# Adaptive Widely Linear Reduced-Rank Beamforming Based on Joint Iterative Optimization

Nuan Song, *Student Member, IEEE*, Waheed Ullah Alokozai, *Student Member, IEEE*, Rodrigo C. de Lamare, *Senior Member, IEEE*, and Martin Haardt, *Senior Member, IEEE*

***Abstract*—We propose a reduced-rank beamformer based on the rank-$D$ Joint Iterative Optimization (JIO) of the modified Widely Linear Constrained Minimum Variance (WLCMV) problem for non-circular signals. The novel WLCMV-JIO scheme takes advantage of both the Widely Linear (WL) processing and the reduced-rank concept, outperforming its linear counterpart as well as the full-rank WL beamformer. We develop an augmented recursive least squares algorithm and present an improved structured version with a much more efficient implementation. It is shown that the improved adaptive scheme achieves the best convergence performance among all the considered methods with a low computational complexity.**

***Index Terms*—Adaptive beamforming, linear constrained minimum variance, non-circular data, recursive least squares algorithms, reduced-rank methods, widely linear processing.**

## I. Introduction

ADAPTIVE beamforming techniques have been widely used in the areas of radar, sonar, speech enhancement, and wireless communications. In general, a beamformer design requires the second-order statistics of the observation data vector $\boldsymbol{r}$, which can be fully described by its covariance matrix $\boldsymbol{R} = \mathbb{E}\{\boldsymbol{r}\boldsymbol{r}^H\}$ and its pseudo-covariance matrix $\check{\boldsymbol{R}} = \mathbb{E}\{\boldsymbol{r}\boldsymbol{r}^T\}$. In the situations when $\boldsymbol{r}$ is second-order non-circular, i.e., $\check{\boldsymbol{R}} \neq \boldsymbol{0}$, Widely Linear (WL) processing can improve the performance as compared to the conventional linear counterpart [1], [2], [3], [4], [5]. Some WL beamforming algorithms based on the Minimum Mean Square Error (MMSE) criterion [6] and the Linearly Constrained Minimum Variance (LCMV) criterion [7], [8], [9], [10] have been discussed and analyzed.

However, in applications with a large number of antennas, the parameter estimation requires a considerable number of data samples. Moreover, WL processing has to consider both the observation data $\boldsymbol{r}$ and its complex conjugate $\boldsymbol{r}^*$ so that the information contained in both $\boldsymbol{R}$ and $\check{\boldsymbol{R}}$ can be fully exploited. This leads to an increased beamformer length and considerably slows down the convergence speed of adaptive algorithms. Reduced-rank techniques can provide a faster convergence by estimating a reduced number of coefficients, which motivates the combination of reduced-rank schemes with the WL processing. Prior work concerning WL reduced-rank techniques is based on the eigen-decomposition [1], the multi-stage Wiener filter (MSWF) [11], or the auxiliary vector filter (AVF) [12]. However, these methods are relatively costly and may suffer from numerical problems. In comparison, the Joint Iterative Optimization (JIO) method proposed in [13] shows a better performance and lends itself to an efficient adaptive implementation.

In this work, we propose a WL JIO beamformer based on the Widely Linear Constrained Minimum Variance (WLCMV) criterion with regularization, namely the WLCMV-JIO. After introducing the WLCMV-JIO algorithm, we develop the corresponding Recursive Least Squares (RLS) adaptive algorithms, namely Augmented RLS (A-RLS) and Structured RLS (S-RLS). The A-RLS directly deals with the concatenation of $\boldsymbol{r}$ and $\boldsymbol{r}^*$. The S-RLS exploits the block conjugate structure of the covariance matrix and the resulting estimation is carried out in a structured manner, yielding a much more efficient implementation than the A-RLS. We evaluate the computational complexity of the proposed schemes in terms of complex additions and multiplications. Simulation results on the convergence and rank-dependent performances are also shown.

## II. Widely Linear Joint Iterative Optimization Beamformer Based on WLCMV

Let us assume that $K$ narrowband signals impinge on an array with an arbitrary geometry, consisting of $M$ $(K \leq M)$ sensor elements. The sources are assumed to be in the far field with Directions-Of-Arrival (DOAs) $\theta_0, \ldots, \theta_{K-1}$. The received vector $\boldsymbol{r}$ can be modeled as

$$\boldsymbol{r} = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s} + \boldsymbol{n} \in \mathbb{C}^M, \tag{1}$$

where $\boldsymbol{\theta} = [\theta_0, \ldots, \theta_{K-1}]^T \in \mathbb{R}^K$ contains the DOAs, $\boldsymbol{A}(\boldsymbol{\theta}) = [\boldsymbol{a}(\theta_0), \ldots, \boldsymbol{a}(\theta_{K-1})] \in \mathbb{C}^{M\times K}$ consists of the steering vectors $\boldsymbol{a}(\theta_k) \in \mathbb{C}^M, k = 0, \ldots, K-1$, $\boldsymbol{s} \in \mathbb{R}^K$ is the data vector from $K$ sources, and $\boldsymbol{n} \in \mathbb{C}^M$ is the additive white Gaussian noise vector with zero mean and power spectrum density $N_0$. The steering vector of the Signal-of-Interest (SOI) is $\boldsymbol{a}(\theta_0)$.

