Robust Adaptive Beamforming Using a Low-Complexity Shrinkage-Based Mismatch Estimation Algorithm

Hang Ruan and Rodrigo C. de Lamare

***Abstract*—In this work, we propose a low-complexity robust adaptive beamforming (RAB) technique which estimates the steering vector using a Low-Complexity Shrinkage-Based Mismatch Estimation (LOCSME) algorithm. The proposed LOCSME algorithm estimates the covariance matrix of the input data and the interference-plus-noise covariance (INC) matrix by using the Oracle Approximating Shrinkage (OAS) method. LOCSME only requires prior knowledge of the angular sector in which the actual steering vector is located and the antenna array geometry. LOCSME does not require a costly optimization algorithm and does not need to know extra information from the interferers, which avoids direction finding for all interferers. Simulations show that LOCSME outperforms previously reported RAB algorithms and has a performance very close to the optimum.**

***Index Terms*—Covariance matrix shrinkage method, low complexity methods, robust adaptive beamforming.**

I. Introduction

In APPLICATIONS like wireless communications, audio signal processing, radar and microphone array processing, adaptive beamforming has been intensively researched and developed in the past years. However, under certain circumstances, adaptive beamformers suffer a performance degradation due to several reasons which include short data records, the presence of the desired signal in the training data, or imprecise knowledge of the steering vector of the desired signal. In order to improve the performance of adaptive beamformers in the presence of steering vector mismatches, RAB techniques have been developed. Different from the standard designs [1], the design principles of RAB MVDR beamformers [6] include: the generalized sidelobe canceller, diagonal loading [4], [5], eigenspace projection, worst-case optimization [3], [14] and steering vector estimation with presumed prior knowledge [7], [8], [15], [16]. However, RAB designs based on these principles have some drawbacks such as their ad hoc nature, high probability of subspace swap at low SNR and high computational cost [7].

Some recent design approaches have considered combining different design principles together to improve RAB performances. The algorithm which jointly estimates the mismatched steering vector using Sequential Quadratic Program (SQP) [8] and the interference-plus-noise covariance (INC) matrix using a shrinkage method [10] has been reported recently. Later, another similar approach which jointly estimates the steering vector using SQP and the INC matrix using a covariance reconstruction method [11], presents outstanding performance compared to other RAB techniques. However, the cost of the algorithm in [11] is high due to the required matrix reconstruction process.

In this paper, we propose an RAB algorithm with low complexity, which requires very little in terms of prior information, and has a superior performance to previously reported RAB algorithms. The proposed technique estimates the steering vector using a Low-Complexity Shrinkage-Based Mismatch Estimation (LOCSME) algorithm. LOCSME estimates the covariance matrix of the input data and the INC matrix using the Oracle Approximating Shrinkage (OAS) method. The only prior knowledge that LOCSME requires is the angular sector in which the desired signal steering vector lies. Given the sector, the subspace projection matrix of this sector can be computed in very simple steps [7]–[11]. In the first step, an extension of the OAS method [12] is employed to perform shrinkage estimation for both the cross-correlation vector between the received data and the beamformer output and the received data covariance matrix. LOCSME is then used to estimate the mismatched steering vector and does not involve any optimization program, which results in a lower computational complexity. In a further step, we estimate the desired signal power using the desired signal steering vector and the received data. As the last step, a strategy which subtracts the covariance matrix of the desired signal from the data covariance matrix estimated by OAS is proposed to obtain the INC matrix. The advantage of this approach is that it circumvents the use of direction finding techniques for the interferers, which are required to obtain the INC matrix.

This paper is structured as follows. The system model is described in Section II. In Section III, the proposed LOCSME algorithm is presented. Section IV shows and discusses the simulation results. Finally, Section V gives the conclusion.

