Massive MIMO Systems: Signal Processing Challenges and Future Trends

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Abstract— **This article presents a tutorial on multiuser multiple-antenna wireless systems with a very large number of antennas, known as massive multi-input multi-output (MIMO) systems. Signal processing challenges and future trends in the area of massive MIMO systems are presented and key application scenarios are detailed. A linear algebra approach is considered for the description of the system and data models of massive MIMO architectures. The operational requirements of massive MIMO systems are discussed along with their operation in time-division duplexing mode, resource allocation and calibration requirements. In particular, transmit and receiver processing algorithms are examined in light of the specific needs of massive MIMO systems. Simulation results illustrate the performance of transmit and receive processing algorithms under scenarios of interest. Key problems are discussed and future trends in the area of massive MIMO systems are pointed out.**

I. Introduction

Wireless networks are experiencing a very substantial increase in the delivered amount of data due to a number of emerging applications that include machine-to-machine communications and video streaming [1]–[3]. This very large amount of data exchange is expected to continue and rise in the next decade or so, presenting a very significant challenge to designers of wireless communications systems. This constitutes a major problem, not only in terms of exploitation of available spectrum resources, but also regarding the energy efficiency in the transmission and processing of each information unit (bit) that has to substantially improve. The Wireless Internet of the Future (WIoF) will have therefore to rely on technologies that can offer a substantial increase in transmission capacity as measured in bits/Hz but do not require increased spectrum bandwidth or energy consumption.

Multiple-antenna or multi-input multi-output (MIMO) wireless communication devices that employ antenna arrays with a very large number of antenna elements which are known as massive MIMO systems have the potential to overcome those challenges and deliver the required data rates, representing a key enabling technology for the WIoF [4]- [6]. Among the devices of massive MIMO networks are user terminals, tablets, and base stations which could be equipped with a number of antenna elements with orders of magnitude higher than current devices. Massive MIMO networks will be structured by the following key elements: antennas, electronic components, network architectures, protocols and signal processing.

The first important ingredient of massive MIMO networks is antenna technology, which allows designers to assemble large antenna arrays with various requirements in terms of spacing of elements and geometries, reducing the number of required radio frequency (RF) chains at the transmit and the receive ends and their implementation costs [7]–[9]. In certain scenarios and deployments, the use of compact antennas with closely-spaced elements will be of great importance to equip devices with a large number of antennas but this will require techniques to mitigate the coupling effects especially at the user terminals [10]. The second key area for innovation is that of electronic components and RF chains, where the use of low-cost amplifiers with output power in the mWatt range will play an important role. Architectures such as the direct-conversion radio (DCR) [11] are very attractive due to their flexibility and ability to operate with several different air interfaces, frequency bands and waveforms. Existing peripherals such as large coaxial cables and power-hungry circuits will have to be replaced with low-energy solutions.

Another key element of massive MIMO networks is the network architecture, which will evolve from homogeneous cellular layouts to heterogeneous architectures that include small cells and the use of coordination between cells [12]. Since massive MIMO technology is likely to be incorporated into cellular and local area networks in the future, the network architecture will necessitate special attention on how to manage the interference created [13] and measurements campaigns will be of fundamental importance [14]- [16]. The coordination of adjacent cells will be necessary due to the current trend towards aggressive reuse factors for capacity reasons, which inevitably leads to increased levels of inter-cell interference and signalling. The need to accommodate multiple users while keeping the interference at an acceptable level will require significant work in scheduling and medium-access protocols.

The last ingredient of massive MIMO networks and the main focus of this article is signal processing. In particular, MIMO signal processing will play a crucial role in dealing with the impairments of the physical medium and in providing cost-effective tools for processing information. Current state-of-the-art in MIMO signal processing requires a computational cost for transmit and receive processing that grows as a cubic or super-cubic function of the number of antennas, which is clearly not scalable with a large number of antenna elements. We advocate the need for simpler solutions for both transmit

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and receive processing tasks, which will require significant research effort in the next years. Novel signal processing strategies will have to be developed to deal with the problems associated with massive MIMO networks like computational complexity and its scalability, pilot contamination effects, RF impairments, coupling effects, delay and calibration issues. Another key point for future massive MIMO technology is the application scenarios, which will become the main object of investigation in the coming years. Amongst the most important scenarios are multi-beam satellite networks, cellular systems beyond LTE-A [2] and local area networks.

This article is structured as follows. Section II reviews the system model including both uplink and downlink and discusses the application scenarios. Section III is dedicated to transmit processing techniques, whereas Section IV concentrated on receive processing. Section V discusses the results of some simulations and Section VI presents some open problems and suggestions for further work. The conclusions of this article are given in Section VII.

