - [\hat{\boldsymbol{R}}_{xx}(i) + \tilde{\epsilon}\hat{\lambda}(i)\boldsymbol{I}]\boldsymbol{v}(i)\\ =& \boldsymbol{a}[1-\mu] + \mu\boldsymbol{g}(i-1)\\ &- [\boldsymbol{x}\boldsymbol{x}^H + \tilde{\epsilon}(\hat{\lambda}(i) - \mu\hat{\lambda}(i-1))\boldsymbol{I}]\boldsymbol{v}(i-1)\\ &- \alpha(i)[\hat{\boldsymbol{R}}_{xx}(i) + \tilde{\epsilon}\hat{\lambda}(i)\boldsymbol{I}]\boldsymbol{p}(i)\end{aligned} \tag{2.61}$$

Premultiplying with $\boldsymbol{p}^H(i)$, taking the expectation from both sides and considering $\boldsymbol{p}(i)$ uncorrelated with $\boldsymbol{a}$, $\boldsymbol{x}(i)$ and $\boldsymbol{v}(i-1)$ leads to

$$\begin{aligned}\mathrm{E}\left\{\boldsymbol{p}^H(i)\boldsymbol{g}(i)\right\} \approx& \mu\mathrm{E}\left\{\boldsymbol{p}^H(i)\boldsymbol{g}(i-1)\right\}\\ &- \mathrm{E}\left\{\alpha(i)\right\}\mathrm{E}\left\{\boldsymbol{p}^H(i)[\hat{\boldsymbol{R}}_{xx}(i) + \tilde{\epsilon}\hat{\lambda}(i)\boldsymbol{I}]\boldsymbol{p}(i)\right\}.\end{aligned} \tag{2.62}$$

Here it is assumed that the algorithm has already converged, which implies $\boldsymbol{a}[1-\mu] - [\mathrm{E}\left\{\boldsymbol{x}\boldsymbol{x}^H\right\} + \tilde{\epsilon}\hat{\lambda}(i)[1-\mu]\boldsymbol{I}]\boldsymbol{v}(i-1) = \boldsymbol{0}$, where equation (2.58) is taken into account and $\hat{\lambda}(i) \approx \hat{\lambda}(i-1)$. Introducing $\boldsymbol{p}_R = [\hat{\boldsymbol{R}}_{xx}(i) + \lambda\hat{(i)}\tilde{\epsilon}\boldsymbol{I}]\boldsymbol{p}(i)$, rearranging (2.62) and plugging into (2.60) determines the stepsize within its boundaries as follows

$$\alpha(i) = \left[\boldsymbol{p}^H(i)\boldsymbol{p}_R\right]^{-1}(\mu - \eta)\,\boldsymbol{p}^H(i)\boldsymbol{g}(i-1), \tag{2.63}$$

where $0 \leq \eta \leq 0.5$. The direction vector is a linear combination from the previous direction vector and the negative gradient.

$$\boldsymbol{p}(i+1) = \boldsymbol{p}(i) + \beta(i)\boldsymbol{g}(i), \tag{2.64}$$

where $\beta(i)$ is computed for avoiding the reset procedure by employing the Polak-Ribiere approach [Lue84].

$$\beta = [\boldsymbol{g}^H(i-1)\boldsymbol{g}(i-1)]^{-1}[\boldsymbol{g}(i) - \boldsymbol{g}(i-1)]^H\boldsymbol{g}(i) \tag{2.65}$$

The proposed algorithm, which is termed Robust-CMV-MCG, is described in Table 2.3.

Table 2.3: Proposed RCMV-MCG Algorithm

$\boldsymbol{v}(0) = \boldsymbol{0}$; $\boldsymbol{p}(1) = \boldsymbol{g}(0) = \boldsymbol{a}$; $\hat{\boldsymbol{R}}(0) = \delta\boldsymbol{I}$; $\hat{\lambda}(0) = \hat{\lambda}(1) = \hat{\lambda}_0$
For each time instant $i = 1, ..., N$
$\hat{\boldsymbol{R}}_{xx}(i) = \mu\hat{\boldsymbol{R}}_{xx}(i-1) + \boldsymbol{x}(i)\boldsymbol{x}^H(i)$
$\boldsymbol{p}_R = [\hat{\boldsymbol{R}}_{xx}(i) + \hat{\lambda}(i)\tilde{\epsilon}\boldsymbol{I}]\boldsymbol{p}(i)$; $\quad \nu = \left[\hat{\lambda}(i) - \mu\hat{\lambda}(i-1)\right]\tilde{\epsilon}$
$\alpha(i) = \left[\boldsymbol{p}^H(i)\boldsymbol{p}_R\right]^{-1}(\mu-\eta)\,\boldsymbol{p}^H(i)\boldsymbol{g}(i-1)$; $\quad (0 \leq \eta \leq 0.5)$
$\boldsymbol{v}(i) = \boldsymbol{v}(i-1) + \alpha(i)\boldsymbol{p}(i)$
$\boldsymbol{g}(i) = [1-\mu]\boldsymbol{a} + \mu\boldsymbol{g}(i-1) - \alpha(i)\boldsymbol{p}_R$
$\quad - \left(\boldsymbol{x}(i)\boldsymbol{x}^H(i) + \nu\boldsymbol{I}\right)\boldsymbol{v}(i-1)$
$\beta(i) = \left[\boldsymbol{g}^H(i-1)\boldsymbol{g}(i-1)\right]^{-1}[\boldsymbol{g}(i) - \boldsymbol{g}(i-1)]^H\,\boldsymbol{g}(i)$
$\boldsymbol{p}(i+1) = \boldsymbol{g}(i) + \beta(i)\boldsymbol{p}(i)$
$\boldsymbol{w}(i) = \lambda(i)\boldsymbol{v}(i)/2$
$\delta_\lambda = \mu_\lambda[\tilde{\epsilon}\,\Vert\boldsymbol{w}(i)\Vert_2^2 - \mathrm{Re}\left\{\boldsymbol{w}^H(i)\boldsymbol{a}\right\} + \delta]$
while $\delta_\lambda \leq -\lambda(i)$ or $\delta_\lambda \geq \delta_{\lambda\max}$
$\quad \delta_\lambda \Rightarrow \delta_\lambda/2$
end
$\hat{\lambda}(i+1) = \hat{\lambda}(i) + \delta_\lambda$

Note that, for the parallel algorithm to adjust the Lagrange multiplier, we divide the update-term by 2, if the Lagrange multiplier is outside a predefined range, as it is described in Table I. The application of the proposed algorithm corresponds to a computational effort which is quadratic with the number of sensor elements M.

Robust-CCM-MCG

The adaptive algorithm in case of the constrained constant modulus criterion is developed analogously to the minimum variance case. The estimates of $\boldsymbol{R}_a$ and $\boldsymbol{d}$ are based on an exponentially decayed data window.

$$\begin{aligned} \hat{\boldsymbol{R}}_{\mathrm{a}}(i) &= \mu\hat{\boldsymbol{R}}_{\mathrm{a}}(i-1) + |y(i)|^2\,\boldsymbol{x}(i)\boldsymbol{x}^H(i) && (2.66)\\ \hat{\boldsymbol{d}}(i) &= \mu\hat{\boldsymbol{d}}(i-1) + \boldsymbol{x}(i)y^*(i) && (2.67)\end{aligned}$$

Following the steps, while taking into account that

$$\begin{aligned} \boldsymbol{R}_{\mathrm{a}} &= [1-\mu]\hat{\boldsymbol{R}}_{\mathrm{a}}(i) && (2.68)\\ \boldsymbol{d} &= [1-\mu]\hat{\boldsymbol{d}}(i) && (2.69)\end{aligned}$$

leads to the adaptive algorithm. Note, in contrast to the CMV case, here the beamforming weight vector is the same as the conjugate gradient weight vector, which means $\boldsymbol{w} = [\hat{\boldsymbol{R}}_{\mathrm{a}}+\tilde{\epsilon}\hat{\lambda}\boldsymbol{I}]^{-1}[\gamma\hat{\boldsymbol{d}}+\hat{\lambda}\boldsymbol{a}/2]$. The negative gradient vector and its recursive expression are defined as

$$\begin{aligned} \boldsymbol{g}(i) =& [\gamma\hat{\boldsymbol{d}} + \hat{\lambda}\boldsymbol{a}/2] - [\hat{\boldsymbol{R}}_{\mathrm{a}} + \tilde{\epsilon}\hat{\lambda}\boldsymbol{I}]\boldsymbol{w}(i)\\ =& \mu\boldsymbol{g}(i-1) - \alpha(i)\boldsymbol{p}_R - \left(|y(i)|^2\,\boldsymbol{x}(i)\boldsymbol{x}^H(i)\right)\boldsymbol{w}(i-1)\\ &+ \gamma\boldsymbol{x}(i)y^*(i) + \nu\left[\boldsymbol{a}/\left(2\tilde{\epsilon}\right) - \boldsymbol{w}(i-1)\right], && (2.70)\end{aligned}$$

where $\nu = \left[\hat{\lambda}(i) - \mu\hat{\lambda}(i-1)\right]\tilde{\epsilon}$. The proposed algorithm, which is termed Robust-CCM-MCG, is described in Table 2.4.

2.3.3 Simulations

For the simulations a uniform linear sensor array with 10 sensor elements is considered. The forgetting factor $\mu = 0.995$ is chosen. The step sizes are $\mu_\lambda(\mathrm{CMV}) = 800$ and $\mu_\lambda(\mathrm{CCM}) = 100$. The update limitation is set to $\delta_{\lambda max} = 200$. For the robust constraints $\epsilon = \tilde{\epsilon} = 2.1$ holds. According to the different constraint functions, the equality is a special case for $M = 10$. For the loaded sample matrix inversion beamformer (Loaded-SMI) the diagonal loading factor is chosen as $10\ \sigma_n^2$. Besides the desired user (user 1) there are 4 interferers, whose relative powers (P) with respect to the desired user and directions of arrival (DoA) in degrees are detailed in Table 2.5. At $i = 1001$ the beamformers are confronted with a scenario change.