II. System Model

Consider a linear antenna array of M sensors and K narrowband signals received at ith snapshot as expressed by

$$\mathbf{x}(i) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(i) + \mathbf{n}(i), \quad (1)$$

where $\mathbf{s}(i) \in \mathbb{C}^{K\times 1}$ presents the uncorrelated source signals, $\boldsymbol{\theta} = [\theta_1, \cdots, \theta_K]^T \in \mathbb{R}^K$ is the vector containing the directions of arrivals (DoAs), $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1) + \mathbf{e}, \cdots, \mathbf{a}(\theta_K)] \in \mathbb{C}^{M\times K}$ is the matrix which contains the steering vector for each DoA and

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$\mathbf{e}$ is the mismatch of the steering vector of the desired signal, $\mathbf{n}(i) \in \mathbb{C}^{M\times 1}$ is assumed to be complex Gaussian noise with zero mean and variance σ_n^2. The beamformer output is given by

$$y(i) = \mathbf{w}^H\mathbf{x}(i), \tag{2}$$

where $\mathbf{w} = [w_1, \cdots, w_M]^T \in \mathbb{C}^{M\times 1}$ is the beamformer weight vector, where $(\cdot)^H$ denotes the Hermitian Transpose. The optimum beamformer can be computed by maximizing the signal-to-interference-plus-noise ratio (SINR) given by

$$SINR = \frac{\sigma_1^2|\mathbf{w}^H\mathbf{a}|^2}{\mathbf{w}^H\mathbf{R}_{i+n}\mathbf{w}}. \tag{3}$$

Assume that the steering vector $\mathbf{a}$ is known precisely ($\mathbf{a} = \mathbf{a}(\theta_1)$), where σ_1^2 is the desired signal power and $\mathbf{R}_{i+n}$ is the INC matrix, then problem (3) can be transformed into an optimization problem

$$\begin{aligned} &\underset{\mathbf{w}}{\text{minimize}} \quad \mathbf{w}^H\mathbf{R}_{i+n}\mathbf{w} \\ &\text{subject to} \quad \mathbf{w}^H\mathbf{a} = 1, \end{aligned} \tag{4}$$

which is known as the MVDR beamformer or Capon beamformer [1]. The optimum weight vector is given by $\mathbf{w}_{opt} = \frac{\mathbf{R}_{i+n}^{-1}\mathbf{a}}{\mathbf{a}^H\mathbf{R}_{i+n}^{-1}\mathbf{a}}$. Since $\mathbf{R}_{i+n}$ is usually unknown in practice, it is estimated by the sample covariance matrix (SCM) of the received data as

$$\hat{\mathbf{R}}(i) = \frac{1}{i}\sum_{k=1}^{i}\mathbf{x}(k)\mathbf{x}^H(k), \tag{5}$$

which will result in the Sample Matrix Inversion (SMI) beamformer $\mathbf{w}_{SMI} = \frac{\hat{\mathbf{R}}^{-1}\mathbf{a}}{\mathbf{a}^H\hat{\mathbf{R}}^{-1}\mathbf{a}}$. However, the SMI beamformer requires a large number of snapshots and is sensitive to steering vector mismatches [10], [11].

III. Proposed LOCSME Algorithm

In this section, the proposed LOCSME algorithm is introduced. The idea of LOCSME is to estimate the steering vector and the INC matrix separately as in previous approaches. The estimation of the steering vector is described as the projection onto a predefined subspace matrix of an iteratively shrinkage-estimated cross-correlation vector between the beamformer output and the array observation. The INC matrix is obtained by subtracting the desired signal covariance matrix from the data covariance matrix estimated by the OAS method.