II. Application Scenarios and Signal Models

In this section, we discuss several application scenarios for multiuser massive MIMO systems which include multibeam satellite systems, cellular and local area networks. Signal models based on elementary linear algebra are then presented to describe the information processing in both uplink and downlink transmissions. These models are based on the assumption of a narrowband signal transmission over flat fading channels which can be easily generalized to broadband signal transmission with the use of multi-carrier systems.

A. Application Scenarios

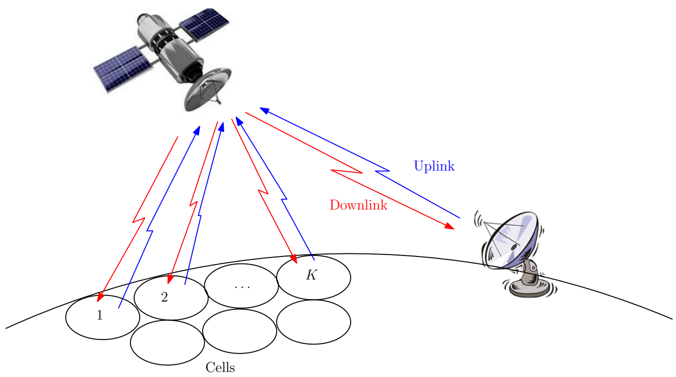

Fig. 1. Multi-beam satellite network.

Amongst the most promising application scenarios of multiuser massive MIMO techniques are multibeam satellite [17], cellular and local area networks. Multibeam satellite systems are perhaps the most natural scenario for massive MIMO because the number of antenna elements is above one hundred. The major benefit of satellite communications is that all users can be served within the coverage region at the same cost. In this context, the next generation of broadband satellite networks will employ multibeam techniques in which the coverage region is served by multiple spot beams intended for the users that are shaped by the antenna feeds forming part of the payload [17], as depicted in Fig 1. A fundamental problem with the multibeam approach is the interference caused by multiple adjacent spot beams that share the same frequency band. This interference between spot beams must be mitigated by suitable signal processing algorithms. Specifically, multiuser interference mitigation schemes such as precoding or multiuser detection can be jointly designed with the beamforming process at the gateway station. The interference mitigation must be applied to all the radiating signals instead of the user beams directly. In the downlink (also known as the forward link in the satellite communications literature), the interference mitigation problem corresponds to designing transmit processing or precoding strategies that require the channel state information (CSI). For the uplink (also known as the reverse link), the interference mitigation problem can be addressed by the design of multiuser detectors.

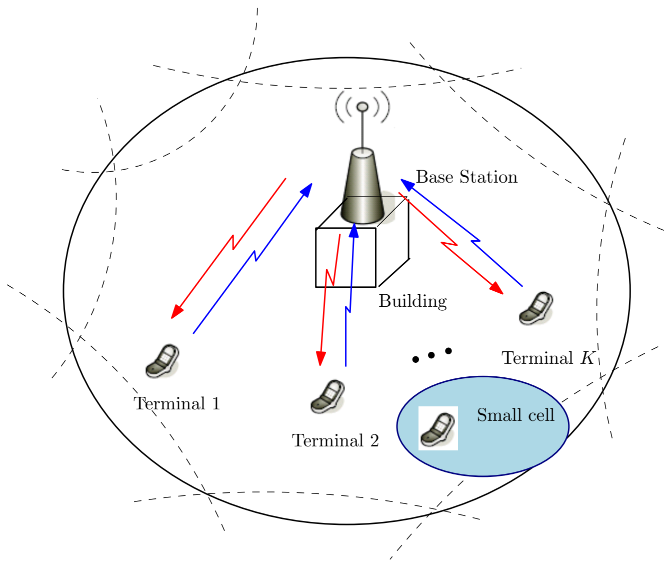

Fig. 2. Mobile cellular network.

The second highly-relevant scenario is that of mobile cellular networks beyond LTE-A [2], which is illustrated in Fig. 2. In such networks, massive MIMO would play a key role with the deployment of hundreds of antenna elements at the base station, coordination between cells and a more modest number of antenna elements at the user terminals. At the base station, very large antenna arrays could be deployed on the roof or on the façade of buildings. With further development in the area of compact antennas and techniques to mitigate mutual coupling effects, it is likely that the number of antenna elements at the user terminals (mobile phones, tablets and other gadgets) might also be significantly increased from $1-4$ elements in current terminals to $10-20$ in future devices. In these networks, it is preferable to employ time-division-duplexing (TDD) mode to perform uplink channel estimation and obtain downlink CSI by reciprocity for signal processing at the transmit side. This operation mode will require cost-effective calibration algorithms. Another critical requirement is the uplink channel estimation, which employs non-orthogonal pilots and due to the existence of adjacent cells and the coherence time of the channel needs to reuse the pilots [18]. Pilot contamination occurs when CSI at the base station in one cell is affected by users from other cells. In particular, the uplink (or multiple-access channel) will need CSI obtained by uplink channel estimation, efficient multiuser detection and decoding algorithms. The downlink (also known as the broadcast channel) will require CSI obtained by reciprocity for transmit processing and the development of cost-effective scheduling and precoding algorithms.