Table 2.4: Proposed RCCM-MCG Algorithm

$\boldsymbol{p}(1) = \boldsymbol{g}(0) = \boldsymbol{a};\ \hat{\boldsymbol{R}}_{\mathrm{a}}(0) = \delta \boldsymbol{I};\ \hat{\boldsymbol{d}}(0) = \boldsymbol{0};$
$\hat{\lambda}(0) = \hat{\lambda}(1) = \hat{\lambda}_0;\ \boldsymbol{w} = \boldsymbol{a}/M$
For each time instant $i = 1, ..., N$
$\hat{\boldsymbol{R}}_{\mathrm{a}}(i) = \mu \hat{\boldsymbol{R}}_{\mathrm{a}}(i-1) + |y(i)|^2 \boldsymbol{x}(i)\boldsymbol{x}^H(i)$
$\boldsymbol{p}_R = [\hat{\boldsymbol{R}}_{\mathrm{a}}(i) + \hat{\lambda}(i)\tilde{\epsilon}\boldsymbol{I}]\boldsymbol{p}(i);\quad \nu = \left[\hat{\lambda}(i) - \mu\hat{\lambda}(i-1)\right]\tilde{\epsilon}$
$\alpha(i) = \left[\boldsymbol{p}^H(i)\boldsymbol{p}_R\right]^{-1}(\mu-\eta)\,\boldsymbol{p}^H(i)\boldsymbol{g}(i-1);\quad (0 \le \eta \le 0.5)$
$\boldsymbol{w}(i) = \boldsymbol{w}(i-1) + \alpha(i)\boldsymbol{p}(i)$
$\boldsymbol{g}(i) = \mu\boldsymbol{g}(i-1) - \alpha(i)\boldsymbol{p}_R - \left(|y(i)|^2 \boldsymbol{x}(i)\boldsymbol{x}^H(i)\right)\boldsymbol{w}(i-1)$
$\quad + \gamma\boldsymbol{x}(i)y^*(i) + \nu\left[\boldsymbol{a}/\left(2\tilde{\epsilon}\right) - \boldsymbol{w}(i-1)\right]$
$\beta(i) = \left[\boldsymbol{g}^H(i-1)\boldsymbol{g}(i-1)\right]^{-1}\left[\boldsymbol{g}(i) - \boldsymbol{g}(i-1)\right]^H \boldsymbol{g}(i)$
$\boldsymbol{p}(i+1) = \boldsymbol{g}(i) + \beta(i)\boldsymbol{p}(i)$

$\delta_{\hat{\lambda}} = \mu_{\hat{\lambda}}[\hat{\epsilon}\,\|\boldsymbol{w}(i)\|_2^2 - \mathrm{Re}\left\{\boldsymbol{w}^H(i)\boldsymbol{a}\right\} + \delta]$
while $\delta_\lambda \le -\hat{\lambda}(i)$ or $\delta_\lambda \ge \delta_{\lambda\max}$
$\quad \delta_\lambda \Rightarrow \delta_\lambda/2$
end
$\hat{\lambda}(i+1) = \hat{\lambda}(i) + \delta_\lambda$

Table 2.5: Interference scenario
P(dB) relative to user1 / DoA(degrees)

snapshot	user 1 (desired user)	user 2	user 3	user 4	user 5
1-1000	0/93°	10/120°	5/140°	10/150°	7/105°
1001-2000	0/93°	30/120°	34/170°	6/104°	9/68°

The signal steering vector is corrupted by local coherent scattering, where

$$\breve{\boldsymbol{a}} = \boldsymbol{a} + \sum_{k=1}^{4} \mathrm{e}^{j\Phi_k}\boldsymbol{a}_{\mathrm{sc}}\left(\theta_k\right) \tag{2.71}$$

and Φ_k is uniformly distributed between zero and 2π and θ_k is uniformly distributed with the assumed direction as the mean and a standard deviation of 2 degrees [VGL03]. The mismatch changes for every realization and is fixed over the snapshots.

Fig. 2.6 shows the SINR performance over the number of snapshots. The suddenly appearing performance degradation is caused by the event happening at $i = 1001$. The simulation is performed at 0dB SNR. The plot shows the average over 1000 simulation

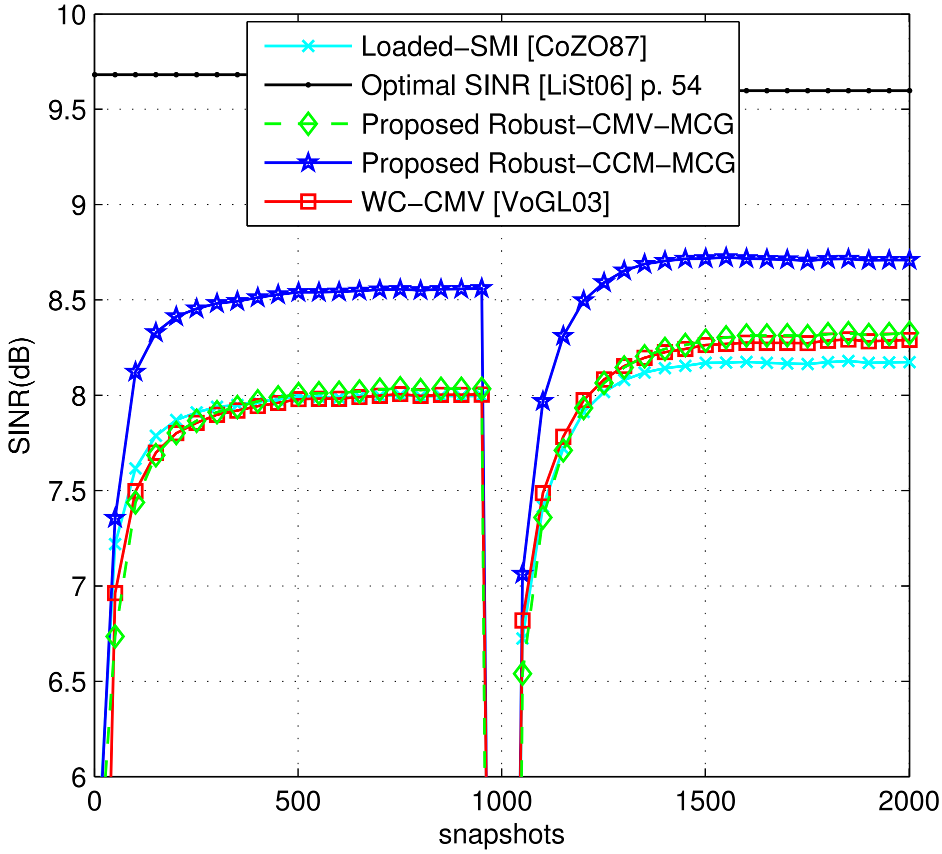

Figure 2.6: SINR performance versus number of snapshots, SNR = 0dB

runs. Fig. 2.7 displays the dependency on the SNR. Here the SINR is recorded for snapshot $i = 1500$. Fig. 2.8 confirms that our proposed low-complexity method is comparable to the worst-case optimization based method.

According to the Fig. 2.6 and Fig. 2.7 the Robust-CMV-MCG algorithm performs equivalently to [VGL03], and the Robust-CCM-MCG outperforms [VGL03] especially at high SNR values. Fig. 2.8 confirms the comparable performance of our proposed low-complexity method to the worst-case optimization based method. In addition, the Robust-CCM-MCG beamformer shows a higher mean as well as a lower standard deviation in terms of SINR performance. Note that the constant modulus design benefits from the knowledge of the constant modulus property of the desired user signal.

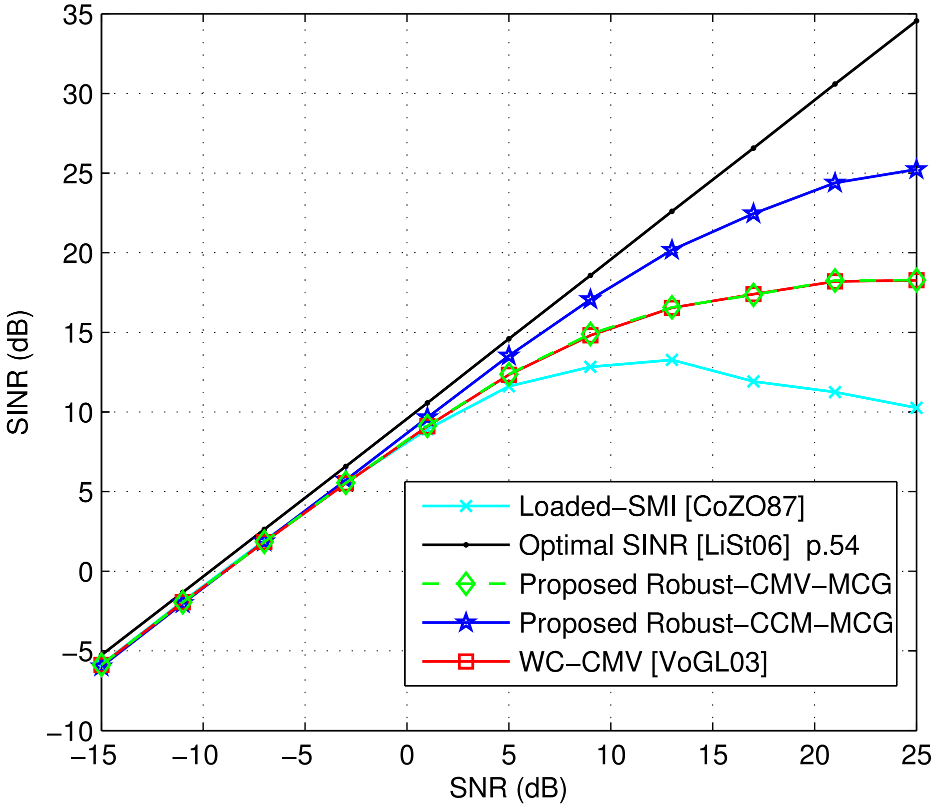

Figure 2.7: SINR performance versus SNR, $i = 1500$

2.3.4 Conclusions and Futurework

Conclusion

This section presents the development of low-comlexity robust adaptive beamforming algorithms, the Robust-CMV-MCG and the Robust-CCM-MCG. They use a constraint similar to the worst-case optimization based approach. It is shown that the joint optimization approach allows the exploitation of highly efficient *on-line* algorithms like the modified conjugate gradient method which performs just one iteration per snapshot. As a result the complexity is reduced by more than an order of magnitude compared to the worst-case optimization based beamformer which is solved with a second order cone program. The proposed Robust-CCM-MCG algorithm based on the Constant Modulus design criterion, shows a better performance which takes advantage of the costant modulus property of the signal amplitude of the desired user.

Futurework

The proposed algorithms are ready to be implemented in a digital hardware.

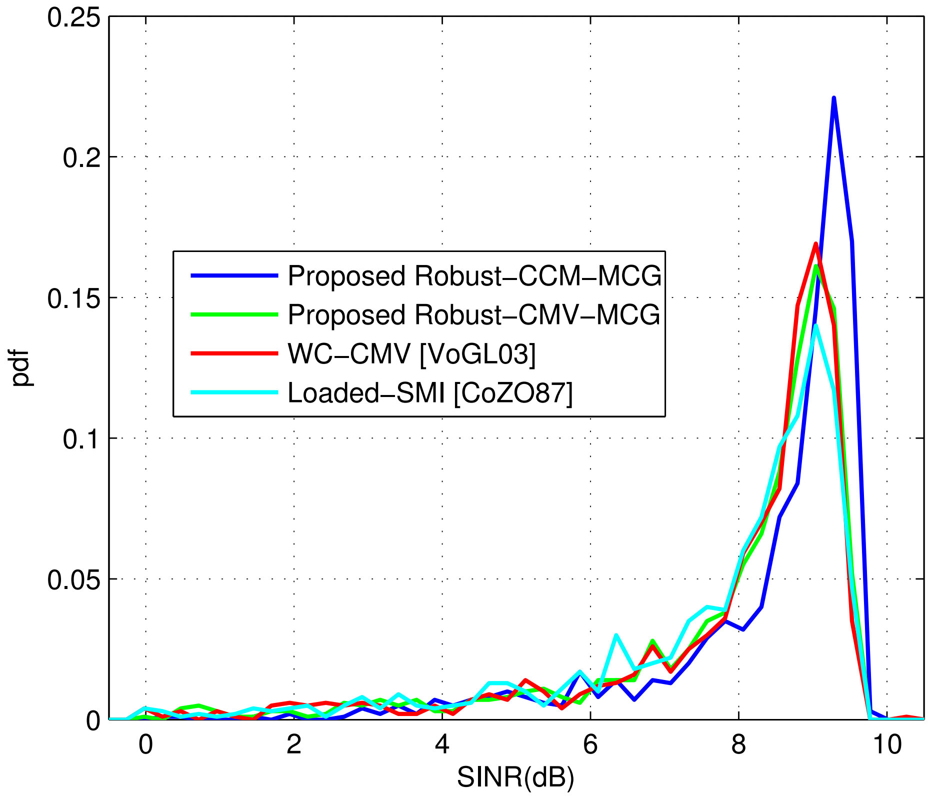

Figure 2.8: Probability density function of the output SINR, $i = 2000$, SNR $= 0$dB

3 Low-Complexity Mismatch Estimation (LOCME) for Robust Beamforming

In this chapter an advanced method is proposed which adapts on the array steering vector mismatch. Unlike the existing method [HV08], which has inapplicable computational requirements in a real-time application, the proposed method termed Low-Complexity Mismatch Estimation (LOCME) requires a low computational burden and does not even imply an optimization algorithm. The LOCME method describes the estimation of the array steering vector as the projection onto a predefined subspace of the correlation between the beamforming output signal and the array observation vector. The proposed algorithms in this chapter are published in [LdLH11a].