A. Steering Vector Estimation Using LOCSME

The cross-correlation between the array observation vector and the beamformer output can be expressed as

$$\mathbf{d} = E\{\mathbf{x}y^*\}. \tag{6}$$

We assume that $|\mathbf{a}_m^H\mathbf{w}| \ll |\mathbf{a}_1^H\mathbf{w}|$ for $m = 2, \cdots, K$, all signal sources and the noise have zero mean, and the desired signal and every interferer are independent from each other. By substituting (1) and (2) into (6), we suppose the interferers are sufficiently canceled such that they fall much below the noise floor and the desired signal power is not affected by the interference so that $\mathbf{d}$ can be rewritten as

$$\mathbf{d} = E\{{\sigma_1}^2\mathbf{a}_1^H\mathbf{w}\mathbf{a}_1 + \mathbf{n}\mathbf{n}^H\mathbf{w}\}. \tag{7}$$

In order to eliminate the unwanted part of $\mathbf{d}$ and obtain an estimate of the steering vector $\mathbf{a}_1$, $\mathbf{d}$ can be projected onto a subspace [9] that collects information about the desired signal. Here the prior knowledge amounts to providing an angular sector range in which the desired signal is located, say $[\theta_1 - \theta_e, \theta_1 + \theta_e]$. The subspace projection matrix $\mathbf{P}$ is given by

$$\mathbf{P} = [\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_p][\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_p]^H, \tag{8}$$

where $\mathbf{c}_1, \cdots, \mathbf{c}_p$ are the p principal eigenvectors vectors of the matrix $\mathbf{C}$, which is defined by [8]

$$\mathbf{C} = \int_{\theta_1-\theta_e}^{\theta_1+\theta_e}\mathbf{a}(\theta)\mathbf{a}^H(\theta)d\theta. \tag{9}$$

At this point, LOCSME will use the OAS method to compute the correlation vector $\mathbf{d}$ iteratively. The aim is to devise a method that estimates $\mathbf{d}$ more accurately with the help of the shrinkage technique. An accurate estimate of $\mathbf{d}$ can help to obtain a better estimate of the steering vector. Let us define

$$\hat{\mathbf{F}} = \hat{\nu}\mathbf{I}, \tag{10}$$

where $\hat{\nu} = \text{tr}(\hat{\mathbf{S}})/M$ and $\hat{\mathbf{S}} = \text{diag}(\mathbf{x}y^*)$. Then, a reasonable tradeoff between covariance reduction and bias increase can be achieved by shrinkage of $\hat{\mathbf{S}}$ towards $\hat{\mathbf{F}}$ [12] and subsequently using it in a vector shrinkage form, which results in

$$\hat{\mathbf{d}} = \hat{\rho}\text{diag}(\hat{\mathbf{F}}) + (1-\hat{\rho})\text{diag}(\hat{\mathbf{S}}), \tag{11}$$

which is parameterized by the shrinkage coefficient $\hat{\rho}$. If we define $\hat{\mathbf{D}} = \text{diag}(\hat{\mathbf{d}})$ then the goal is to find the optimal value of $\hat{\rho}$ that minimizes the mean square error (MSE) of $E[\|\hat{\mathbf{D}}(i) - \hat{\mathbf{F}}(i-1)\|^2]$ in the ith snapshot, which leads to (12) and (13), shown at the bottom of the page, where the derivation is shown in the Appendix and $\hat{\mathbf{S}}(i)$ is the sample correlation vector (SCV) given by

$$\hat{\mathbf{S}}(i) = \text{diag}\left(\frac{1}{i}\sum_{k=1}^{i}\mathbf{x}(k)y^*(k)\right). \tag{14}$$

As long as the initial value of $\hat{\rho}(0)$ is between 0 and 1, the iterative process in (12) and (13) is guaranteed to converge [12]. Once the correlation vector $\hat{\mathbf{d}}$ is obtained by the above OAS method, the steering vector is estimated by

$$\hat{\mathbf{a}}_1(i) = \frac{\mathbf{P}\hat{\mathbf{d}}(i)}{\|\mathbf{P}\hat{\mathbf{d}}(i)\|_2}, \tag{15}$$

where $\hat{\mathbf{a}}_1(i)$ gives the final estimate of the steering vector.