### A. WLCMV Beamformer

Given a received signal $\boldsymbol{r} \in \mathbb{C}^M$, the original vector $\boldsymbol{r}$ and its complex conjugate $\boldsymbol{r}^*$ are often combined into an augmented vector using a bijective transformation $\mathcal{T}$

$$\boldsymbol{r} \xrightarrow{\mathcal{T}} \boldsymbol{r}_a : \qquad \boldsymbol{r}_a = [\,\boldsymbol{r}^T, \quad \boldsymbol{r}^H\,]^T \in \mathbb{C}^{2M}, \tag{2}$$

Manuscript received October 11, 2013; accepted December 10, 2013. Date of publication December 23, 2013; date of current version January 17, 2014. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Sascha Spors.

N. Song, W. U. Alokozai, and M. Haardt are with the Communications Research Laboratory, Ilmenau University of Technology, D-98684 Ilmenau, Germany, (e-mail: nuan.song@tu-ilmenau.de; waheed-ullah.alokozai@tu-ilmenau.de; martin.haardt@tu-ilmenau.de; http://www.tu-ilmenau.de/crl).

R. C. de Lamare is with CETUC/PUC-Rio, Rio de Janeiro Brazil, and also with the Communications Research Group, Department of Electronics, University of York, Heslington, North Yorkshire, York Y010 5DD, U.K. (e-mail: rodrigo.delamare@york.ac.uk).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/LSP.2013.2295943

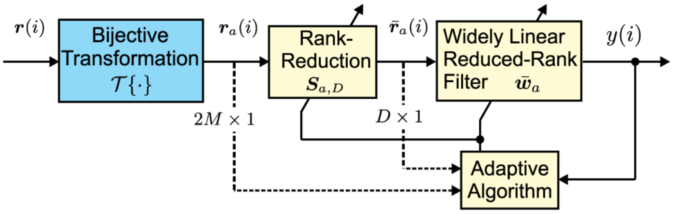


Fig. 1. Block diagram of the WL reduced-rank scheme.

in order to exploit the information contained in both the covariance matrix $\boldsymbol{R}$ and the pseudo-covariance matrix $\check{\boldsymbol{R}}$. The output of a WL beamformer is $y = \boldsymbol{w}_a^H \boldsymbol{r}_a$, where the complex weight vector $\boldsymbol{w}_a \in \mathbb{C}^{2M}$ is designed for the augmented received vector $\boldsymbol{r}_a$.

The WLCMV beamformer $\boldsymbol{w}_a$ is calculated by solving the following constrained optimization problem [7], [8], [9], [10]

$$\begin{aligned} &\text{minimize } \mathbb{E}\{|y(i)|^2\} = \boldsymbol{w}_a^H \boldsymbol{R}_a \boldsymbol{w}_a \\ &\text{s. t. } \boldsymbol{w}^H \boldsymbol{a}_a(\theta_0) = \gamma, \end{aligned} \tag{3}$$

where $\boldsymbol{a}_a(\theta_0) = \mathcal{T}\{\boldsymbol{a}(\theta_0)\}$ is the augmented array steering vector of the SOI and $\gamma$ is a constant. The augmented covariance matrix $\boldsymbol{R}_a$ with a block structure is represented as

$$\boldsymbol{R}_a = \mathbb{E}\{\boldsymbol{r}_a(i)\boldsymbol{r}_a^H(i)\} = \begin{bmatrix} \boldsymbol{R} & \check{\boldsymbol{R}} \\ \check{\boldsymbol{R}}^* & \boldsymbol{R}^* \end{bmatrix} \in \mathbb{C}^{2M\times 2M}. \tag{4}$$

For the non-circular data sources, $\check{\boldsymbol{R}} \neq \boldsymbol{0}$, which means that $\boldsymbol{r}$ is second-order non-circular. The weight vector designed from (3) minimizes the output power while preserving the response in the direction of the augmented SOI. The optimum solution is written as

$$\boldsymbol{w}_{a,\mathrm{opt}} = \frac{\gamma^* \boldsymbol{R}_a^{-1} \boldsymbol{a}_a(\theta_0)}{\boldsymbol{a}_a^H(\theta_0)\boldsymbol{R}_a^{-1}\boldsymbol{a}_a(\theta_0)}. \tag{5}$$

It is shown in [14], [15] that if the data to be estimated are real, i.e., strictly non-circular such as Binary Phase Shift Keying (BPSK) signals, and the MMSE criterion [15] or the minimum output-energy criterion [16] is used, it follows that $\boldsymbol{w}_a = [\check{\boldsymbol{w}}^T, \check{\boldsymbol{w}}^H]^T = \mathcal{T}\{\check{\boldsymbol{w}}\}$, where $\check{\boldsymbol{w}} \in \mathbb{C}^M$. Therefore, a key property of the WL filtering is the conjugate symmetry defined as $\boldsymbol{w}_a^H \boldsymbol{r}_a = \boldsymbol{r}_a^T \boldsymbol{w}_a^* = 2 \cdot \Re\{\check{\boldsymbol{w}}^H \boldsymbol{r}\}$.