3.1 Proposed Low-Complexity Mismatch Estimation

The LOCME approach is based on the following relations

$$\begin{aligned} \boldsymbol{d} &= \mathrm{E}\left\{\boldsymbol{x}y^*\right\} &(3.1)\\ &= \mathrm{E}\left\{\left(\boldsymbol{A}\boldsymbol{s}+\boldsymbol{n}\right)\left(\boldsymbol{A}\boldsymbol{s}+\boldsymbol{n}\right)^H\boldsymbol{w}\right\}, &(3.2)\end{aligned}$$

where $\boldsymbol{x}$ is the array observation vector, y represents the output of the beamformer and $\boldsymbol{A} = [\boldsymbol{a}_1, ..., \boldsymbol{a}_D]$ contains the array steering vector of the desired user $\boldsymbol{a}_1$ and the array steering vectors from other impinging signals. The array steering vector can be modeled as $\boldsymbol{a}_1 = \boldsymbol{a}(\theta_1) + \boldsymbol{e}$, where $\boldsymbol{a}(\theta_1)$ is known by the system and $\boldsymbol{e}$ is the mismatch vector. Furthermore it is assumed that $\left|\boldsymbol{a}_m^H\boldsymbol{w}\right| \ll \left|\boldsymbol{a}_1^H\boldsymbol{w}\right|$ for $m = 2...D$. So, the vector $\boldsymbol{d}$ can be rewritten as

$$\boldsymbol{d} = \mathrm{E}\left\{\left(\boldsymbol{A}\boldsymbol{s}+\boldsymbol{n}\right)\left(s_1^*\boldsymbol{a}_1^H\boldsymbol{w}+\boldsymbol{n}^H\boldsymbol{w}\right)\right\}, \qquad (3.3)$$

where $\boldsymbol{a}_1$ is the value to be estimated by the LOCME method. In addition, the desired signal is assumed to be independent from the noise and the other users, which allows the following notation

$$\boldsymbol{d} = \mathrm{E}\left\{|s_1|^2\, c\, \boldsymbol{a}_1 + \boldsymbol{n}\boldsymbol{n}^H \boldsymbol{w}\right\}, \tag{3.4}$$

where $c = \boldsymbol{a}_1^H \boldsymbol{w}$. Obviously a scaled version of the unknown array steering vector is a component of $\boldsymbol{d}$. To eliminate the unwanted part, $\boldsymbol{d}$ can be projected onto a predefined subspace. Here some prior information can be used. In our example we assume that the desired array steering vector is a superposition of different array steering vectors in a predefined range like in [HV08]. Let us consider a range from $\theta_1 \pm \theta_{\mathrm{E}}$. With that knowledge we compute a matrix as follows

$$\boldsymbol{C} = \int_{\theta_1 - \theta_{\mathrm{E}}}^{\theta_1 + \theta_{\mathrm{E}}} \boldsymbol{a}(\theta)\boldsymbol{a}^H(\theta)\mathrm{d}\theta. \tag{3.5}$$

Depending on the application this matrix needs to be recomputed. The K normalized principal eigenvectors of $\boldsymbol{C}$ form the projection matrix

$$\boldsymbol{P} = [\boldsymbol{c}_1, \boldsymbol{c}_2, ..., \boldsymbol{c}_K]\,[\boldsymbol{c}_1, \boldsymbol{c}_2, ..., \boldsymbol{c}_K]^H \tag{3.6}$$

The resulting estimate of the array steering vector is given by

$$\hat{\boldsymbol{a}}_1 = \sqrt{M}\frac{\boldsymbol{P}\boldsymbol{d}}{\|\boldsymbol{P}\boldsymbol{d}\|_2} \tag{3.7}$$

Since, the proposed technique depends on the fact that the beamformer is already working, it is reasonable to combine it with already existing robust adaptive beamforming algorithms like the worst-case optimization based algorithm.

3.2 Proposed Adaptive Agorithms using LOCME

This section presents the LOCME technique using the worst-case optimization based approach in the constrained minimum variance design, which is a SOCP and also the LOCME using the robust constrained constant modulus design, which is designed as a QCQP. In each adaptive algorithm the LOCME method is implemented with the following recursions

$$\hat{\boldsymbol{d}}(i) = \mu\hat{\boldsymbol{d}}(i-1) + \boldsymbol{x}(i)y^*(i), \tag{3.8}$$

$$\hat{\boldsymbol{a}}_1(i) = \sqrt{M}\frac{\boldsymbol{P}\hat{\boldsymbol{d}}(i)}{\left\|\boldsymbol{P}\hat{\boldsymbol{d}}(i)\right\|_2}, \tag{3.9}$$

where $\boldsymbol{P}$ is the projection matrix with rank K given by (3.6) and $\hat{\boldsymbol{a}}_1(i)$ is the estimate of the array steering vector.

3.2.1 WC-CMV-LOCME

The adaptive worst-case optimization based beamformer [VGL03] using the LOCME feature is cast as follows

$$\begin{aligned} \min_{\boldsymbol{w}} \quad \boldsymbol{w}^H\hat{\boldsymbol{R}}_{xx}(i)\boldsymbol{w} \;\; \text{s. t. } & \operatorname{Re}\left\{\boldsymbol{w}^H\hat{\boldsymbol{a}}_1(i)\right\} - \delta \geq \epsilon\left\|\boldsymbol{w}\right\|_2 \\ & \operatorname{Im}\left\{\boldsymbol{w}^H\hat{\boldsymbol{a}}_1(i)\right\} = 0, \end{aligned} \tag{3.10}$$

where $\hat{\boldsymbol{a}}_1(i)$ is the adaptive LOCME estimate according to (3.9), the signal covariance matrix is replaced by its exponentially decayed data window estimate $\hat{\boldsymbol{R}}_{xx}(i) = \mu\hat{\boldsymbol{R}}_{xx}(i-1) + \boldsymbol{x}(i)\boldsymbol{x}^H(i)$ and ϵ is related to the mismatch level. Since the convex optimization problem does not change compared to [VGL03], it is a SOCP which can be solved using interior point methods with a complexity of $\mathcal{O}(M^{3.5})$.

3.2.2 RCCM-LOCME

For received signals which are constant modulus, the constant modulus design criterion [dLHSN08] [dLSN05] exploits the knowledge of the modulus of the desired signal. The robust constrained constant modulus (RCCM) algorithm can be obtained from the following optimization problem

$$\min_{\boldsymbol{w}} \quad \mathrm{E}\left\{\left(|y|^2-\gamma\right)^2\right\} \quad \text{s. t. } \operatorname{Re}\left\{\boldsymbol{w}^H\boldsymbol{a}\right\} - \delta \geq \tilde{\epsilon}\left\|\boldsymbol{w}\right\|_2^2, \tag{3.11}$$

where $\gamma \geq 0$ is a parameter related to the energy of the desired signal, which is assumed as known. A closed-form solution is not possible because it is a fourth order function with a more complicated structure but it can be solved iteratively as follows. While the presumed array steering vector is replaced with its LOCME estimate $\hat{\boldsymbol{a}}_1(i)$ (3.9) the optimization problem can be cast as

$$\begin{aligned} \min_{\boldsymbol{w}} \quad & \boldsymbol{w}^H \hat{\boldsymbol{R}}_a(i)\boldsymbol{w} - 2\gamma \ \mathrm{Re}\left\{\hat{\boldsymbol{d}}^H(i)\boldsymbol{w}\right\} \\ \text{s. t.} \quad & \mathrm{Re}\left\{\boldsymbol{w}^H\hat{\boldsymbol{a}}_1(i)\right\} - \delta \geq \tilde{\epsilon}\,\|\boldsymbol{w}\|_2^2\,, \end{aligned} \tag{3.12}$$

where $\hat{\boldsymbol{R}}_a(i) = \mu\hat{\boldsymbol{R}}_a(i-1) + |y|^2\,\boldsymbol{x}(i)\boldsymbol{x}^H(i)$, $\hat{\boldsymbol{d}}(i)$ is described in (3.8) and $\tilde{\epsilon}$ is the parameter controlling the robustness. The optimization problem is a convex QCQP which can be solved using interior point methods with a complexity of $\mathcal{O}(M^{3.5})$.

3.3 LOCME using Algorithms based on the MCG

It is reasonable to combine LOCME with other low-complexity approaches. We integrate the LOCME in the recently developed low-complexity robust algorithms [LdLWH11] based on joint optimization and modified conjugate gradient [CW00], which reduces the complexity by more than an order of magnitude to $\mathcal{O}(M^2)$. Unlike the algorithms in [LdLWH11], that belong to the class of diagonal loading techniques, the proposed algorithms employ LOCME. The resulting algorithms are termed the Robust Constrained Minimum Variance design based on the Modified Conjugate Gradient using the LOCME (RCMV-MCG-LOCME) and the Robust Constrained Constant Modulus design based on the Modified Conjugate Gradient Algorithm using the LOCME (RCCM-MCG-LOCME).