$$\hat{\mathbf{d}}(i) = \hat{\rho}(i)\text{diag}(\hat{\mathbf{F}}(i)) + (1-\hat{\rho}(i))\text{diag}(\hat{\mathbf{S}}(i)), \tag{12}$$

$$\hat{\rho}(i+1) = \frac{(1-\frac{2}{M})\text{tr}(\hat{\mathbf{D}}(i)\hat{\mathbf{S}}^*(i)) + \text{tr}(\hat{\mathbf{D}}(i))\text{tr}(\hat{\mathbf{D}}^*(i))}{(i+1-\frac{2}{M})\text{tr}(\hat{\mathbf{D}}(i)\hat{\mathbf{S}}^*(i)) + (1-\frac{i}{M})\text{tr}(\hat{\mathbf{D}}(i))\text{tr}(\hat{\mathbf{D}}^*(i))}, \tag{13}$$

TABLE I
PROPOSED LOCSME ALGORITHM

Initialize:
$\mathbf{C} = \int_{\theta_1-\theta_e}^{\theta_1+\theta_e} \mathbf{a}(\theta)\mathbf{a}^H(\theta)d\theta$
$[\mathbf{c}_1, \cdots, \mathbf{c}_p]$: p princical eigenvectors of $\mathbf{C}$
Subspace projection $\mathbf{P} = [\mathbf{c}_1, \cdots, \mathbf{c}_p][\mathbf{c}_1, \cdots, \mathbf{c}_p]^H$
$\hat{\mathbf{R}}(0) = \mathbf{0}$; $\hat{\mathbf{S}}(0) = \mathbf{0}$; $\mathbf{w}(0) = \mathbf{1}$;
$\hat{\rho}(1) = \rho(0) = \hat{\rho}_0(1) = \rho_0(0) = 1$;
For each snapshot index $i = 1, 2, \cdots$:
$\hat{\mathbf{R}}(i) = \frac{1}{i}\sum_{k=1}^{i} \mathbf{x}(k)\mathbf{x}^H(k)$
$\hat{\mathbf{S}}(i) = \mathrm{diag}(\frac{1}{i}\sum_{k=1}^{i} \mathbf{x}(k)y^*(k))$
$\hat{\nu}(i) = \mathrm{tr}(\hat{\mathbf{S}}(i))/M$
$\hat{\mathbf{F}}(i) = \hat{\nu}(i)\mathbf{I}$
$\hat{\mathbf{d}}(i) = \hat{\rho}(i)\mathrm{diag}(\hat{\mathbf{F}}(i)) + (1-\hat{\rho}(i))\mathrm{diag}(\hat{\mathbf{S}}(i))$
$\hat{\mathbf{D}}(i) = \mathrm{diag}(\hat{\mathbf{d}}(i))$
$\hat{\rho}(i+1) = \frac{(1-\frac{2}{M})\mathrm{tr}(\hat{\mathbf{D}}(i)\hat{\mathbf{S}}^*(i)) + \mathrm{tr}(\hat{\mathbf{D}}(i))\mathrm{tr}(\hat{\mathbf{D}}^*(i))}{(i+1-\frac{2}{M})\mathrm{tr}(\hat{\mathbf{D}}(i)\hat{\mathbf{S}}^*(i)) + (1-\frac{i}{M})\mathrm{tr}(\hat{\mathbf{D}}(i))\mathrm{tr}(\hat{\mathbf{D}}^*(i))}$
$\hat{\mathbf{a}}_1(i) = \frac{\mathbf{P}\hat{\mathbf{d}}(i)}{\|\mathbf{P}\hat{\mathbf{d}}(i)\|_2}$
$\hat{\nu}_0(i) = \mathrm{tr}(\hat{\mathbf{R}}(i))/M$
$\hat{\mathbf{F}}_0(i) = \hat{\nu}_0(i)\mathbf{I}$
$\tilde{\mathbf{R}}(i) = \hat{\rho}_0(i)\hat{\mathbf{F}}_0(i) + (1-\hat{\rho}_0(i))\hat{\mathbf{R}}(i)$
$\hat{\rho}_0(i+1) = \frac{(1-\frac{2}{M})\mathrm{tr}(\tilde{\mathbf{R}}(i)\hat{\mathbf{R}}(i)) + \mathrm{tr}^2(\tilde{\mathbf{R}}(i))}{(i+1-\frac{2}{M})\mathrm{tr}(\tilde{\mathbf{R}}(i)\hat{\mathbf{R}}(i)) + (1-\frac{i}{M})\mathrm{tr}^2(\tilde{\mathbf{R}}(i))}$
$\hat{\sigma}_1^2(i) = \frac{|\hat{\mathbf{a}}_1^H(i)\mathbf{x}(i)|^2 - \hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)\sigma_n^2}{|\hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)|^2}$
$\tilde{\mathbf{R}}(i) = \tilde{\mathbf{R}}(i) + \|\tilde{\mathbf{R}}(i)\|_2\mathbf{I}$
$\tilde{\mathbf{R}}_{i+n}(i) = \tilde{\mathbf{R}}(i) - \hat{\sigma}_1^2(i)\hat{\mathbf{a}}_1(i)\hat{\mathbf{a}}_1^H(i)$
$\tilde{\mathbf{R}}_{i+n}(i) = \tilde{\mathbf{R}}_{i+n}(i)\frac{2\sigma_n^2}{\|\tilde{\mathbf{R}}_{i+n}(i)\|_2}$
$\hat{\mathbf{w}}(i) = \frac{\tilde{\mathbf{R}}_{i+n}^{-1}(i)\hat{\mathbf{a}}_1(i)}{\hat{\mathbf{a}}_1^H(i)\tilde{\mathbf{R}}_{i+n}^{-1}(i)\hat{\mathbf{a}}_1(i)}$