### B. WLCMV-JIO Design

The block diagram of the proposed WL reduced-rank beamformer is shown in Fig. 1. After the augmented received vector $\boldsymbol{r}_a(i) \in \mathbb{C}^{2M}$ is obtained, it is transformed by a rank-reduction matrix $\boldsymbol{S}_{a,D} \in \mathbb{C}^{2M\times D}$ into a subspace with dimension $D$ ($D \ll M$). The beamformer $\bar{\boldsymbol{w}}_a \in \mathbb{C}^D$ is designed by processing the reduced-rank vector $\bar{\boldsymbol{r}}_a(i) = \boldsymbol{S}_{a,D}^H \boldsymbol{r}_a(i) \in \mathbb{C}^D$ and its output is expressed as $y(i) = \bar{\boldsymbol{w}}_a^H \bar{\boldsymbol{r}}_a(i)$.

Both $\boldsymbol{S}_{a,D}$ and $\bar{\boldsymbol{w}}_a$ can be calculated according to the following proposed rank-$D$ WLCMV optimization criterion,

$$\begin{aligned} &\min\ \mathbb{E}\{|y(i)|^2 + \delta\|\bar{\boldsymbol{r}}_a(i)\|^2\} \\ &\text{s.t.} \bar{\boldsymbol{w}}_a^H \boldsymbol{S}_{a,D}^H \boldsymbol{a}_a(\theta_0) + \beta \sum_{d=1}^{D} \boldsymbol{e}_d^H \boldsymbol{S}_{a,D}^H \boldsymbol{I}_{2M,D} \boldsymbol{e}_d = \gamma, \end{aligned} \tag{6}$$

where $\beta$ and $\delta$ are small positive constants used for regularization and to ensure that $\boldsymbol{S}_{a,D}$ has rank $D$ and $\gamma$ is a constant corresponding to the constraint. The augmented steering vector of the SOI is expressed as $\boldsymbol{a}_a(\theta_0) = \mathcal{T}\{\boldsymbol{a}(\theta_0)\} \in \mathbb{C}^{2M}$. Furthermore, $\boldsymbol{e}_d$ is a $D \times 1$ pinning vector, which is the $d$-th column of the identity matrix $\boldsymbol{I}_D$. The matrix $\boldsymbol{I}_{N,K}$ represents an $N$-by-$K$ matrix with 1s on the diagonal and zeros elsewhere. Thus, $\boldsymbol{I}_{2M,D} = [\boldsymbol{I}_D,\ \ \boldsymbol{0}_{D\times(2M-D)}]^T$.

The above problem can be solved by the method of Lagrange multipliers. The unconstrained Lagrangian can be written as

$$\begin{aligned} \mathcal{L}(\bar{\boldsymbol{w}}_a, \boldsymbol{S}_{a,D}) = {}& \bar{\boldsymbol{w}}_a^H \boldsymbol{S}_{a,D}^H \boldsymbol{R}_a \boldsymbol{S}_{a,D} \bar{\boldsymbol{w}}_a + \delta \boldsymbol{r}_a^H \boldsymbol{S}_{a,D} \boldsymbol{S}_{a,D}^H \boldsymbol{r}_a \\ &+ \zeta \Bigg( \bar{\boldsymbol{w}}_a^H \boldsymbol{S}_{a,D}^H \boldsymbol{a}_a(\theta_0) \\ &+ \beta \sum_{d=1}^{D} \boldsymbol{e}_d^H \boldsymbol{S}_{a,D}^H \boldsymbol{I}_{2M,D} \boldsymbol{e}_d - \gamma \Bigg), \end{aligned} \tag{7}$$

where $\zeta$ is a scalar corresponding to the Lagrange multiplier.

A two-step optimization procedure is applied to derive $\boldsymbol{S}_{a,D}$ and $\bar{\boldsymbol{w}}_a$. Firstly, we fix $\boldsymbol{S}_{a,D}$ and minimize (7) by taking its gradient with respect to $\bar{\boldsymbol{w}}_a^*$. The resulting reduced-rank beamforming vector can be expressed as

$$\bar{\boldsymbol{w}}_a = \frac{(\gamma^* - \beta\rho^*)\bar{\boldsymbol{R}}_a^{-1}\bar{\boldsymbol{a}}_a(\theta_0)}{\bar{\boldsymbol{a}}_a^H(\theta_0)\bar{\boldsymbol{R}}_a^{-1}\bar{\boldsymbol{a}}_a(\theta_0)}, \tag{8}$$

where $\rho = \sum_{d=1}^{D} \boldsymbol{e}_d^H \boldsymbol{S}_{a,D}^H \boldsymbol{I}_{2M,D} \boldsymbol{e}_d$ is a scalar, $\bar{\boldsymbol{a}}_a(\theta_0) = \boldsymbol{S}_{a,D}^H \boldsymbol{a}_a(\theta_0)$ is the reduced-rank augmented array steering vector of the SOI, and $\bar{\boldsymbol{R}}_a = \mathbb{E}\{\bar{\boldsymbol{r}}_a(i)\bar{\boldsymbol{r}}_a^H(i)\} = \boldsymbol{S}_{a,D}^H \boldsymbol{R}_a \boldsymbol{S}_{a,D} \in \mathbb{C}^{D\times D}$ is the reduced-rank augmented covariance matrix. Secondly, by fixing $\bar{\boldsymbol{w}}_a$, the solution to the minimization of (7) with respect to $\boldsymbol{S}_{a,D}^*$ is given by