3.3.1 Proposed RCMV-LOCME-MCG

Here the low-complexity algorithm corresponding to the minimum variance criterion, whose constraint is similar to the worst-case based optimization is presented. It is based on the optimization problem defined as

$$\min_{\boldsymbol{w}} \quad \boldsymbol{w}^H\hat{\boldsymbol{R}}_{xx}(i)\boldsymbol{w} \quad \text{s. t.} \quad \mathrm{Re}\left\{\boldsymbol{w}^H\hat{\boldsymbol{a}}_1(i)\right\} - \delta \geq \tilde{\epsilon}\,\|\boldsymbol{w}\|_2^2\,, \tag{3.13}$$

where $\hat{\boldsymbol{R}}_{xx}(i)$ is the exponentially decayed data window estimate of the signal covariance matrix and $\hat{\boldsymbol{a}}_1(i)$ is the LOCME estimate from (3.9). Using the method of Lagrange multipliers, taking the gradient of the Lagrangian with respect to $\boldsymbol{w}^*$, and equating it to zero gives

$$\boldsymbol{w} = \left[\hat{\boldsymbol{R}}_{xx}(i) + \tilde{\epsilon}\lambda\boldsymbol{I}\right]^{-1} \lambda\hat{\boldsymbol{a}}_1(i)/2, \tag{3.14}$$

which can be effectively solved by *on-line* algorithms with one iteration per snapshot. Here we choose a modified conjugate gradient [CW00]. In a parallel algorithm the Lagrange multiplier λ is adjusted as follows

$$\lambda(i) = \lambda(i-1) + \mu_\lambda\left[\tilde{\epsilon}\,\|\boldsymbol{w}(i)\|_2^2 - \mathrm{Re}\left\{\boldsymbol{w}^H(i)\hat{\boldsymbol{a}}(i)\right\} + \delta\right]\ , \tag{3.15}$$

where μ_λ is the stepsize. The algorithm increases λ in case of a not fullfilled constraint. Table shows the algorithm.

Table 3.1: Proposed RCMV-MCG-LOCME Algorithm

$\boldsymbol{C} = \int_{\theta_1-\theta_{\mathrm{E}}}^{\theta_1+\theta_{\mathrm{E}}} \boldsymbol{a}(\theta)\boldsymbol{a}^H(\theta)$
$[\boldsymbol{c}_1, \boldsymbol{c}_2, ..., \boldsymbol{c}_K] =$ principal eigenvectors$(\boldsymbol{C})$
$\boldsymbol{P} = [\boldsymbol{c}_1, \boldsymbol{c}_2, ..., \boldsymbol{c}_K]\,[\boldsymbol{c}_1, \boldsymbol{c}_2, ..., \boldsymbol{c}_K]^H$
$\hat{\boldsymbol{R}}_{xx}(0) = \sigma_n^2\boldsymbol{I};\ \hat{\boldsymbol{d}}(0) = \boldsymbol{0};\ \lambda(0) = \lambda_0$
For each time instant $i = 1, ..., N$
$\quad\hat{\boldsymbol{R}}_{xx}(i) = \mu\hat{\boldsymbol{R}}_{xx}(i-1) + \boldsymbol{x}(i)\boldsymbol{x}^H(i)$
$\quad\hat{\boldsymbol{d}}(i) = \mu\hat{\boldsymbol{d}}(i-1) + \boldsymbol{x}(i)y^*(i)$
$\quad\hat{\boldsymbol{a}}_1(i) = \sqrt{M}\frac{\boldsymbol{Pd}(i)}{\|\boldsymbol{Pd}(i)\|_2}$
to solve with an online MCG [CW00]:
$\quad\boldsymbol{w}(i) = [\hat{\boldsymbol{R}}_{xx}(i) + \tilde{\epsilon}\lambda(i)\boldsymbol{I}]^{-1}\lambda(i)\hat{\boldsymbol{a}}_1(i)/2$
$\quad\delta_\lambda = \mu_\lambda[\tilde{\epsilon}\,\|\boldsymbol{w}(i)\|_2^2 - \mathrm{Re}\left\{\boldsymbol{w}^H(i)\hat{\boldsymbol{a}}_1(i)\right\} + \delta]$
while $\delta_\lambda \leq -\lambda(i)$ or $\delta_\lambda \geq \delta_{\lambda\max}$
$\quad\quad\delta_\lambda \Rightarrow \delta_\lambda/2$
end
$\quad\lambda(i+1) = \lambda(i) + \delta_\lambda$

3.3.2 Proposed RCCM-MCG-LOCME

In this section we present the constant modulus design of the low-complexity algorithm shown in the previous section. The optimization problem is the QCQP of (3.12) but now it is solved with a low computational effort. Using the method of Lagrange multipliers, taking the gradient of the Lagrangian with respect to $\boldsymbol{w}^*$, and equating it to zero gives

$$\boldsymbol{w} = \left[\hat{\boldsymbol{R}}_a(i) + \tilde{\epsilon}\lambda \boldsymbol{I}\right]^{-1} \left[\hat{\boldsymbol{d}}(i) + \lambda \hat{\boldsymbol{a}}_1(i)/2\right]. \tag{3.16}$$

The solution is obtained in an analogous form to the minimum variance case, which implies a modified conjugate gradient method with one iteration per snapshot and an adjustment of the Lagrange multiplier in a parallel algorithm (3.15). The RCCM-MCG-LOCME algorithm is given in Table 3.2.

Table 3.2: Proposed RCCM-MCG-LOCME Algorithm

$\boldsymbol{C} = \int_{\theta_1-\theta_{\mathrm{E}}}^{\theta_1+\theta_{\mathrm{E}}} \boldsymbol{a}(\theta)\boldsymbol{a}^H(\theta)$
$[\boldsymbol{c}_1, \boldsymbol{c}_2, ..., \boldsymbol{c}_K]$ = principal eigenvectors($\boldsymbol{C}$)
$\boldsymbol{P} = [\boldsymbol{c}_1, \boldsymbol{c}_2, ..., \boldsymbol{c}_K]\,[\boldsymbol{c}_1, \boldsymbol{c}_2, ..., \boldsymbol{c}_K]^H$
$\hat{\boldsymbol{R}}_a(0) = \sigma_n^2 \boldsymbol{I};\ \hat{\boldsymbol{d}}(0) = \boldsymbol{0};\ \lambda(0) = \lambda_0;\ \boldsymbol{w}(0) = \boldsymbol{a}/M$
For each time instant $i = 1, ..., N$
  $\hat{\boldsymbol{R}}_a(i) = \mu \hat{\boldsymbol{R}}_a(i-1) + |y(i)|^2\, \boldsymbol{x}(i)\boldsymbol{x}^H(i)$
  $\hat{\boldsymbol{d}}(i) = \mu \hat{\boldsymbol{d}}(i-1) + \boldsymbol{x}(i)y^*(i)$
  $\hat{\boldsymbol{a}}_1(i) = \sqrt{M}\frac{\boldsymbol{P}\boldsymbol{d}(i)}{\|\boldsymbol{P}\boldsymbol{d}(i)\|_2}$

  to solve with an online MCG [CW00]:
  $\boldsymbol{w}(i) = [\hat{\boldsymbol{R}}_a(i) + \tilde{\epsilon}\lambda(i)\boldsymbol{I}]^{-1}[\hat{\boldsymbol{d}}(i) + \lambda(i)\hat{\boldsymbol{a}}_1(i)/2]$

  $\delta_\lambda = \mu_\lambda[\tilde{\epsilon}\,\|\boldsymbol{w}(i)\|_2^2 - \mathrm{Re}\left\{\boldsymbol{w}^H(i)\hat{\boldsymbol{a}}_1(i)\right\} + \delta]$
  while $\delta_\lambda \leq -\lambda(i)$ or $\delta_\lambda \geq \delta_{\lambda\max}$
    $\delta_\lambda \Rightarrow \delta_\lambda/2$
  end
  $\lambda(i+1) = \lambda(i) + \delta_\lambda$

3.4 Simulations

In this section the performance of the proposed and existing algorithms is assessed. The following parameters are choosen: $\epsilon = \tilde{\epsilon} = 2.1$, $K = 2$, $M = 10$, $\delta_{\lambda\max} = 200$, $\mu = 0.995$, $\mu_\lambda(\mathrm{CMV}) = 800$, $\mu_\lambda(\mathrm{CCM}) = 100$, $\delta = 1$, $\gamma = 1$, $|s_1|^2 = 1$, $\theta_{\mathrm{E}} = 3$ degree. While the low-complexity solutions, the proposed RCMV-MCG-LOCME algorithm and the proposed RCCM-MCG-LOCME algorithm, are solved as described in Tables I and II, the proposed WC-LOCME algorithm and the proposed RCCM-LOCME algorithm are solved with SeDuMi [**?**] or alternatively with [GB11]. The algorithms are compared with the worst-case optimization based approach (WC-CMV) [VGL03] using $\epsilon = 2.1$, the loaded sample-matrix-inversion (Loaded-SMI) algorithm with 10 σ_n^2 as the diagonal

Table 3.3: Interference scenario, P(dB) relative to user1 / DoA(degrees)

user 1	user 2	user 3	user 4	user 5
0/96°	13/120°	10/140°	0/-67°	0/157°

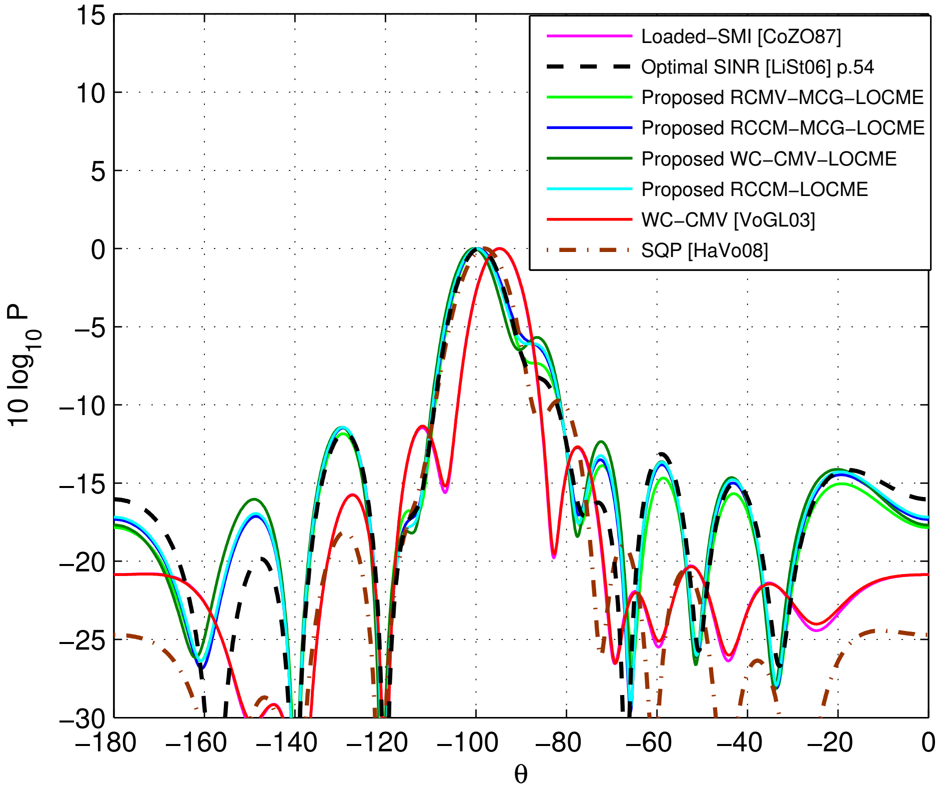

Figure 3.1: Beampattern, mismatch due to LCS, SNR = 0 dB, $K = 2$, $D = 5$

loading factor, the SQP approach which is implemented as in [HV08] with $K_{\text{SQP}} = 6$ and the optimal solution (Opt-SINR) according to [LS06] p.54. Besides the desired user (user 1) there are four interferers, the powers (P) relative to user 1 and directions of arrival (DoA) in degrees of which are detailed in Table 3.3.