B. Interference-Plus-Noise Covariance Matrix Estimation

In order to estimate the INC matrix, the data covariance matrix (which contains the desired signal) is required. The SCM in (5) is necessary as a preliminary approximation. In the next step, similar to using OAS to estimate the cross-correlation vector $\hat{\mathbf{d}}$, the SCM is also processed with the OAS method as a further shrinkage estimation step. Let us define the following quantity

$$\hat{\mathbf{F}}_0 = \hat{\nu}_0\mathbf{I}, \tag{16}$$

where $\hat{\nu}_0 = \mathrm{tr}(\hat{\mathbf{R}})/M$. Then, we use the shrinkage form again

$$\tilde{\mathbf{R}} = \hat{\rho}_0\hat{\mathbf{F}}_0 + (1-\hat{\rho}_0)\hat{\mathbf{R}}. \tag{17}$$

By minimizing the MSE described by $E[\|\tilde{\mathbf{R}}(i) - \hat{\mathbf{F}}_0(i-1)\|^2]$, we obtain the following recursion

$$\tilde{\mathbf{R}}(i) = \hat{\rho}_0(i)\hat{\mathbf{F}}_0(i) + (1-\hat{\rho}_0(i))\hat{\mathbf{R}}(i), \tag{18}$$

$$\hat{\rho}_0(i+1) = \frac{(1-\frac{2}{M})\mathrm{tr}(\tilde{\mathbf{R}}(i)\hat{\mathbf{R}}(i)) + \mathrm{tr}^2(\tilde{\mathbf{R}}(i))}{(i+1-\frac{2}{M})\mathrm{tr}(\tilde{\mathbf{R}}(i)\hat{\mathbf{R}}(i)) + (1-\frac{i}{M})\mathrm{tr}^2(\tilde{\mathbf{R}}(i))}. \tag{19}$$