$$\boldsymbol{S}_{a,D} = \frac{\gamma^* \boldsymbol{R}_a^{-1} \boldsymbol{T}_a \boldsymbol{R}_{\bar{w}_a}^{-1}}{\boldsymbol{a}_a^H(\theta_0)\boldsymbol{R}_a^{-1}\boldsymbol{T}_a \boldsymbol{R}_{\bar{w}_a}^{-1}\bar{\boldsymbol{w}}_a + \beta\tau^*}, \tag{9}$$

where $\boldsymbol{T}_a = \boldsymbol{a}_a(\theta_0)\bar{\boldsymbol{w}}_a^H + \beta \boldsymbol{I}_{2M,D} \in \mathbb{C}^{2M\times D}$ is a rank $D$ matrix, $\tau = \sum_{d=1}^{D} \boldsymbol{e}_d^H \boldsymbol{R}_{\bar{w}_a}^{-1} \boldsymbol{T}_a^H \boldsymbol{R}_a^{-1} \boldsymbol{I}_{2M,D} \boldsymbol{e}_d$ is a scalar and $\boldsymbol{R}_{\bar{w}_a} = \bar{\boldsymbol{w}}_a \bar{\boldsymbol{w}}_a^H + \delta \boldsymbol{I}_D \in \mathbb{C}^{D\times D}$ is the reduced-rank weight matrix.

It is worth remarking that by using such a joint optimization, $\bar{\boldsymbol{w}}_a$ and $\boldsymbol{S}_{a,D}$ expressed in (8) and (9) depend on each other and thus they are not closed-form solutions. Therefore, the computation of $\bar{\boldsymbol{w}}_a$ and $\boldsymbol{S}_{a,D}$ should be carried out in an iterative fashion with the corresponding initial values. The rank-reduction matrix $\boldsymbol{S}_{a,D}$ designed in WLCMV-JIO transforms the augmented vector $\boldsymbol{r}_a(i)$ into a subspace with a much smaller dimension to improve the convergence performance. One advantage lies in the iterative exchange of the information between the rank-reduction matrix and the WL reduced-rank beamformer, which leads to a faster convergence. It offers a simpler implementation as compared to the existing WL reduced-rank schemes such as the MSWF or the AVF [13], because it is possible to devise efficient adaptive algorithms to solve (7). The WLCMV-JIO also benefits from fully exploiting the second-order statistics of the non-circular signals, leading to a better estimation performance.

### C. Adaptive Algorithms

Two adaptive algorithms, namely Augmented RLS (A-RLS) and Structured RLS (S-RLS), are developed for the WLCMV-JIO scheme to estimate $\boldsymbol{S}_{a,D}(i)$ and $\bar{\boldsymbol{w}}_a(i)$[1].

*1) Augmented RLS:* One straightforward way is to apply the RLS adaptation based on the augmented received vector $\boldsymbol{r}_a(i)$, i.e., the A-RLS algorithm. Either (8) or the adaptation of the rank-reduction matrix $\boldsymbol{S}_{a,D}(i)$ in (9) requires estimating the inverse of a matrix. According to the matrix inversion lemma, for example, we can update $\boldsymbol{R}_a^{-1}(i)$ as

$$\boldsymbol{R}_a^{-1}(i) = \lambda^{-1}\boldsymbol{R}_a^{-1}(i-1) - \lambda^{-1}\boldsymbol{k}(i)\boldsymbol{r}_a^H(i)\boldsymbol{R}_a^{-1}(i-1), \quad (10)$$

where the gain vector is

$$\boldsymbol{k}(i) = \frac{\lambda^{-1}\boldsymbol{R}_a^{-1}(i-1)\boldsymbol{r}_a(i)}{1+\lambda^{-1}\boldsymbol{r}_a^H(i)\boldsymbol{R}_a^{-1}(i-1)\boldsymbol{r}_a(i)} \quad (11)$$

and $\lambda$ is the forgetting factor which is a positive constant close to but less than 1. Similarly, the updates of $\bar{\boldsymbol{R}}_a^{-1}(i)$ can be performed by replacing the relevant variables with $\bar{\boldsymbol{r}}_a(i)$.

To estimate $\boldsymbol{R}_{\bar{w}_a}^{-1}(i)$, we avoid the direct matrix inversion by applying the matrix inversion lemma and obtain

$$\begin{aligned}\boldsymbol{R}_{\bar{w}_a}^{-1}(i) &= (\bar{\boldsymbol{w}}_a(i)\bar{\boldsymbol{w}}_a^H(i)+\delta\boldsymbol{I}_D)^{-1}\\ &= \frac{1}{\delta}\left(\boldsymbol{I}_D - \frac{\bar{\boldsymbol{w}}_a(i)\bar{\boldsymbol{w}}_a^H(i)}{\delta+\|\bar{\boldsymbol{w}}_a(i)\|^2}\right). \end{aligned} \quad (12)$$