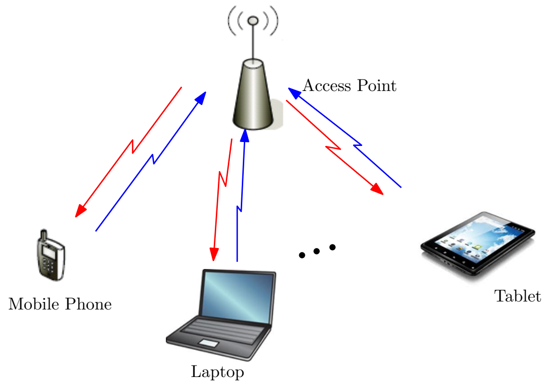

Fig. 3. Wireless local area network.

The third and last highly-relevant scenario is represented by wireless local area networks (WLANs) [3], which are shown in Fig. 3. The deployment of WLANs has increased tremendously in the last few years with the proliferation of hot spots and home users. These systems have adopted orthogonal frequency-division multiplexing (OFDM) for their air interface and are equipped with a number of antennas of up to 8 at the access point and up to 4 antennas at the user terminals [3]. Massive MIMO could play an important role in the incorporation of a substantial number of antenna elements at the access point using compact antennas and planar array geometries to keep the size of the access point at reasonable physical dimensions. The user terminals (laptops, tablets and smart phones) could also rely on compact antennas to accommodate a substantial number of radiating elements. In the future, it is possible that the number of antenna elements at the user terminals will be significantly increased from 8 to over 100 elements at the access points terminals and from 4 to over 40 in future devices.

A key challenge in all the three scenarios is how to deal with a very large number of antenna elements and develop cost-effective algorithms, resulting in excellent performance in terms of the metrics of interest, namely, bit error rate (BER), sum-rate and throughput. In what follows, signal models that can describe the processing and transmission will be detailed.

B. Donwlink Model

In our description, we consider a multiuser massive MIMO system with a number of antenna elements equal to N_A at the transmitter, which could be a satellite gateway, a base station of a cellular network or an access point of a WLAN. The transmitter communicates with K users in the system, where each user is equipped with N_U antenna elements and $N_A > KN_U$. It should be noted that in massive MIMO systems, it is desirable to have an excess of degrees of freedom [4], which means N_A should exceed KN_U by a significant margin in order to leverage the array gain. At each time instant $[i]$, the transmitter applies a precoder to the KN_U data vector $\boldsymbol{s}[i]$ intended for the K users. The KN_U data vector $\boldsymbol{s}[i]$ consists of the stacking of the $N_U \times 1$ vectors $\boldsymbol{s}_k[i] = \big[s_{k,1}[i],\ s_{k,2}[i],\ \ldots,\ s_{k,N_U}[i]\big]^T$ of the K users, where each entry is a data symbol taken from a modulation constellation $A = \{a_1,\ a_2,\ \ldots,\ a_N\}$ with zero mean and variance σ_s^2, where $(\cdot)^T$ denotes transpose. The $N_A \times 1$ precoded data vector for user k is given by $\boldsymbol{x}_k[i] = \mathcal{P}(\boldsymbol{s}_k[i])$, where $\mathcal{P}(\cdot)$ is the mathematical mapping applied by the precoder, and is then transmitted over flat fading channels.

The received signal at each user after demodulation, matched filtering and sampling is collected in an $N_U \times 1$ vector $\boldsymbol{r}_k[i] = \big[r_{k,1}[i],\ r_{k,2}[i],\ \ldots,\ r_{k,N_U}[i]\big]^T$ with sufficient statistics for processing and given by

$$\boldsymbol{r}_k[i] = \sum_{k=1}^{K} \boldsymbol{H}_k \boldsymbol{x}_k[i] + \boldsymbol{n}_k[i], \qquad (1)$$

where the $N_U \times 1$ vector $\boldsymbol{n}_k[i]$ is a zero mean complex circular symmetric Gaussian noise with covariance matrix $E\big[\boldsymbol{n}_k[i]\boldsymbol{n}_k^H[i]\big] = \sigma_n^2 \boldsymbol{I}$, where $E[\cdot]$ stands for expected value, $(\cdot)^H$ denotes the Hermitian operator, σ_n^2 is the noise variance and $\boldsymbol{I}$ is the identity matrix. The $N_A \times 1$ precoded data vectors $\boldsymbol{x}_k[i]$ have covariance matrices $E\big[\boldsymbol{x}_k[i]\boldsymbol{x}_k^H[i]\big] = \sigma_{x_k}^2 \boldsymbol{I}$, where $\sigma_{x_k}^2$ is the signal power. The elements h_{n_U,n_A} of the $N_U \times N_A$ channel matrices $\boldsymbol{H}_k$ are the complex channel gains from the n_Ath transmit antenna to the n_Uth receive antenna.