Provided that $0 < \hat{\rho}_0(0) < 1$, the iterative process in (18) and (19) is guaranteed to converge [12]. In order to eliminate the unwanted information of the desired signal in the covariance matrix and obtain the INC matrix, the desired signal power σ_1^2 must be estimated. Let us rewrite the received data as

$$\mathbf{x} = \sum_{k=1}^{K} \mathbf{a}_k s_k + \mathbf{n}. \tag{20}$$

Pre-multiplying the above equation by $\mathbf{a}_1^H$, we have

$$\mathbf{a}_1^H\mathbf{x} = \mathbf{a}_1^H\mathbf{a}_1 s_1 + \mathbf{a}_1^H\left(\sum_{k=2}^{K} \mathbf{a}_k s_k + \mathbf{n}\right). \tag{21}$$

Assuming that $\mathbf{a}_1$ is uncorrelated with the interferers, we obtain

$$\mathbf{a}_1^H\mathbf{x} = \mathbf{a}_1^H\mathbf{a}_1 s_1 + \mathbf{a}_1^H\mathbf{n}. \tag{22}$$

Taking the expectation of $E[|\mathbf{a}_1^H\mathbf{x}|^2]$, we obtain

$$|\mathbf{a}_1^H\mathbf{x}|^2 = E[(\mathbf{a}_1^H\mathbf{a}_1 s_1 + \mathbf{a}_1^H\mathbf{n})^*(\mathbf{a}_1^H\mathbf{a}_1 s_1 + \mathbf{a}_1^H\mathbf{n})]. \tag{23}$$

If the noise is statistically independent of the desired signal, we have

$$|\mathbf{a}_1^H\mathbf{x}|^2 = |\mathbf{a}_1^H\mathbf{a}_1|^2|s_1|^2 + \mathbf{a}_1^H\mathbf{n}\mathbf{n}^H\mathbf{a}_1, \tag{24}$$

where $|s_1|^2$ is the desired signal power which can be replaced by its estimate $\hat{\sigma}_1^2$, $\mathbf{n}\mathbf{n}^H$ represents the noise covariance matrix $\mathbf{R}_n$ which can be replaced by $\sigma_n^2\mathbf{I}_M$. Replacing $\mathbf{a}_1$ by its estimate $\hat{\mathbf{a}}_1(i)$ the desired signal power estimate is given by

$$\hat{\sigma}_1^2(i) = \frac{|\hat{\mathbf{a}}_1^H(i)\mathbf{x}(i)|^2 - \hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)\sigma_n^2}{|\hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)|^2}. \tag{25}$$

As the last step, the desired signal covariance matrix is subtracted and the INC matrix is given by

$$\tilde{\mathbf{R}}_{i+n}(i) = \tilde{\mathbf{R}}(i) - \hat{\sigma}_1^2(i)\hat{\mathbf{a}}_1(i)\hat{\mathbf{a}}_1^H(i). \tag{26}$$

The advantage of this step compared to SMI and existing methods is that it does not require direction finding and is suitable for real-time applications. With the estimates for the steering vector and the INC matrix, the beamformer is computed by

$$\hat{\mathbf{w}}(i) = \frac{\tilde{\mathbf{R}}_{i+n}^{-1}(i)\hat{\mathbf{a}}_1(i)}{\hat{\mathbf{a}}_1^H(i)\tilde{\mathbf{R}}_{i+n}^{-1}(i)\hat{\mathbf{a}}_1(i)}. \tag{27}$$

Table I summarizes LOCSME in steps. From a complexity point of view, the main computational cost is due to the following steps: SCM of the observation data, OAS estimation for SCM, norm computations of the covariance matrix and the INC matrix. Each of these steps has a complexity of $\mathcal{O}(M^3)$. Additionally, compared to the previous RAB algorithms in [7], [8], [10] and [11] which have complexity equal or higher than $\mathcal{O}(M^{3.5})$, LOCSME has a lower cost ($\mathcal{O}(M^3)$).