*2) Structured RLS:* In A-RLS, the calculation of $\boldsymbol{R}_a^{-1}(i)$ requires the calculation of parameters with a dimension of $2M$, which is computationally inefficient especially when $M$ is large. By exploiting the structured property of the augmented covariance matrix $\boldsymbol{R}_a$ as shown in (4), the adaptive estimation algorithm can be implemented in a much more efficient way [6]. Let us rewrite $\boldsymbol{R}_a^{-1}(i)$ as

$$\boldsymbol{R}_a^{-1}(i) = \begin{bmatrix}\boldsymbol{P}(i) & \boldsymbol{Q}(i)\\ \boldsymbol{Q}^*(i) & \boldsymbol{P}^*(i)\end{bmatrix}, \quad (13)$$

where it follows that $\boldsymbol{P} = \boldsymbol{P}^H$ and $\boldsymbol{Q} = \boldsymbol{Q}^T$. Thereby, the estimation of $\boldsymbol{R}_a^{-1}(i)$ can be broken down into the calculation of $\boldsymbol{P}(i)$ and $\boldsymbol{Q}(i)$, respectively, so as to reduce the computational complexity. By inserting (13) into (10), we can obtain

$$\boldsymbol{P}(i) = \lambda^{-1}(\boldsymbol{P}(i-1) - c^{-1}(i)\boldsymbol{x}(i)\boldsymbol{x}^H(i)) \quad (14)$$

$$\boldsymbol{Q}(i) = \lambda^{-1}(\boldsymbol{Q}(i-1) - c^{-1}(i)\boldsymbol{x}(i)\boldsymbol{x}^T(i)), \quad (15)$$

where

$$\boldsymbol{x}(i) = \boldsymbol{P}(i-1)\boldsymbol{r}(i) + \boldsymbol{Q}(i-1)\boldsymbol{r}^*(i) \quad (16)$$

$$c(i) = \lambda + 2\cdot\Re\{\boldsymbol{x}^H(i)\boldsymbol{r}(i)\}. \quad (17)$$

Moreover, applying (13) to (9) and using the property of conjugate symmetry, we get

$$\boldsymbol{S}_{a,D}(i) = \frac{\left\{\begin{bmatrix}\boldsymbol{v}(i)\\ \boldsymbol{v}^*(i)\end{bmatrix}\bar{\boldsymbol{w}}_a^H(i) + \beta\begin{bmatrix}\boldsymbol{P}(i)\\ \boldsymbol{Q}^*(i)\end{bmatrix}\boldsymbol{I}_{M,D}\right\}\boldsymbol{R}_{\bar{w}_a}^{-1}(i)}{\left(\boldsymbol{a}(\theta_0)^T\boldsymbol{P}(i) + \boldsymbol{a}^H(\theta_0)\boldsymbol{Q}^*(i)\right)\boldsymbol{T}\boldsymbol{R}_{\bar{w}_a}^{-1}(i)\bar{\boldsymbol{w}}_a(i) + \beta\tau^*}, \quad (18)$$

where $\boldsymbol{v}(i) = \boldsymbol{P}(i)\boldsymbol{a}(\theta_0) + \boldsymbol{Q}(i)\boldsymbol{a}^*(\theta_0)$ and $\boldsymbol{T} = 2\boldsymbol{a}(\theta_0)\bar{\boldsymbol{w}}_a^H(i) + \beta\boldsymbol{I}_{M,D}$. The expression for $\boldsymbol{S}_{a,D}(i)$ in (18) breaks the calculation of matrices in the denominator from $2M$ down to $M$, which reduces the computational complexity.

The A-RLS and S-RLS algorithms of WLCMV-JIO are summarized in Tables I and II, where $\delta_a$, $\bar{\delta}$, $\delta_p$, $\delta_q$ are initialization scalars to ensure the numerical stability.

In what follows, we compare the proposed algorithms with the full-rank LCMV-RLS algorithm [17], the JIO-RLS scheme based on the LCMV criterion (denoted by LCMV-JIO-RLS) [13], as well as the full-rank WLCMV methods in terms of both A-RLS and S-RLS adaptations.

TABLE I
THE A-RLS ADAPTIVE ALGORITHM FOR WLCMV-JIO

| |
|---|
| Initialization with a chosen rank $D$: $\bar{\boldsymbol{R}}_a^{-1}(0) = \bar{\delta}\boldsymbol{I}_D, \boldsymbol{R}_{\bar{w}_a}^{-1}(0) = \delta\boldsymbol{I}_D$, $\boldsymbol{R}_a^{-1}(0) = \delta_a\boldsymbol{I}_{2M}, \boldsymbol{S}_{a,D}(0) = \boldsymbol{I}_{2M,D}$ |
| For the time index $i = 1, 2, \cdots$ |
| $\bar{\boldsymbol{r}}_a(i) = \boldsymbol{S}_{a,D}^H(i-1)\boldsymbol{r}_a(i)$, $\bar{\boldsymbol{a}}_a(\theta_0) = \boldsymbol{S}_{a,D}^H(i-1)\boldsymbol{a}_a(\theta_0)$ |
| Update $\bar{\boldsymbol{R}}_a^{-1}(i)$ similar to (10) |
| Estimate $\bar{\boldsymbol{w}}_a(i) = \frac{(1-\beta\rho^*)\bar{\boldsymbol{R}}_a^{-1}(i)\bar{\boldsymbol{a}}_a(\theta_0)}{\bar{\boldsymbol{a}}_a^H(\theta_0)\bar{\boldsymbol{R}}_a^{-1}(i)\bar{\boldsymbol{a}}_a(\theta_0)}$ |
| Update $\boldsymbol{T}_a(i) = \boldsymbol{a}_a(\theta_0)\bar{\boldsymbol{w}}_a^H(i) + \beta\boldsymbol{I}_{2M,D}$ |
| Update $\boldsymbol{R}_a^{-1}(i)$ by (10) and $\boldsymbol{R}_{\bar{w}_a}^{-1}(i)$ by (12) |
| Estimate $\boldsymbol{S}_{a,D}(i) = \frac{\boldsymbol{R}_a^{-1}(i)\boldsymbol{T}_a(i)\boldsymbol{R}_{\bar{w}_a}^{-1}(i)}{\boldsymbol{a}_a^H(\theta_0)\boldsymbol{R}_a^{-1}(i)\boldsymbol{T}_a(i)\boldsymbol{R}_{\bar{w}_a}^{-1}(i)\bar{\boldsymbol{w}}_a(i) + \beta\tau^*}$ |
| End |