C. Uplink Model

Let us now consider the uplink of a multiuser massive MIMO system with K users that are equipped with N_U antenna elements and communicate with a receiver with N_A antenna elements, where $N_A > KN_U$. At each time instant, the K users transmit N_U symbols which are organized into a $N_U \times 1$ vector $\boldsymbol{s}_k[i] = \big[s_{k,1}[i],\ s_{k,2}[i],\ \ldots,\ s_{k,N_U}[i]\big]^T$ taken from a modulation constellation $A = \{a_1,\ a_2,\ \ldots,\ a_N\}$. The data vectors $\boldsymbol{s}_k[i]$ are then transmitted over flat fading channels. The received signal after demodulation, matched filtering and sampling is collected in an $N_A \times 1$ vector $\boldsymbol{r}[i] = \big[r_1[i],\ r_2[i],\ \ldots,\ r_{N_R}[i]\big]^T$ with sufficient statistics for processing as described by

$$\boldsymbol{r}[i] = \sum_{k=1}^{K} \boldsymbol{H}_k \boldsymbol{s}_k[i] + \boldsymbol{n}[i], \qquad (2)$$

where the $N_A \times 1$ vector $\boldsymbol{n}[i]$ is a zero mean complex circular symmetric Gaussian noise with covariance matrix $E\big[\boldsymbol{n}[i]\boldsymbol{n}^H[i]\big] = \sigma_n^2 \boldsymbol{I}$. The data vectors $\boldsymbol{s}_k[i]$ have zero mean and covariance matrices $E\big[\boldsymbol{s}_k[i]\boldsymbol{s}_k^H[i]\big] = \sigma_{s_k}^2 \boldsymbol{I}$, where $\sigma_{s_k}^2$ is the signal power. The elements h_{n_A,n_U} of the $N_A \times N_U$ channel matrices $\boldsymbol{H}_k$ are the complex channel gains from the n_Uth transmit antenna to the n_Ath receive antenna.

III. Transmit Processing

In this section, we discuss several aspects related to transmit processing in massive MIMO systems. Fundamental results in information theory have shown that the optimum transmit strategy for the multiuser massive MIMO downlink channel involves a theoretical dirty paper coding (DPC) technique that performs interference cancellation combined with an implicit user scheduling and power loading algorithm [37]. However, this optimal approach is extremely costly and unlikely to be used in any practical deployment. In what follows, we consider several aspects of transmit processing in massive MIMO systems which include TDD operation, pilot contamination, resource allocation and precoding, and related signal processing tasks.

A. TDD operation

One of the key problems in modern wireless systems is the acquisition of CSI in a timely way. In time-varying channels, TDD offers the most suitable alternative to obtain CSI because the training requirements in a TDD system is independent of the number of antennas at the base station (or access point) [18] and there is no need for CSI feedback. In particular, TDD systems rely on reciprocity by which the uplink channel is used as an estimate of the downlink channel. An issue in this operation mode is the difference in the transfer characteristics of the amplifiers and the filters in the two directions. This can be addressed through measurements and appropriate calibration [5]. In contrast, in a frequency division duplexing (FDD) system the training requirements is proportional to the number of antennas and CSI feedback is essential. For this reason, massive MIMO systems will most likely operate in TDD mode and will require further investigation in calibration methods.

B. Pilot contamination

The adoption of TDD mode and uplink training in massive MIMO systems with multiple cells results in a phenomenon called pilot contamination. In multi-cell scenarios, it is difficult to employ orthogonal pilot sequences because the duration of the pilot sequences depends on the number of cells and this duration is severely limited by the channel coherence time due to mobility. Therefore, non-orthogonal pilot sequences must be employed and this affects the CSI employed at the transmitter. Specifically, the channel estimate is contaminated by a linear combination of channels of other users that share the same pilot [18]. Consequently, the precoders and resource allocation algorithms will be highly affected by the contaminated CSI. Strategies to control or mitigate pilot contamination and its effects are very important for massive MIMO networks. Possible approaches include work on optimization of waveforms, blind channel estimation techniques, implicit training approaches and precoding and resource allocation techniques that take into account pilot contamination to mitigate its effects.