In our simulations, the array steering vector mismatch is corrupted due to coherent local scattering $\boldsymbol{a}_1 = \boldsymbol{a} + \sum_{l=1}^{4} \mathrm{e}^{j\Phi_l} \boldsymbol{a}_{\text{sc}}\left(\theta_l\right)$, where θ_l is uniformly distributed with the presumed direction as the mean and 2 degree standard deviation Φ_l and is uniformly distributed in the intervall $[0, 2\pi]$. The mismatch changes for every realization and is fixed over the snapshots.

Fig. 3.1 shows the beampattern of the compared algorithms for a mismatch realization. In Fig. 3.2 the average over 500 realizations of the signal to interference-plus-noise ratio (SINR) is presented over the snapshots for SNR=0 dB. Here the convergence speed of the proposed algorithms is demonstrated. Note for 500 realizations the LOCME algorithms always provide above 7 dB SINR for $i = 500$. Fig. 4.1 shows the SINR performance over the SNR. The proposed algorithms outperform the existing algorithms especially in high SNR. However, the existing algorithms have an advantage in terms of resolution.

3.5 Conclusions and Futurework

Conclusions

This section introduces the proposed LOCME method and robust beamforming algorithms. The four proposed algorithms based on the CMV and CCM criterion benefit from the LOCME feature. All the proposed algorithms outperform the conventional worst-case optimization based algorithm and the approaches based on the conjugate gradient method have a reduction in complexity by more than an order of magnitude compared to the worst-case optimization based approach. LOCME does not require any additional information from the system (e.g. statistics of the mismatch, ellipsoidal parameter).

Futurework

- It remains to be explored how to extend the LOCME scheme to provide a higher reliability in special cases where interferers are close to the desired user. It is an open problem how to incooperate additional constraints in the design which take that into account and how to implement it with a low computational effort.
- To make sure that the LOCME method does not adapt on interferers theoretically the $\boldsymbol{d}$ vector can be obtained while using the output function computed by a conventional minimum variance beamforming algorithm which is processed in a parallel way. The resulting performance degradations could be shown in a simulation.

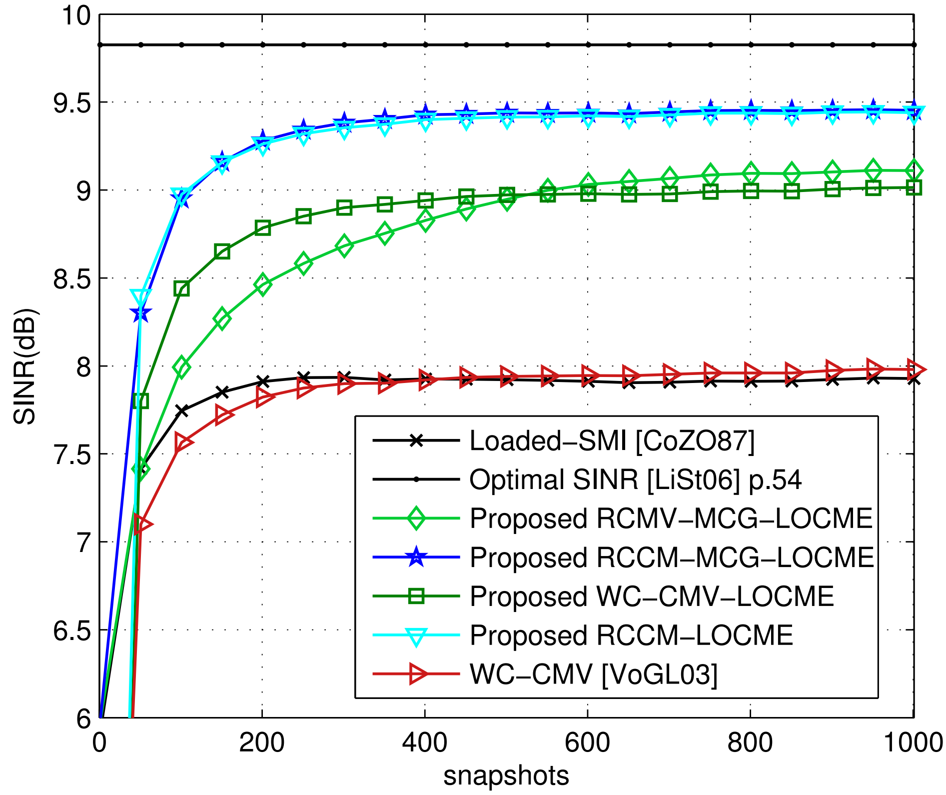

Figure 3.2: SINR versus i, SNR = 0 dB, $K = 2$

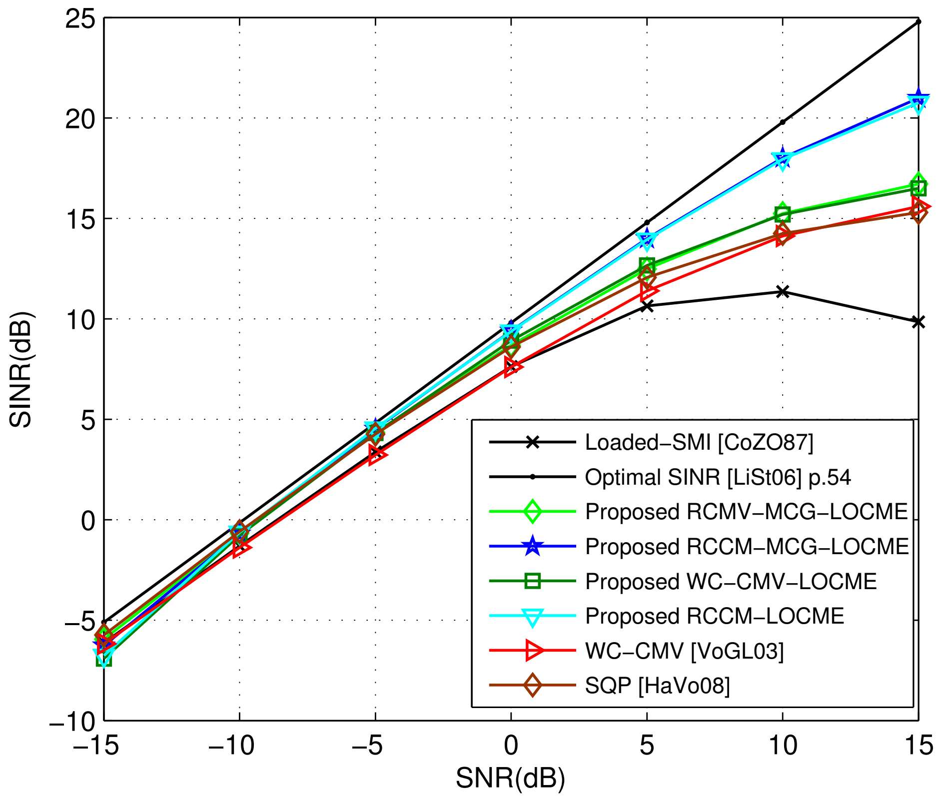

Figure 3.3: SINR performance versus SNR, local coherent scattering, $i = 500$

4 Beamforming in Relay Networks

Beamforming in relay networks has become an important topic during the last few years. This part especially focuses on relay networks using single-antenna receivers and the amplify and forward protocol. This chapter points out issues that are observated in practical circumstances. The theoretically optimal solution needs to be centrally processed, which requires extra signalling in the network. In addition the CSI can be particullary outdated. Robust worst-case optimization based approaches [GKKPO09a] have been considered for adressing situations with imperfect CSI, but the simulations show that they do not provide an impact on the average performance. To solve the CSI mismatch problem collaborative algorithms can be used [JCJ08],[RNB05] where local CSI is used, which exploits a significantly higher accuracy. The MMSE based consensus algorithm [Cho11] does not even require any central processing but requires a significant number of iterations to converge. However, these algorithms are based on the linear MMSE criterion which is not optimal in a sense of minimizing the BER. In this chapter a new method based on Pseudo-SNR is proposed. Each relay can compute its own weight autonomous with a very low requirement of communication between the relays. The proposed method shows a better performance compared to the MMSE based Consensus algorithm [Cho11], and its performance is comparable to the MSNR.

4.1 Different Relay Strategies

The existing algorithms use either global channel state information (CSI), its statistics or local CSI. Besides the algorithms based on the maximum signal to noise ratio (MSNR) , which is described in the example in the introduction, techniques based on the minimum mean squared error (MMSE), which can provide comparable results have also been reported. In addition, there is often a power constraint due to the relays or the whole network. To give a brief overview some remarkable relay strategies with their properties are summarized in Table 4.1. While the minimum error probability can be achieved with the relay weights due to the maximum SNR at the receiver there is no direct relation between the MMSE based design and the error probability but

Table 4.1: Relay Strategies for Distributed Beamforming

strategy	Criterion	Power Constraint (PC)	CSI
[KS07]	MMSE	average total PC	local CSI
[OP06]	MMSE	instan. individual PC	local CSI
[YK07],[JJ08]	MSNR	instan. total PC	global CSI
[ZWPO09]	MSNR	instan. individual PC	global CSI
[HNSGL08]	MSNR	average total PC	statistics of CSI

its performance is still good and established. The main adavantage of the algorithms based on local CSI is the significantly higher accuracy of the CSI compared to the CSI quality in a central processed algorithm. To have a fair comparison this chapter compares a central processed robust distributed beamforming algoritm [GKKPO09a] in the presence of CSI mismatch, the recently reported distributed consensus algorithm [Cho11] in the absence of CSI mismatch and the proposed distributed beamforming algorithm based on Pseudo-SNR.

4.2 A Consensus Algorithm for Cooperative Relay Networks

According to the amplify and forward protocol the transmission is divided into two phases. During the first phase the signal is transmitted to the relay nodes and during the second phase the signal including noise is forwarded at the relay nodes to the receiving node. The channels between the transmitter and the D relays are denoted as $f_1, ..., f_D$ and the channels between the relays and the receiver are denoted as $g_1, ..., g_D$. The received signal at the mth relay can be cast as

$$x_m = \sqrt{P_0} f_m s + n_m, \tag{4.1}$$

where n_m is the received noise at the relay with its variance σ_m^2 and P_0 denotes the transmission power of the signal. Considering the coefficient $\alpha_m = \frac{f_m^* P_0}{P_0 |f_m|^2 + \sigma_{m1}^2}$ the MMSE estimate of the signal at each relay can be expressed as

$$\hat{s}_m = \alpha_m x_m \tag{4.2}$$

and its normed notation is given by $\tilde{s}_m = \frac{\hat{s}_m}{\mathrm{E}\left\{|\hat{s}_m|^2\right\}}$. Using that notation the global formulation of the MMSE design with the total relay power constraint can be formulated as

$$\begin{aligned}
&\min_{\boldsymbol{w}} \sum_{m=1}^{D} \kappa_m \mathrm{E}\left\{|s - g_m w_m \tilde{s}_m|^2\right\} \\
&\text{s. t. } \|\boldsymbol{w}\|^2 \leq P_{\mathrm{T}},
\end{aligned} \tag{4.3}$$

where P_{T} is the total relay power budget. The resulting relay weights are given by

$$w_m = \frac{g_m^*}{\frac{\lambda}{\kappa_m} + |g_m|^2} \sqrt{\frac{|f_m|^2 P_0^2}{|f_m|^2 P_0 + \sigma_{m1}^2}} = \frac{g_m^*}{\frac{\lambda}{\kappa_m} + |g_m|^2} \sqrt{\frac{\gamma_m}{\gamma_m + 1}} \sqrt{P_0}, \tag{4.4}$$

where λ is the Lagrange multiplier and and γ_m is the signal to noise ratio at the corresponding relay node. In addition, there are the design parameters κ_m, which can be useful in scenarios where a single relay run out of the available energy. In that case the corresponding κ_m need to be choosen to a small value.