TABLE II
THE S-RLS ADAPTIVE ALGORITHM FOR WLCMV-JIO

| |
|---|
| Initialization with a chosen rank $D$: $\bar{\boldsymbol{R}}_a^{-1}(0) = \bar{\delta}\boldsymbol{I}_D, \boldsymbol{R}_{\bar{w}_a}^{-1}(0) = \delta\boldsymbol{I}_D$, $\boldsymbol{P}(0) = \delta_p\boldsymbol{I}_M, \boldsymbol{Q}(0) = \delta_q\boldsymbol{I}_M, \boldsymbol{S}_{a,D}(0) = \boldsymbol{I}_{2M,D}$ |
| For the time index $i = 1, 2, \cdots$ |
| $\bar{\boldsymbol{r}}_a(i) = \boldsymbol{S}_{a,D}^H(i-1)\boldsymbol{r}_a(i)$, $\bar{\boldsymbol{a}}_a(\theta_0) = \boldsymbol{S}_{a,D}^H(i-1)\boldsymbol{a}_a(\theta_0)$ |
| Update $\bar{\boldsymbol{R}}_a^{-1}(i)$ similar to (10) |
| Estimate $\bar{\boldsymbol{w}}_a(i) = \frac{(1-\beta\rho^*)\bar{\boldsymbol{R}}_a^{-1}(i)\bar{\boldsymbol{a}}_a(\theta_0)}{\bar{\boldsymbol{a}}_a^H(\theta_0)\bar{\boldsymbol{R}}_a^{-1}(i)\bar{\boldsymbol{a}}_a(\theta_0)}$ |
| Update $\boldsymbol{T}_a(i) = \boldsymbol{a}_a(\theta_0)\bar{\boldsymbol{w}}_a^H(i) + \beta\boldsymbol{I}_{2M,D}$ |
| Update $\boldsymbol{P}(i)$ and $\boldsymbol{Q}(i)$ via (14) - (17) and $\boldsymbol{R}_{\bar{w}_a}^{-1}(i)$ by (12) |
| Estimate $\boldsymbol{S}_{a,D}(i)$ using (18) |
| End |

## III. COMPLEXITY ANALYSIS

The computational complexity of the proposed WLCMV-JIO algorithms and other considered schemes is estimated and compared in Table III. Fig. 2 illustrates the total number of complex additions and multiplications per iteration per symbol for each algorithm as a function of $M$, where the rank of the JIO schemes is chosen as $D = 6$. It can be observed that the complexity of WLCMV-JIO-S-RLS is only slightly higher than the full-rank LCMV-RLS, but it exhibits a lower complexity than the A-RLS algorithms, which are based on both the WLCMV-JIO and the full-rank WLCMV.

## IV. SIMULATION RESULTS

This section presents the Signal-to-Interference plus Noise Ratio (SINR) performance of the proposed algorithms and the other considered schemes. The output SINR of the reduced-rank algorithms can be calculated by

$$\text{SINR}(i) = \frac{\bar{\boldsymbol{w}}_a^H(i)\boldsymbol{S}_{a,D}^H(i)\boldsymbol{R}_{a,\text{ss}}\boldsymbol{S}_{a,D}(i)\bar{\boldsymbol{w}}_a(i)}{\bar{\boldsymbol{w}}_a^H(i)\boldsymbol{S}_{a,D}^H(i)\boldsymbol{R}_{a,\text{in}}\boldsymbol{S}_{a,D}(i)\bar{\boldsymbol{w}}_a(i)}, \quad (19)$$

[1]For simplicity, we consider the constraint $\gamma = 1$ and assume that all the users transmit real-valued data, i.e., strictly non-circular.