C. Resource allocation

Prior work on multiuser MIMO [32]–[34] has shown that resource allocation techniques are fundamental to obtain further capacity gains. In massive MIMO this will be equally important and will have the extra benefit of more accurate CSI. From a multiuser information theoretic perspective, the capacity region boundary is achieved by serving all K active users simultaneously. The resources (antennas, users and power) that should be allocated to each user depend on the instantaneous CSI which may vary amongst users. Since the total number of users Q that could be served is often much higher than the number of transmit antennas N_A, the system needs a resource allocation algorithm to select the best set of users according to a chosen criterion such as the sum rate or a user target rate. The resource allocation task is then to choose a set of users and their respective powers in order to satisfy a given performance metric. In massive MIMO systems, the spatial signatures of the users to be scheduled might play a fundamental role thanks to the very large number of antennas and an excess of degrees of freedom [4], [5]. The multiuser diversity [32] along with high array gains might be exploited by resource allocation algorithm along with timely CSI. In particular, the problem of user selection, i.e., scheduling, corresponds to a combinatorial problem equivalent to the combination of K choosing Q. Hence, it is clear that the exhaustive search over all possible combinations is computationally prohibitive when the K in the system is reasonably large, and thus cost-effective user selection algorithms will be required. Strategies based on greedy, low-cost and discrete optimization methods [33], [34], [36] are very promising for massive MIMO networks because they could reduce the cost of resource allocation algorithms.

D. Precoding and Related Techniques

Strategies for mitigating the multiuser interference at the transmit side include transmit beamforming [5] and precoding based on linear minimum mean square error (MMSE) [38] or zero-forcing (ZF) [39] techniques and nonlinear approaches such as DPC, Tomlinson-Harashima precoding (THP) [58] and vector perturbation [43]. Transmit matched filtering (TMF) is the simplest method for processing data at the transmit side and has been recently advocated by several works for massive MIMO systems [4], [5]. The basic idea is to apply the conjugate of the channel matrix to the data symbol vector $\boldsymbol{s}[i]$ prior to transmission as described by

$$\boldsymbol{x}[i] = \boldsymbol{H}^H \boldsymbol{s}[i], \tag{3}$$

where the $N_A \times KN_U$ matrix $\boldsymbol{H}$ contains the parameters of all the channels and the $N_A \times 1$ vector $\boldsymbol{x}[i]$ represents the data processed by TMF.

Linear precoding techniques such as ZF and MMSE precoding are based on channel inversion operations and are attractive due to their relative simplicity for MIMO systems with a small to moderate number of antennas. However, channel inversion based precoding requires a higher average transmit power than other precoding algorithms especially for ill conditioned channel matrices, which could result in poor performance. A linear precoder applies a linear transformations to the data symbol vector $\boldsymbol{s}[i]$ prior to transmission as described by

$$\boldsymbol{x}[i] = \boldsymbol{W}_k \boldsymbol{s}_k[i] + \sum_{l=1, l\neq k}^{K} \boldsymbol{W}_l \boldsymbol{s}_l[i], \tag{4}$$

where the $N_A \times N_U$ matrix $\boldsymbol{W}_l$ contains the parameters of the channels and the $N_U \times 1$ data symbol vectors $\boldsymbol{s}_k[i]$ represent the data processed by the linear precoder. The linear MMSE precoder is described by $\boldsymbol{W}_{\mathrm{MMSE}} = \boldsymbol{H}^H(\boldsymbol{H}\boldsymbol{H}^H + \gamma \boldsymbol{I})^{-1}$, where γ is a gain factor, and the linear ZF precoder is expressed by $\boldsymbol{W}_{\mathrm{ZF}} = \boldsymbol{H}^H(\boldsymbol{H}\boldsymbol{H}^H)^{-1}$.

Block diagonalization (BD) type precoding algorithms have been proposed in [39]–[41] for MU-MIMO systems. The main advantage of BD type algorithms is the sum-rate performance that is not far from that obtained by DPC techniques and the relative simplicity for implementation in systems with a modest number of antennas. However, existing BD solutions are unlikely to be used in massive MIMO systems due to the cost associated with their implementation in antenna arrays with hundreds of elements. This suggests that there is need for cost-effective BD type strategies for very large antenna arrays. THP [58] is a non-linear precoding technique that employs feedforward and feedback matrices along with a modulo operation to cancel the multiuser interference in a more effective way than a standard linear precoder. With THP, the $N_A \times 1$ precoded data vector is given by

$$\boldsymbol{x}[i] = \boldsymbol{F}\tilde{\boldsymbol{x}}[i], \tag{5}$$

where $\boldsymbol{F}$ is the $N_A \times KN_U$ feedforward precoding matrix which can be obtained by an LQ decomposition of the channel matrix $\boldsymbol{H}$ and the input data $\tilde{\boldsymbol{x}}[i]$ is computed element-by-element by

$$\tilde{x}_l[i] = \mathrm{mod}\big\{s_l[i] - \sum_{q=1}^{l-1} b_{lq} x_q[i]\big\}, \quad l = 1, \ldots, KN_U, \tag{6}$$

where b_{lq} are the elements of the $KN_U \times KN_U$ lower triangular matrix $\boldsymbol{B}$ that can also be obtained by an LQ decomposition. Amongst the appealing features of THP are its excellent BER and sum-rate performances which are not far from DPC and its flexibility to incorporate channel coding. Future work on THP for massive MIMO networks should concentrate on the reduction of the computational cost to compute the feedforward and feedback matrices since existing factorization algorithms would be too costly for systems with hundreds of antenna elements.