Since the computation of the Lagrange multiplier requires global information the approach can not be applied in a fully distributed way but the consensus algorithm recently reported in [Cho11] describes a fully distributed solution. In this approach the relay nodes exchange information with their neighbors without using any central information. While using the dual decomposition method, each relay solves a certain suboptimization problem

$$\begin{aligned}
&\min_{\boldsymbol{w}_m} \kappa_m \mathrm{E}\left\{|s - g_m w_m \tilde{s}_m|^2\right\} \\
&\text{s. t. } \|\boldsymbol{w}\|^2 \leq P_{\mathrm{T}}, \boldsymbol{w}_m = \boldsymbol{w}_q,\ q \in \mathcal{M}_m,
\end{aligned} \tag{4.5}$$

where $\mathcal{M}_m$ is the set of nodes in the neighborhood of the mth relay. The phases of each relay weight are already determined as the inverse of the channels, which are known at the node. Therefore it is sufficient to exchange just the absolute values of the weight vectors. For that case the author in [Cho11] devises an algorithm to adjust the mth relay node in the following way

$$w_{l,m} = \begin{cases} \frac{g_m^*}{\frac{\lambda_m}{\kappa_m} + |g_m|^2} \left(\sqrt{\frac{\gamma_m}{\gamma_m+1}} \sqrt{P_0} - \frac{\sum_{q \in \mathcal{M}_m} \beta_{m,q,m}}{2} \right) & \text{, if } m = l \\ -\frac{\sum_{q \in \mathcal{M}_m} \beta_{m,q,l}}{2\lambda_m} & \text{, if } m \neq l, \end{cases} \tag{4.6}$$

where λ_m and $\boldsymbol{\beta}_{m,q}$ can be achieved iteratively as follows

$$\lambda_m(i+1) = \max\left\{0, \lambda_m(i) + \mu_\lambda\left(\|\boldsymbol{w}_m\|^2 - P_{\mathrm{T}}\right)\right\} \tag{4.7}$$

$$\boldsymbol{\beta}_{m,q}(i+1) = \boldsymbol{\beta}_{m,q}(i) + \mu_\beta(\boldsymbol{u}_m - \boldsymbol{u}_q), \tag{4.8}$$

where μ_λ and μ_β are the stepsizes and $\boldsymbol{u}_m = [|w_{1,m}| \dots |w_{D,m}|]^{\mathrm{T}}$.

While the relay weight strategy in equation (4.4) appears to be disadvantageous, because of the fact that the computed weight is inversely proportional to the channel, which is not desirable, the consensus algorithm (4.6) converges to a different solution which shows good performance.

4.3 Worst-Case SNR Maximization

In this subsection the robust distributed beamforming algorithm reported in [GKKPO09a] is shown. In this approach global CSI at the source node is considered, where the relay weights are computed. Due to the amplify and forward protocol and including the relay network the received signal can be modeled as

$$y = \sum_{m=1}^{D} g_m w_m l_m (f_m x + n_m) + n_0, \tag{4.9}$$

where f_m is the complex channel gain between the source node and the mth relay node, while g_m represents the channel between the relay node and the destination and $l_m = (|f_m|^2 P_0 + \sigma_m^2)^{-\frac{1}{2}}$ scales the received signals at the relays. Furthermore it is assumed that the channel gains between the relay nodes and the source node are known nearly perfectly because they can be directly estimated using training data sequences. In case of the estimated channel gains between relay nodes and destination node the uncertainty is significantly higher, because the estimation which is done at the relays still need to be forwarded to the source node. Therefore the corrupted CSI is modeled as

$$\boldsymbol{g} = \hat{\boldsymbol{g}} + \Delta\boldsymbol{g}, \tag{4.10}$$

where $\hat{\boldsymbol{g}} = [\hat{g_1} \dots \hat{g_D}]^T$ are the channel estimates at the source node and $\Delta\hat{\boldsymbol{g}}$ is the corresponding error vector. The error is assumed to be a point in the sphere, $\Delta\boldsymbol{g} \in \mathcal{S}$,

$$\mathcal{S} = \left\{\Delta\boldsymbol{g}, \|\Delta\boldsymbol{g}\|_2 \leq D\rho^2\right\}. \tag{4.11}$$

Involving the mismatch, the SNR at the destination node can be cast as

$$\Gamma = \frac{P_0 \left| \sum_{m=1}^{D} (\hat{\boldsymbol{g}} + \Delta \boldsymbol{g}) f_m l_m w_m \right|^2}{\sum_{m=1}^{D} \left| \hat{\boldsymbol{g}} + \Delta \boldsymbol{g} \right|^2 l_m^2 \left| w_m \right|^2 \sigma_m^2 + \sigma_0^2}. \tag{4.12}$$

The corresponding worst-case optimization problem is given by

$$\max_{|\boldsymbol{w}|^2 \leq \boldsymbol{p}} \; \min_{\Delta \boldsymbol{g} \in \mathcal{S}} \Gamma, \tag{4.13}$$

which is a SNR maximization over the relay weights and at the same time a SNR minimization over the set of CSI mismatch. Because it is a quasi convex optimization problem the solution can be obtained by repeatly solving

$$\begin{aligned} &\min_{|\boldsymbol{w}|^2 \leq \boldsymbol{p}} \; \|\boldsymbol{w}\|^2 \quad \text{s. t.} \\ &\qquad \min_{\Delta \boldsymbol{g} \in \mathcal{S}} \Gamma \geq \gamma \end{aligned} \tag{4.14}$$

for defined values for γ (SNR value). The problem in (4.14) can be solved using rank relaxation and S-Procedure as follows.

For the formulation it is defined $\tilde{w}_m = f_m l_m w_m$, $\boldsymbol{G} = \mathrm{Diag}\left\{ \frac{1}{|f_1|^2 l_1^2}, ..., \frac{1}{|f_D|^2 l_D^2} \right\}$ and $\boldsymbol{v}$ is a phase shifted version of $\tilde{\boldsymbol{w}}$ due to $\tilde{w}_m = \frac{v_m \hat{g}_m^*}{|\hat{g}_m|}$ and the real valued (phase shifted) estimated CSI as $\tilde{\boldsymbol{g}} = |\hat{\boldsymbol{g}}|$. Furthermore the phase shifted uncertainty vector is defined as

$$\Delta \tilde{\boldsymbol{g}} = \left[\Delta g_1 \frac{\hat{g}_1^*}{|\hat{g}_1|} \; ... \; \Delta g_D \frac{\hat{g}_D^*}{|\hat{g}_D|} \right]^T. \tag{4.15}$$

Then, the constraint in (4.14) is reformulated as

$$(\tilde{\boldsymbol{g}} + \Delta \tilde{\boldsymbol{g}})^H \boldsymbol{Q} (\tilde{\boldsymbol{g}} + \Delta \tilde{\boldsymbol{g}}) \geq \gamma \sigma_0^2, \quad \forall \Delta \boldsymbol{g} \in \mathcal{S}, \tag{4.16}$$

where

$$\boldsymbol{Q} = P_0 \boldsymbol{v} \boldsymbol{v}^H - \gamma \mathrm{Diag}\left\{ \frac{|v|_1^2 \sigma_1^2}{|f_1|^2}, ..., \frac{|v|_D^2 \sigma_D^2}{|f_D|^2} \right\}. \tag{4.17}$$

While using the S-Procedure the constraint can be written as

$$\begin{bmatrix} \tilde{\boldsymbol{g}}^T \boldsymbol{Q} \tilde{\boldsymbol{g}} - \gamma\sigma_0^2 - sD\rho^2 & \tilde{\boldsymbol{g}}^T \boldsymbol{Q} \\ \boldsymbol{Q}\tilde{\boldsymbol{g}} & \boldsymbol{Q} + s\boldsymbol{I} \end{bmatrix} \succeq \boldsymbol{0}, \quad s \geq 0. \tag{4.18}$$

Including rank relaxation as $\boldsymbol{V} = \boldsymbol{v}\boldsymbol{v}^H$ the optimization problem in (4.14) can be cast as

$$\min_{\boldsymbol{V} \succeq \boldsymbol{0}, s \geq 0} \text{trace}\{\boldsymbol{G}\boldsymbol{V}\}$$
$$\text{s. t.} \begin{cases} \begin{bmatrix} \tilde{\boldsymbol{g}}^T \boldsymbol{Q} \tilde{\boldsymbol{g}} - \gamma\sigma_0^2 - sD\rho^2 & \tilde{\boldsymbol{g}}^T \boldsymbol{Q} \\ \boldsymbol{Q}\tilde{\boldsymbol{g}} & \boldsymbol{Q} + s\boldsymbol{I} \end{bmatrix} \succeq \boldsymbol{0} \\ p_m \left|f_m\right|^2 l_m^2 \geq \boldsymbol{V}_{mm}, \quad \forall m, \end{cases}$$

The corresponding solution is the vector which fulfills $\boldsymbol{V} = \boldsymbol{v}\boldsymbol{v}^H$ and the resulting relay weights are given by

$$w_m = \frac{v_m (g_m f_m)^*}{l_m \left|\hat{g_m}\right| \left|f_m\right|^2}. \tag{4.19}$$

The conditions which garantiee rank 1 solutions are detailed in [GKKPO09b]. To obtain a consistent comparison of the relay network beamforming algorithms, the total relay power constraint given by trace$\{\boldsymbol{V}\boldsymbol{G}\} \leq P_T$ is used instead of the single relay power from above. The non-robust counterpart can be easily obtained by setting ρ to a small value.

4.4 Proposed Consensus Algorithm using Pseudo-SNR

The accuracy of the CSI in systems with dynamically changing channel conditions especially depends on the age of the measurement data. Therefore it is advantageous to place the CSI estimation and also the relay weight processing directly on the corresponding relay node in combination with a fast consensus method. This section introduces an alternative approach providing a solution to the mentioned requirements. The method is not directly MSNR based but it shows comparable results.