TABLE III
ESTIMATED COMPUTATIONAL COMPLEXITY ACCORDING TO THE NUMBER OF COMPLEX OPERATIONS

| Algorithms | Additions | Multiplications |
|---|---|---|
| LCMV-RLS | $4M^2 - M - 1$ | $5M^2 + 5M$ |
| WLCMV-A-RLS | $16M^2 - 2M - 1$ | $20M^2 + 10M$ |
| WLCMV-S-RLS | $7M^2 + 3M$ | $9M^2 + 10M + 3$ |
| LCMV-JIO-RLS | $4M^2 - 2M + 8D^2 + 6DM - 3D - 3$ | $5M^2 + 6M + 10D^2 + 7DM + 10D + 2$ |
| WLCMV-JIO-A-RLS | $16M^2 - 4M + 8D^2 + 12DM - 3D - 3$ | $20M^2 + 12M + 10D^2 + 14DM + 10D + 2$ |
| WLCMV-JIO-S-RLS | $7M^2 + M + 8D^2 + 12DM - 3D - 2$ | $9M^2 + 12M + 10D^2 + 14DM + 10D + 5$ |

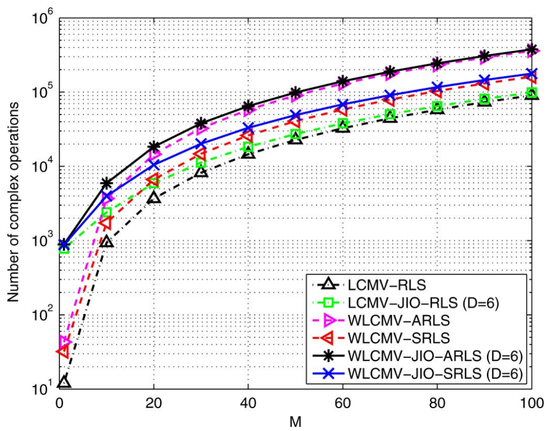


Fig. 2. Computational complexity in terms of complex additions and multiplications per iteration per symbol versus $M$.

where $\boldsymbol{R}_{a,\text{ss}}$ and $\boldsymbol{R}_{a,\text{in}}$ are the augmented covariance matrices of the SOI and the interference plus noise, respectively. A uniform linear array consisting of $M = 32$ sensors is considered. We assume that among $K$ sources, the DOA of the SOI is known a priori at the receiver and let $\theta_0 = 0^\circ$ without loss of generality. The interfering signals impinge on the array with DOAs of $(\pm 10^\circ \cdot [1, \cdots, \frac{K-1}{2}])$. The source signals ($K = 9$) are assumed to be BPSK-modulated with an input Signal-to-Noise Ratio (SNR) of 10 dB and the Signal-to-Interference Ratio (SIR) of $-20$ dB. The calculation of the reduced-rank beamforming vector $\bar{\boldsymbol{w}}_a(i)$ is achieved by initializing the rank-reduction matrix $\boldsymbol{S}_{a,D}(0) = \boldsymbol{I}_{2M,D}$ with a chosen rank $D$. The initialization of the other matrices is chosen so that the best performance of each method is achieved in order to ensure a fair comparison.

Fig. 3 shows the convergence performance of various adaptive schemes in terms of the SINR, where the maximum achievable SINRs for LCMV and WLCMV (cf. [12]) are included. We can observe that the WLCMV-JIO-S-RLS and the WLCMV-JIO-A-RLS outperform their linear counterpart (i.e., LCMV-JIO-RLS) as well as the full-rank schemes. Since the WLCMV-JIO-S-RLS estimates the parameters in a structured way, it converges faster than the WLCMV-JIO-A-RLS, which has to deal with the augmented received vector of size $2M$.

The performance of the WLCMV-JIO algorithms also depends on the rank $D$. We analyze such a rank-dependent performance as a function of $D$ and depict the corresponding performance using $N = 250$ snapshots in Fig. 4. It is shown that the best performance for both RLS versions of the WL-JIO can be achieved when $D = 3, 4$ or 5.

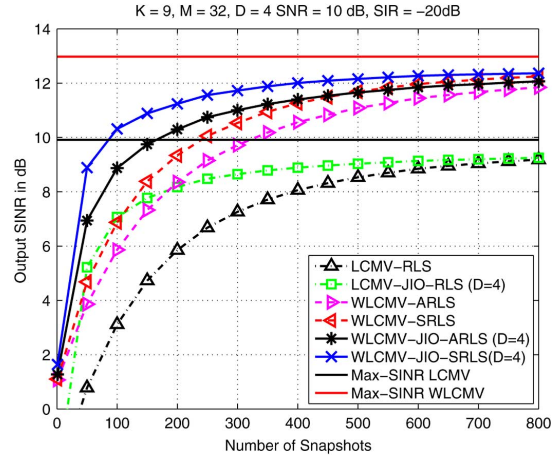


Fig. 3. The output SINR versus the number of snapshots.

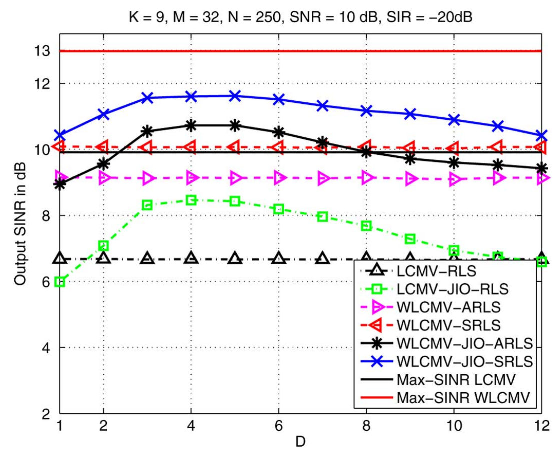


Fig. 4. The output SINR versus the rank $D$ using $N = 250$ snapshots.