Vector perturbation employs a modulo operation at the transmitter to perturb the transmitted signal vector and to avoid the transmit power enhancement incurred by ZF or MMSE methods [43]. The task of finding the optimal perturbation involves solving a minimum distance type problem that can be implemented using sphere encoding or full search-based algorithms. Let $\boldsymbol{H}$ denote a $N_A \times KN_U$ multiuser composite channel. The idea of perturbation is to find a perturbing vector $\boldsymbol{p}$ from an extended constellation to minimize the transmit power. The perturbation $\boldsymbol{p}$ is obtained by solving

$$\boldsymbol{p}[i] = \arg \min_{\boldsymbol{p}'[i] \in ACZ^K} ||\boldsymbol{W}(\boldsymbol{s}[i] + \boldsymbol{p}'[i]||^2 \tag{7}$$

where $\boldsymbol{W}$ is some linear transformation or precoder such that $Tr(\boldsymbol{W}^H\boldsymbol{W}) \leq P$, the scalar A is chosen depending on the constellation size (e.g., $A = 2$ for QPSK), and CZ^K is the K-dimensional complex lattice. The transmit matched filter, linear ZF or MMSE precoders can be used for $\boldsymbol{W}$. After pre-distortion using a linear precoder, the resulting constellation region also becomes distorted and thus a modulo operation is employed. This problem can be regarded as K-dimensional integer-lattice least squares problem, which can be solved by search based algorithms [43].

IV. Receive Processing

In this section, we discuss receive processing in massive MIMO systems. In particular, we examine parameter estimation and detection algorithms, iterative detection and decoding techniques, mitigation of RF impairments and related signal processing tasks.

A. Parameter Estimation and Detection Algorithms

Amongst the key problems in the uplink of multiuser massive MIMO systems are the estimation of parameters such as channels gains and receive filter coefficients, and the detection of the transmitted symbols $\boldsymbol{s}_k$ of each user as described by the signal model in (2). The parameter estimation task usually relies on pilot (or training) sequences and signal processing algorithms. In multiuser massive MIMO networks, non-orthogonal training sequences are likely to be used in most application scenarios and the estimation algorithms must be able to provide the most accurate estimates and to track the variations due to mobility. Standard MIMO linear MMSE and least-squares (LS) channel estimation algorithms [44] can be used for obtaining CSI. However, the cost associated with these algorithms is often cubic in the number of antenna elements at the receiver, i.e., N_A in the uplink. Moreover, in scenarios with mobility the receiver will need to employ adaptive algorithms [73] which can track the channel variations. Interestingly, massive MIMO systems have an excess of degrees of freedom that translates into a reduced-rank structure to perform parameter estimation. This is an excellent opportunity that massive MIMO offers to apply reduced-rank algorithms [28]- [31] and further develop these techniques.

In order to separate the data streams transmitted by the different users in a multiuser massive MIMO network, a designer must resort to detection techniques, which are similar to multiuser detection methods [45]. The optimal maximum likelihood (ML) detector is described by

$$\hat{\boldsymbol{s}}_{\mathrm{ML}}[i] = \arg \min_{\boldsymbol{s}[i]} ||\boldsymbol{r}[i] - \boldsymbol{H}\boldsymbol{s}[i]||^2 \tag{8}$$

where the $KN_U \times 1$ data vector $\boldsymbol{s}[i]$ contains the symbols of all users. The ML detector has a cost that is exponential in the number of data streams and the modulation order that is too complex to be implemented in systems with a large number of antennas. Even though the ML solution can be alternatively computed using sphere decoder (SD) algorithms [46]- [50] that are very efficient for MIMO systems with a small number of antennas, the cost of SD algorithms depends on the noise variance, the number of data streams to be detected and the signal constellation, resulting in high computational costs for low signal-to-noise ratios (SNR), high-order constellations and a large number of data streams.

The high computational complexity of the ML detector and the SD algorithms in the scenarios described above have motivated the development of numerous alternative strategies for MIMO detection, which often rely on signal processing with receive filters. The key advantage of these approaches with receive filters is that the cost is typically not dependent on the modulation and the receiver can compute the receive filter only once per data packet and perform detection. Algorithms that can compute the parameters of receive filters with low cost are of central importance to massive MIMO systems. In what follows, we will briefly review some relevant suboptimal detectors, which include linear and decision-driven strategies.