The design is based on the assumption that the other relays are absence and the total power would be allocated to the relay of interest. The corresponding SNR expression can be described as

$$\check{\gamma}_m = \frac{P_0 \left|g_m f_m l_m w_m\right|^2}{\left|g_m l_m w_m\right|^2 \sigma_m^2 + \sigma_0^2}, \tag{4.20}$$

where $l_m = (\left|f_m\right|^2 P_0 + \sigma_m^2)^{-\frac{1}{2}}$ normalizes the signal, f_m denotes the channel coefficent between source and the mth relay node,g_m denotes the channel coefficent the mth relay node and the destination node, σ_m^2 is the relay noise power, σ_0^2 is the noise power at the destination, P_0 is the transmitting power and w_m is the relay weight. While the whole power is allocated theoretically to the mth relay, $\frac{|w_m|^2}{P_\mathrm{T}} = 1$ holds. It can be added to (4.20) at a suitable place and that allows the following notation

$$\check{\gamma}_m = \frac{P_0 \left|g_m f_m l_m\right|^2}{\left|g_m l_m\right|^2 \sigma_m^2 + \frac{\sigma_0^2}{P_\mathrm{T}}}, \tag{4.21}$$

which provides a metric for the quality of the relay node. Introducing a function of this coefficient, the relay weight can be generally computed as

$$w_m = \frac{(g_m f_m)^*}{|g_m f_m|} \; f\left(\check{\gamma}_m\right), \tag{4.22}$$

and in what follows the relay Power $\left|w_m\right|^2$ is chosen as directly proportional to the SNR metric as

$$w_m = \sqrt{P_\mathrm{T}} \frac{(g_m f_m)^*}{|g_m f_m|} \frac{\sqrt{\check{\gamma}_m}}{\eta}, \tag{4.23}$$

where $\eta = \sqrt{\sum_{m=1}^{D} \check{\gamma}_m}$ is defined in order to fullfill a total power constraint. The method implies that the sum of the SNR metric is known by each node which can be achieved with a simple consensus algorithm and the computation of the SNR metrics requires the knowledge about the noise level at the receiver σ_0^2. Since both values can be assumed as just smoothly changing, the adaptation depends on the CSI and therefore a quick adaptation can be expected. In contrast to the existing Consensus algorithm based on MMSE criterion the proposed method just need a low numer of iterations to converge.

One simple method to obtain the sum over all Pseudo-SNR in a collaborative way is given by the following strategy. Considering that each node has an estimate of all the Pseudo-SNR values in the network which is given by $\check{\boldsymbol{\gamma}}_m(i) = [\check{\gamma}_{m,1}(i), ..., \check{\gamma}_m(i), ..., \check{\gamma}_{m,D}(i)]$, where $\check{\gamma}_m(i)$ denotes its own precisely known Pseudo-SNR and i is the time index. The set of nodes in the neighborhood of the mth relay node including the node itself is given by $\mathcal{M}_m$. Once the nodes have exchanged their Pseudo-SNR vectors the estimates can be updated according to

$$\check{\gamma}_{m,k}(i) = \begin{cases} \frac{1}{D_m} \sum_{q \in \mathcal{M}_m} \check{\gamma}_{q,k}(i-1) & \text{, if } k \neq m \\ \check{\gamma}_m(i) & \text{, if } k = m, \end{cases} \tag{4.24}$$

where D_m is the number of nodes in the set $\mathcal{M}_m$. The initialization vector is given by $\check{\boldsymbol{\gamma}}_m(0) = [\check{\gamma}_m(0), ..., \check{\gamma}_m(0)]$.

4.5 Simulations

For all the simulations the channels are considered as independent circular symmetric complex gaussian random variables with zero mean and unit variance. The step sizes for the consensus algorithm are choosen as $\mu_\lambda = 0.08$ and $\mu_\beta = 0.05$. To compare the convergence properties of the consensus algorithm and the Pseudo-SNR based algorithm Figure 4.1 shows the power allocation and the total power as a function of the iterations for a relay network containing $D = 6$ nodes. Moreover, the BER has been simulated. In the simulations QPSK signals are considered. All the algorithms are subject to the total relay power constraint. Figure 4.3 compares the BER for the MSNR approach, with $\rho = 0$ with the Consensus algorithm and with the approach based on Pseudo-SNR in the presence of perfect CSI.

The CSI mismatch between relay nodes and destination nodes is introduced in Figure 4.4. The mismatch is uniformly distributed in a sphere with radius $r = (\rho^2 D)$ as it is described in equation (4.11). Caused by the fact, that the CSI accuracy is significantly higher for algorithms processed at the relay nodes, here perfect CSI is assumed for

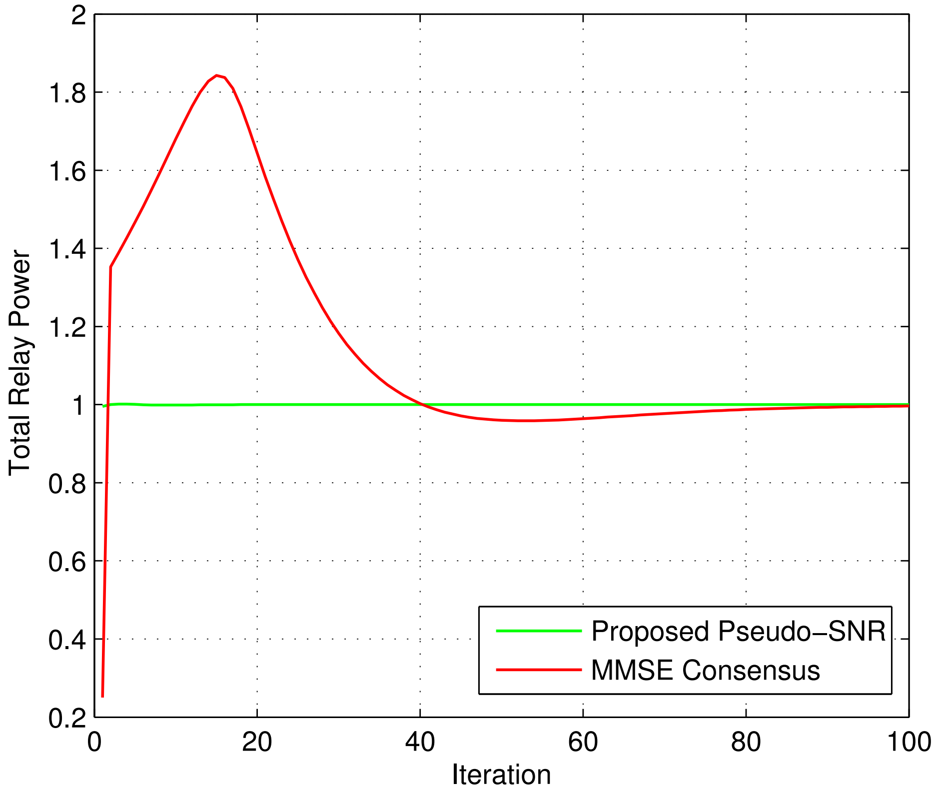

Figure 4.1: Total power versus iterations, $D = 6$, SNR $= 10$ dB

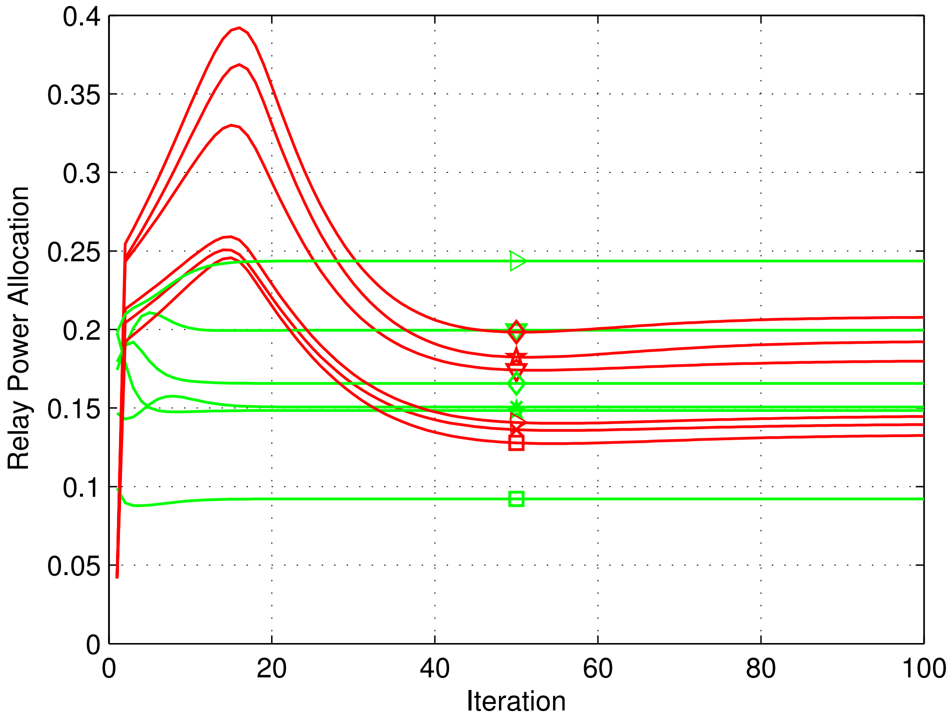

Figure 4.2: Relay Powers for the example from above, the marker specifies the node

the Consensus and the Pseudo-SNR based algorithm. The MSNR based approach is simulated with preknowledge of the mismatch level and also as the non robust algorithm which means $\rho = 0$. The simulation shows that especially the averaged BER performance of the non robust algorithm is comparable and slightly better to the worst case optimization based one. While perfect CSI for the relay processed algorithms is assumed, they outperform the MSNR based algorihm from a specific mismatch level.

4.6 Conclusions and Futurework

Conclusion

In this chapter the Pseudo-SNR based distributed beamforming algorithm is proposed. Each relay chooses its relay weight based on the SNR for the theoretically absence of all the other relay nodes. In this method each relay node can directly compute its own relay weight, using local CSI and a very low level of collaboration with the other nodes. Because of the fast adaptation and the immediate processing with the measured CSI the proposed method is an alternative to existing approaches in that field. Even in the case of the perfect CSI assumption the new method provides comparable performance compared to the optimium which is given by the MSNR approach which is optimal in the absense of multiple access interference and performs significantly better than the MMSE-based consensus algorithm. Furthermore it is demonstrated that the so-called robust MSNR approach based on worst-case optimization does not provide performance gain in the averaged BER. As a general conclusion the author advises fast subobtimal solutions which exploits high accuracy of the CSI instead of optimal solutions computed with uncertainties in the CSI.

Futurework

- The Pseudo-SNR based approach is introduced for the total relay power constraint and for the single relay power constraint it is still to develop.
- The small level of collaboration in the Pseudo-SNR based approach is the joint computation of the sum of all Pseudo-SNRs. For that particular problem the best solution still needs to be found. Here it is not clear if a consenus algorithm or a centralized algorithm is more advantageous.