## V. CONCLUSION

We propose a novel reduced-rank WLCMV beamformer based on the rank-$D$ JIO concept for non-circular signals. The WLCMV-JIO scheme aims at minimizing the output power of the sensor array while preserving the desired response in the direction of the "augmented" SOI. As the second-order statistics are fully exploited, it outperforms its linear counterpart. The rank-$D$ JIO is performed according to the modified WLCMV criterion such that the information between the reduced-rank beamforming vector and the rank-reduction matrix can be iteratively exchanged. In this way, the proposed scheme yields a better convergence performance with a small rank than the full-rank case. Two adaptive algorithms, namely A-RLS and S-RLS, are developed for the WLCMV-JIO beamformer. Thanks to the structured property of the augmented covariance matrix $\boldsymbol{R}_a$, the S-RLS method converges faster and has a much lower complexity than the A-RLS version.

## REFERENCES


[1] P. J. Schreier and L. L. Scharf, "Second-order analysis of improper complex random vectors and processes," *IEEE Trans. Signal Process.*, vol. 51, no. 3, pp. 714–725, Mar. 2003.

[2] D. Mandic *et al., Complex Valued Nonlinear Adaptive Flters: Noncircularity, Widely Linear and Neural Models*. Hoboken, NJ, USA: Wiley, 2009, vol. 59.

[3] P. J. Schreier and L. L. Scharf, *Statistical Signal Processing of Complex-Valued Data: The Theory of Improper and Noncircular Signals*. Cambridge, U.K.: Cambridge Univ. Press, 2010.

[4] P. Chevalier, A. Oukac, and J. P. Delmas, "Glrt-based array receivers for the detection of a known signal with unknown parameters corrupted by noncircular interferences," *EURASIP J. Adv. Signal Process.*, vol. 2011, no. 1, pp. 1–20, 2011.

[5] J. P. Delmas, A. Oukaci, and P. Chevalier, "On the asymptotic distribution of glr for impropriety of complex signals," *Signal Processing (Elsevier)*, vol. 91, no. 10, pp. 2259–2267, 2011.

[6] S. C. Douglas, "Widely-linear recursive least-squares algorithm for adaptive beamforming," in *IEEE Int. Conf. Acoustics, Speech and Signal Processing, ICASSP'09*, 2009, pp. 2041–2044.

[7] P. Chevalier and A. Maurice, "Constrained beamforming for cyclostationary signals," in *Proc. Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP 1997)*, 1997, vol. 5, pp. 3789–3792.

[8] T. McWhorter and P. Schreier, "Widely-linear beamforming," in *the Thirty-Seventh Asilomar Conf. Signals, Systems and Computers*, 2003, vol. 1, pp. 753–759.

[9] P. Chevalier and A. Blin, "Widely linear MVDR beamformers for the reception of an unknown signal corrupted by noncircular interferences," *IEEE Trans. Signal Process.*, vol. 55, no. 11, pp. 5323–5336, 2007.

[10] P. Chevalier, J. P. Delmas, and A. Oukaci, "Optimal widely linear MVDR beamforming for noncircular signals," in *IEEE Int. Conf. Acoustics, Speech and Signal Processing, ICASSP'09*, 2009, pp. 3573–3576.

[11] N. Song, R. C. de Lamare, M. Haardt, and M. Wolf, "Adaptive widely linear reduced-rank interference suppression based on the multi-stage wiener filter," *IEEE Trans. Signal Process.*, vol. 60, no. 8, 2012.

[12] N. Song, J. Steinwandt, L. Wang, R. C. de Lamare, and M. Haardt, "Non-data-aided adaptive beamforming algorithm based on the widely linear auxiliary vector filter," in *Proc. Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP 2011)*, Prague, Czech Republic, May 2011.

[13] R. C. de Lamare, L. Wang, and R. Fa, "Adaptive reduced-rank lcmv beamforming algorithms based on joint iterative optimization of filters: Design and analysis," *Signal Process.*, vol. 90, no. 2, pp. 640–652, 2010.

[14] S. Buzzi, M. Lops, and A. M. Tulino, "A new family of MMSE multiuser receivers for interference suppression in DS/CDMA systems employing BPSK modulation," *IEEE Trans. Commun.*, vol. 49, no. 1, pp. 154–167, Jan 2001.

[15] B. Picinbono and P. Chevalier, "Widely linear estimation with complex data," *IEEE Trans. Signal Process.*, vol. 43, no. 8, pp. 2030–2033, 1995.

[16] A. S. Cacciapuoti, G. Gelli, L. Paura, and F. Verde, "Finite-sample performance analysis of widely linear multiuser receivers for DS-CDMA systems," *IEEE Trans. Signal Process.*, vol. 56, no. 4, pp. 1572–1588, 2008.

[17] L. S. Resende, J. M. T. Romano, and M. G. Bellanger, "A fast least-squares algorithm for linearly constrained adaptive filtering," *IEEE Trans. Signal Process.*, vol. 44, no. 5, pp. 1168–1174, 1996.