IV. SIMULATIONS

In our simulations, a uniform linear array (ULA) of $M = 12$ omnidirectional sensors with a spacing of half wavelength is considered. Three source signals include the desired signal which is presumed to arrive at $\theta_1 = 10°$ and two interferers which are impinging on the antenna array from directions $\theta_2 = 50°$ and $\theta_3 = 90°$. The signal-to-interference ratio (SIR) is fixed at 20 dB. Only one iteration is performed per snapshot and we employ $i = 50$ snapshots and 100 repetitions to obtain each point of the curves. The beamformer computed with LOCSME is compared to existing beamformers in terms of the output SINR. For the beamformers of [7], [8], [10], [11] and the beamformer with LOCSME, the angular sector is chosen as $[\theta_1 - 5°, \theta_1 + 5°]$ and $p = 8$ principal eigenvectors are used. The number of eigenvectors of the subspace projection matrix p is selected manually with the help of simulations. For the beamformers of [7], [8], [10] and

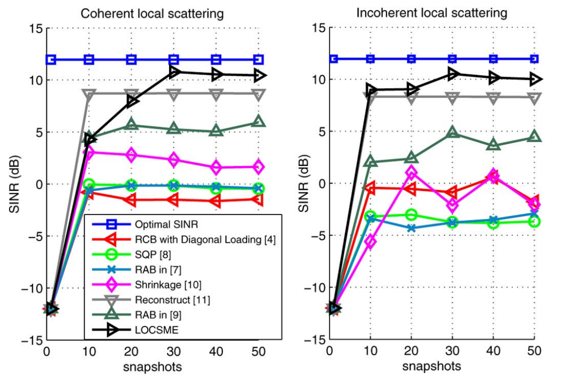

Fig. 1. SINR versus snapshots.

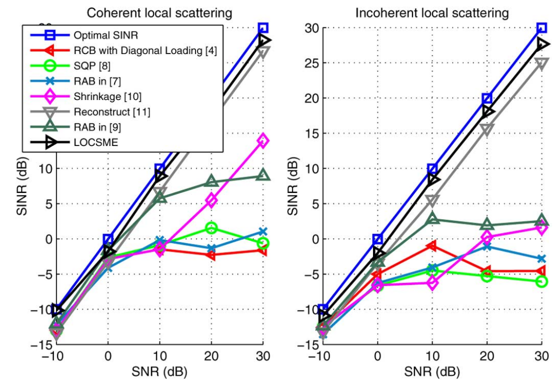

Fig. 2. SINR versus SNR.

[11] which also require an optimization technique, the CVX software is used. The SINR performance versus snapshots and SNR of the algorithms is shown in Figs. 1 and 2 and the number of snapshots is 50 for the SINR versus SNR plots. The average execution time of the algorithms in [7], [8], [10] and [11] is around 0.3 sec/snapshot, while LOCSME only requires 0.021 sec/snapshot.

A. Mismatch due to Coherent Local Scattering

In this case, the steering vector of the desired signal is affected by a local scattering effect and modeled as

$$\mathbf{a} = \mathbf{p} + \sum_{k=1}^{4} e^{j\varphi_k}\mathbf{b}(\theta_k), \tag{28}$$

where $\mathbf{p}$ corresponds to the direct path while $\mathbf{b}(\theta_k)(k=1,2,3,4)$ corresponds to the scattered paths. The angles $\theta_k(k=1,2,3,4)$ are randomly and independently drawn in each simulation run from a uniform generator with mean 10° and standard deviation 2°. The angles $\varphi_k(k=1,2,3,4)$ are independently and uniformly taken from the interval $[0,2\pi]$ in each simulation run. Notice that θ_k and φ_k change from trials while remaining constant over snapshots [3]. Figs. 1(a) and 2(a) illustrate the SINR performance versus snapshots and SNR under the coherent scattering case. LOCSME outperforms the other algorithms and is close to the optimum SINR.