Linear detectors [51] include approaches based on the receive matched filter (RMF), ZF and MMSE designs and are described by

$$\hat{\boldsymbol{s}}[i] = Q\big(\boldsymbol{W}^H\boldsymbol{r}[i]\big), \tag{9}$$

where the receive filters are $\boldsymbol{W}_{\mathrm{RMF}} = \boldsymbol{H}$ for the RMF,

$\boldsymbol{W}_{\mathrm{MMSE}} = (\boldsymbol{H}\boldsymbol{H}^H + \sigma_s^2/\sigma_n^2\boldsymbol{I})^{-1}\boldsymbol{H}$ for the MMSE and $\boldsymbol{W}_{\mathrm{ZF}} = (\boldsymbol{H}\boldsymbol{H}^H)^{-1}\boldsymbol{H}$ for the ZF design, and $Q(\cdot)$ represents the slicer used for detection.

Decision-driven detection algorithms such as successive interference cancellation (SIC) approaches used in the Vertical-Bell Laboratories Layered Space-Time (VBLAST) systems [52]- [56] and decision feedback (DF) [57] detectors are techniques that can offer attractive trade-offs between performance and complexity. Prior work on SIC and DF schemes has been reported with DF detectors with SIC (S-DF) [57], [63] and DF receivers with parallel interference cancellation (PIC) (P-DF) [66], [67], combinations of these schemes [24], [66], [70] and mechanisms to mitigate error propagation [71], [72]. DF detectors [57], [63], [66] employ feedforward and feedback matrices that can be based on the receive matched filter (RMF), ZF and MMSE designs as described by

$$\hat{\boldsymbol{s}} = Q\big(\boldsymbol{W}^H\boldsymbol{r}[i] - \boldsymbol{F}^H\hat{\boldsymbol{s}}_o[i]\big), \tag{10}$$

where $\hat{\boldsymbol{s}}_o$ corresponds to the initial decision vector that is usually performed by the linear section of the DF receiver (e.g., $\hat{\boldsymbol{s}}_o = Q(\boldsymbol{W}^H\boldsymbol{r})$) prior to the application of the feedback section. The receive filters $\boldsymbol{W}$ and $\boldsymbol{F}$ can be computed using design criteria and optimization algorithms.

An often criticized aspect of these sub-optimal schemes is that they typically do not achieve the full receive-diversity order of the ML algorithm. This led to the investigation of detection strategies such as lattice-reduction (LR) schemes [58]- [59], QR decomposition, M-algorithm (QRD-M) detectors [60], probabilistic data association (PDA) [61], [62] and multi-branch [24], [26] detectors, which can approach the ML performance at an acceptable cost for small to moderate systems. The development of cost-effective detection algorithms for massive MIMO systems is a formidable task that calls for new approaches and ideas in this exciting area.

B. Iterative Detection and Decoding Techniques

Iterative detection and decoding (IDD) schemes have received considerable attention in the last years following the discovery of Turbo codes [74] and the use of the Turbo principle for mitigation of several sources of interference [75]-[83]. More recently, work on IDD schemes has been extended to low-density parity-check codes (LDPC) [79], [81] and their variants which rival Turbo codes in terms of performance. The basic idea of an IDD system is to combine an efficient soft-input soft-output (SISO) detection algorithm and a SISO decoding technique. In particular, the detector produces log-likelihood ratios (LLRs) associated with the encoded bits and these LLRs serve as input to the decoder. Then, in the second phase of the detection/decoding iteration, the decoder generates a posteriori probabilities (APPs) after a number of (inner) decoding iterations for encoded bits of each data stream. These APPs are fed to the detector to help in the next iterations between the detector and the decoder, which are called outer iterations. The joint process of detection/decoding is then repeated in an iterative manner until the maximum number of (inner and outer) iterations is reached. In massive MIMO systems, it is likely that either Turbo or LDPC codes will be adopted in IDD schemes for mitigation of multiuser, multipath, intercell and other sources of interference. LDPC codes exhibit some advantages over Turbo codes that include simpler decoding and implementation issues. However, LDPC codes often require a higher number of decoding iterations which translate into delays or increased complexity. The development of IDD schemes and decoding algorithms that perform message passing with reduced delays [84]- [86] are of paramount importance in future wireless systems.