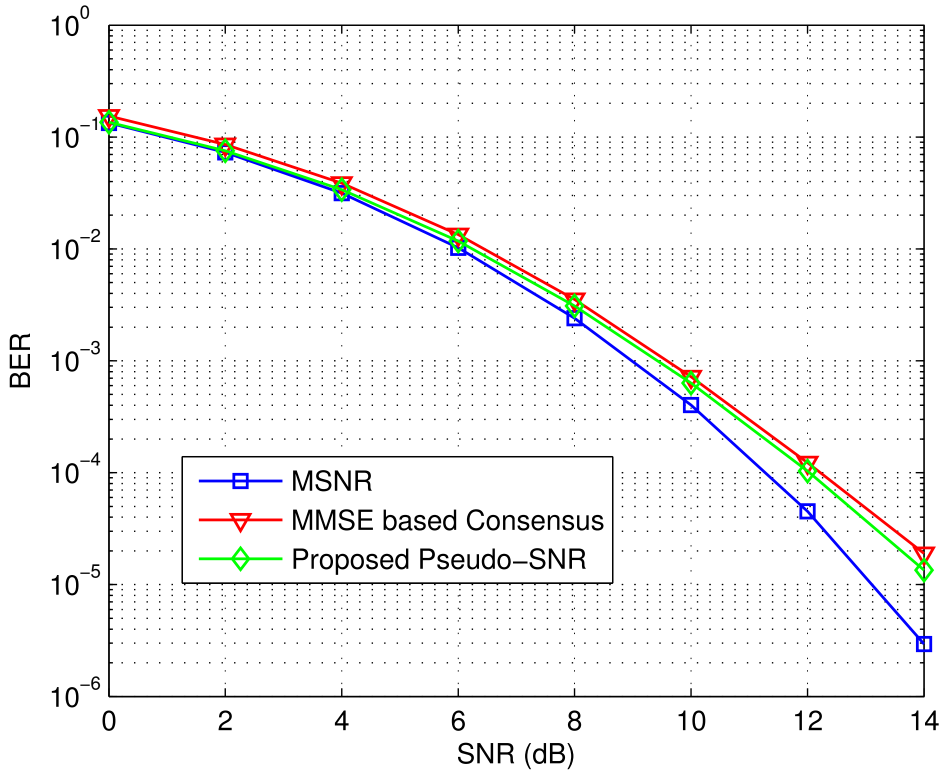

Figure 4.3: BER versus SNR, perfect CSI, $\rho = 0$, $D = 6$

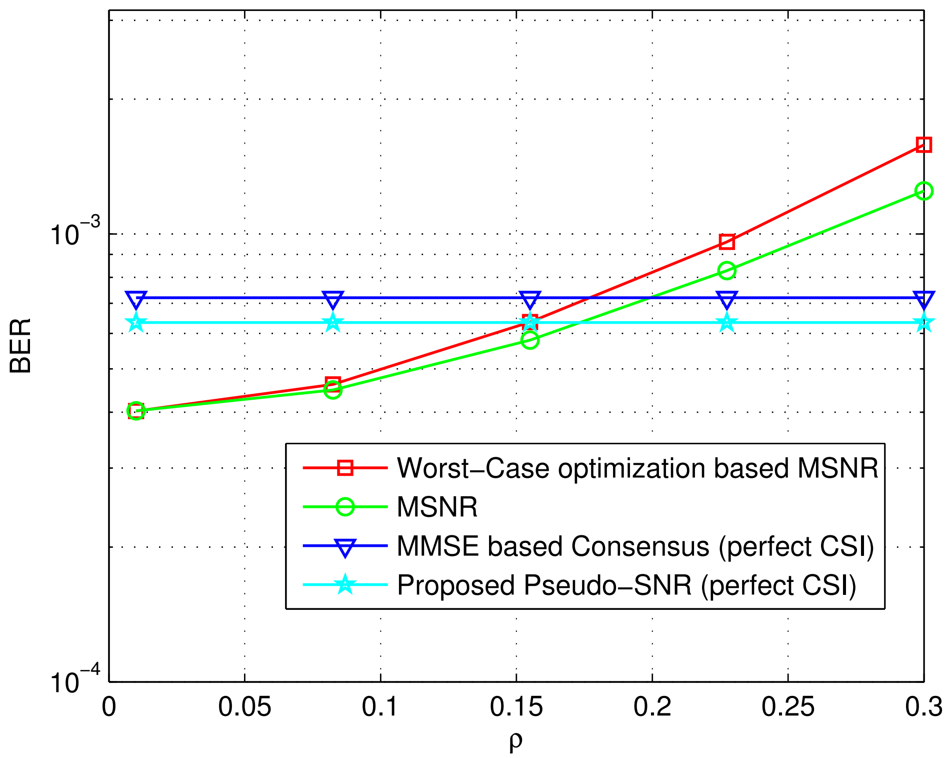

Figure 4.4: BER versus CSI mismatch level, SNR = 10 dB, $D = 6$

- The methods can be compared in a time varying channel conditions to demonstrate the superiority of the proposed method in a realistic scenario.
- It is not proven that the function which connects the Pseudo-SNR metric with the resulting relay weight is optimally choosen.
- The deviation to the optimal solution is possibly within a specific bound. This bound cooresponds to an maximization problem over a constrained parameter set (CSI), which maximizes the error between the optimal solution and the proposed suboptimal solution. The maximization problem corresponds possibly itself to a convex optimization problem, which would imply that the result is a numerical. The set of nodes which are in the neighboorhood

5 Conclusions

Receive Beamforming

In practice, circumstances like imprecisely calibration, local coherent scattering, direction errors or unknown wavefield propagation effects leads to a mismatch in the presumed array steering vector. This can lead to the so-called signal self nulling effect and dramatical performance degradation. Several robust approaches have been reported in the last decades and the most remarkable is the worst-case optimization based approach employing convex optimization. Later more advanced methods have been published to estimate the array steering vector.

However, this work proposes a set of algorithms which provides better performances while the computational complexity is more than an order of magnitude lower compared to the existing approaches. The proposed algorithms are published on international conferences.

The proposed worst-case optimization based constrained constant modulus beamforming algorithm is introduced. It provides robustness against array steering vector mismatch and exploits the constant modulus property of the desired signal. The algorithm is solved iteratively where each iteration contains a second order cone program. A condition which garantiees convexity of the optimization problem has been found. The proposed constrained constant modulus algorithm shows a significant performance gain compared to its minimum variance design counterpart. In addition an analysis of the robust constraint discovers a link beween the SNR and the choice the worst-case parameter.

The proposed low-complexity beamforming algorithms termed the Robust Constrained Minimum Variance Algorithm based on the Modified Conjugate Gradient (RCMV-MCG) and the Robust Constrained Constant Modulus Algorithm based on the Modified Conjugate Gradient (RCCM-MCG) are introduced. While the algorithms exploit previous computations the computational complexity is reduced by more than an order of magnitude compared to the worst-case based algorithms which are second order cone programs. The algorithms are based on a joint optimization strategy including the modified conjugate gradient method which performs just one iteration per snapshot and a parallel adjustment to fullfill the robust constraint. The simulations demonstrate that the proposed algorithms perform equivalently or outperform the worst-case optimization based algorithm.

The proposed robust adaptive beamforming algorithms using Low-Complexity Mismatch Estimation (LOCME) are introduced. The LOCME method provides a mismatch estimate of the steering vector which is just known imprecisely by the system. To achieve the estimate a correlation based rough estimate is projected into a predefined subspace. The new method is shown in four different algorithms. The algorithms are developed according to the constant modulus design and to the minimum variance design criteria. While the algorithms can be solved using convex optimzation toolboxes, their low complexity counterparts are also shown. The simulations of all the proposed beamforming algorithms show superior performances and outperform the existing solutions.

Relay Network Beamforming

Relay Network Beamforming algorithms to achieve the optimal BER are based on the MSNR criterion. However, in practice these BER can not be achieved. Mainly it is caused by the fact that the available CSI is partially outdated. Some worst-case optimization based approaches have been reported recently which provides a robust design against CSI mismatch, but the simulations show that there is no impact on the average performance. As an alternative some suboptimal MMSE based algorithms have been reported. Even if they do not have the direct relation to the minimum BER, they can be applied as a consensus algorithm using local CSI which achieves a significantly higher accuracy. However, the existing MMSE based approaches are wether dependend on a centralized processing which is not desirable or the processing is done using a collaborative algorithm which requires a large number of iterations.
A solution is given by the proposed Pseudo-SNR based approach. Using this approach each relay node computes its own relay weight autonomous with a very low level of network collaboration. In the proposed method the relay power is proportional to the maximum achievable SNR at the destination node in the theoretically absence of all other relay nodes. The sum over all pseudo-SNRs needs to be computed in a consensus algorithm, which is expected as nearly constant over time. According to the simulation the proposed method outclasses the MMSE based consensus algorithm while the performance is comparable to the MSNR based approach. Introducing a certain level of CSI mismatch to the centralized MSNR approach the proposed method provides the best performance.

Appendix

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List of Abbreviations and Formulas

$\boldsymbol{A}$	matrix of array steering vectors
$\boldsymbol{a}$	arrray steering vector
$\boldsymbol{e}$	mismatch vector
$\boldsymbol{n}$	noise
$\boldsymbol{R}_{\mathrm{a}}$	constrained constant modulus covariance matrix
$\boldsymbol{R}_{xx}$	signal covariance matrix
$\boldsymbol{w}$	beamforming weight vector
$\boldsymbol{x}$	array observation vector
ϵ	mismatch level
λ	Lagrangian multiplier
σ^2	noise power
i	timeindex
M	number of array sensor elements
P	power
s	signal
y	beamforming output
ASV	**A**rray **S**teering **V**ector
BER	**B**it **E**rror **R**ate
CCM	**C**onstrained **C**onstant **M**odulus
CSI	**C**hannel **S**tate **I**information
DoA	**D**irection **o**f **A**rrival
LCS	**L**ocal **C**oherent **S**cattering
LOCME	**LO**w **C**omplexity **M**ismatch **E**stimation
MCG	**M**odified **C**onjugate **G**radient
MMSE	**M**inimum **M**ean to **S**quared **E**rror
MSE	**M**ean **S**quared **E**rror
MSNR	**M**aximum **S**ignal to **N**oise **R**atio
PDF	**P**robability **D**ensity **F**unction
PSD	**P**ositive **S**emi **D**efinite

QCQP .... **Q**uadratically **C**onstrained **Q**uadratic **P**rogram
SDP .... **S**emi **D**efinite **P**rogramming
SINR .... **S**ignal to **I**nterference plus **N**oise **R**atio
SNR .... **S**ignal to **N**oise **R**atio
SOC .... **S**econd **O**rder **C**one
SOCP .... **S**econd **O**rder **C**one **P**rogramming
SQP .... **S**equential **Q**uadratic **P**rogramming
WC-CCM .... **W**orst **C**ase **C**onstrained **C**onstant **M**odulus
WC-CMV .... **W**orst **C**ase **C**onstrained **M**inimum **V**ariance

Theses

1. Robust beamforming algorithms using the Constrained Constant Modulus criterion can exploit the constant modulus property of the desired signal.

2. The robust adaptive beamforming algorithm using joint optimization strategy based on the Modified Conjugate Gradient and the parallel adjustment of the robust constraint performs equivalent to the conventional algorithm based on second order cone programming and reduces the computational complexity by more than an order of magnitude.

3. The Low-Complexity Mismatch Estimation (LOCME) method provides an estimate of the true array steering vector which projects the correlation of array observation and beamforming output onto a predefined subspace.

4. Distributed beamforming based on the Pseudo-SNR algorithm, allows each relay node to compute its own relay weight with a low requirement of network collaboration. The method performs better than the existing consensus algorithm based on MMSE cirterion and performs comparable to the MSNR based approach.

Ilmenau, den 09. 05. 2011 Lukas Landau

Erklärung

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