B. Mismatch due to Incoherent Local Scattering

In the incoherent local scattering case, the desired signal has a time-varying signature and the steering vector is modeled by

$$\mathbf{a}(i) = s_0(i)\mathbf{p} + \sum_{k=1}^{4} s_k(i)\mathbf{b}(\theta_k), \tag{29}$$

where $s_k(i)(k = 0,1,2,3,4)$ are i.i.d zero mean complex Gaussian random variables independently drawn from a random generator. The angles $\theta_k(k=0,1,2,3,4)$ are drawn independently in each simulation run from a uniform generator with mean 10° and standard deviation 2°. This time, $s_k(i)$ changes both from run to run and from snapshot to snapshot. Figs. 1(b) and 2(b) depict the SINR performance versus snapshots and SNR. Compared to the coherent scattering results, all the algorithms have a performance degradation due to the effect of incoherent local scattering. However, LOCSME is able to outperform the remaining robust beamformers over a wide range of input SNR. The reason for the improved performance of LOCSME is the combined use of accurate estimates of the INC matrix and of the steering vector mismatch.

Further testing with a larger number of antenna array elements indicates that the performance of all algorithms degrades (e.g. LOCSME has around 2 dB degradation when $M=60$). In addition, inappropriate choice for the angular sector in which the desired signal is assumed to be located will lead to obvious performance degradation.

V. Conclusion

We have proposed LOCSME that only requires prior knowledge of the angular sector of the desired signal and is less costly than existing methods. Simulation results have shown that LOCSME outperforms prior art in both coherent local scattering and incoherent local scattering cases.

Appendix

Derivation of $\hat{\rho}(i)$: Equation (12) can be rewritten in an alternative way in matrix version as $\hat{\mathbf{D}}(i) = \hat{\rho}(i)\hat{\mathbf{F}}(i) + (1-\hat{\rho}(i))\hat{\mathbf{S}}(i)$. By using (10), then the shrinkage intensity $\hat{\rho}(i)$ can be computed from the following optimization problem

$$\begin{aligned} &\min_{\hat{\rho}(i),\hat{\nu}(i)} \quad E[\|\hat{\mathbf{D}}(i) - \hat{\mathbf{F}}(i-1)\|^2] \\ &\text{subject to} \quad \hat{\mathbf{D}}(i) = \hat{\rho}(i)\hat{\nu}(i)\mathbf{I} + (1-\hat{\rho}(i))\hat{\mathbf{S}}(i). \end{aligned} \tag{30}$$

Since $E[\hat{\mathbf{S}}(i)] = E[\hat{\mathbf{F}}(i-1)]$, the objective function in (30) can be rewritten as $\hat{\rho}^2(i)\|\hat{\mathbf{F}}(i-1) - \hat{\nu}(i)\mathbf{I}\|^2 + (1-\hat{\rho}(i))^2 E[\|\hat{\mathbf{S}}(i) - \hat{\mathbf{F}}(i-1)\|^2]$ [13]. The optimal value of $\hat{\nu}(i)$ is obtained as the solution to a problem that does not depend on $\hat{\rho}(i)$ as given by $\min_{\hat{\nu}(i)} \|\hat{\mathbf{F}}(i-1) - \hat{\nu}(i)\mathbf{I}\|^2$, which can be solved by computing the partial derivative of the argument with respect to $\hat{\nu}(i)$ and equating the terms to zero. By substituting the optimal value of $\hat{\nu}(i)$ into (30), computing the partial derivative of the argument with respect to $\hat{\rho}(i)$, equating the terms to zero and solving for $\hat{\rho}(i)$, we obtain

$$\hat{\rho}(i) = \frac{E[\|\hat{\mathbf{S}}(i) - \hat{\mathbf{F}}(i-1)\|^2]}{\|\hat{\mathbf{F}}(i-1) - \mu(i)\mathbf{I}\|^2 + E[\|\hat{\mathbf{S}}(i) - \hat{\mathbf{F}}(i-1)\|^2]}. \tag{31}$$

By further Gaussian assumptions as in [12], replacing $\hat{\mathbf{F}}(i-1)$ by its estimate $\hat{\mathbf{D}}(i)$ and the data sample number n by the snapshot index i, equation (13) can be obtained.

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