C. Mitigation of RF Impairments

The large antenna arrays used in massive MIMO systems will pose several issues to system designers such as coupling effects, in-phase/quadrature (I/Q) imbalances [87], and failures of antenna elements, which will need to be addressed. The first potential major impairment in massive MIMO systems is due to reduced spacing between antenna elements which result in coupling effects. In fact, for compact antenna arrays a reduction of the physical size of the array inevitably leads to reduced spacing between antenna elements, which can severely reduce the multiplexing gain. In order to address these coupling effects, receive processing approaches will have to work with transmit processing techniques to undo the coupling induced by the relatively close spacing of radiating elements in the array. Another major impairment in massive MIMO systems is I/Q imbalances in the RF chains of the large arrays. This problem can be addressed by receive or transmit processing techniques and require modelling of the impairments for subsequent mitigation. When working with large antenna arrays, a problem that might also occur is the failure of some antenna elements. Such sensor failures are responsible for a reduction in the degrees of freedom of the array and must be dealt by signal processing algorithms.

V. Simulation Results

In this section, we illustrate some of the techniques outlined in this article using massive MIMO configurations, namely, a very large antenna array, an excess of degrees of freedom provided by the array and a large number of users with multiple antennas. We consider QPSK modulation and channels that are fixed during a data packet and that are modeled by complex Gaussian random variables with zero mean and variance equal to unity. The signal-to-noise ratio (SNR) in dB is defined as $\mathrm{SNR} = 10\log_{10}\frac{N_T\sigma_s^2}{\sigma^2}$, where σ_s^2 is the variance of the symbols, σ_n^2 is the noise variance, and we consider data packets of 1000 QPSK symbols.

In the first example, we compare the BER performance against the SNR of several detection algorithms, namely, the RMF with $K = 8$ users and with a single user, the linear MMSE detector [51] and the DF MMSE detector using a successive interference cancellation [24], [57], [66]. In particular, a scenario with $N_A = 128$ antenna elements at the receiver, $K = 8$ users and $N_U = 8$ antenna elements at the user devices is considered, which corresponds to an excess of degrees of freedom equal to $N_A - KN_U = 64$. The results shown in Fig. 4 indicate that the RMF with a single user has the best performance, followed by the DF MMSE, the linear MMSE and the RMF detectors. Unlike previous works [5] that advocate the use of the RMF, it is clear that the BER performance loss experienced by the RMF should be avoided and more advanced receivers should be considered. However, the cost of linear and DF receivers is dictated by the matrix

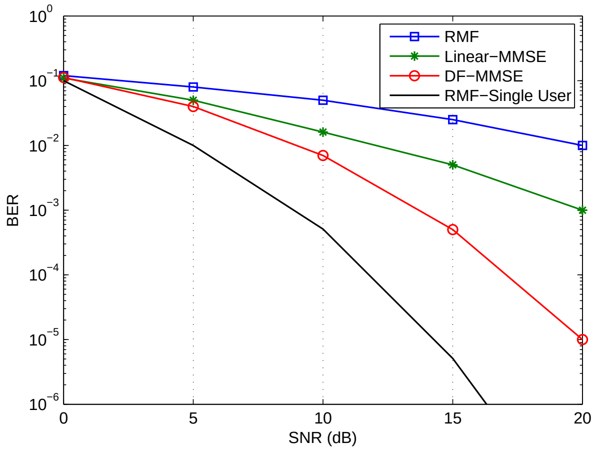

Fig. 4. BER performance against SNR of detection algorithms in a scenario with $N_A = 128$, $K = 8$ users and $N_U = 8$ antenna elements.

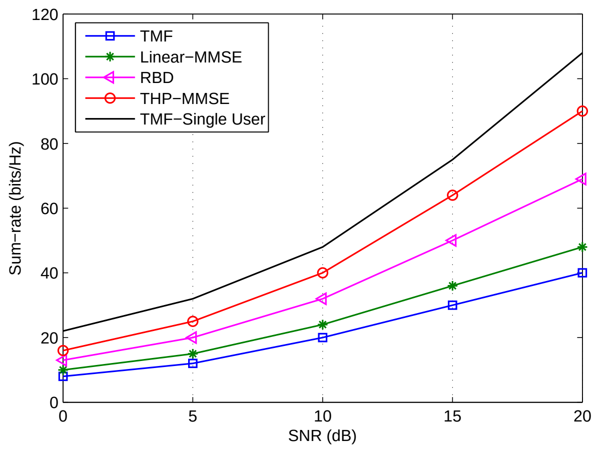

Fig. 5. Sum-rate performance against SNR of precoding algorithms in a scenario with $N_A = 128$, $K = 8$ users and $N_U = 8$ antenna elements.

inversion of $N_A \times N_A$ matrices which must be reduced for large systems.

In the second example, we compare the sum-rate performance against the SNR of several precoding algorithms, namely, the TMF with a varying number of users and with a single user, the linear MMSE precoder and the THP MMSE precoder. The sum-rate is calculated using [90]:

$$C = \log(\det(\boldsymbol{I} + \sigma_n^{-2} \boldsymbol{H}\boldsymbol{P}\boldsymbol{P}^H \boldsymbol{H}^H))(\text{bits/Hz}). \quad (11)$$

We consider a similar scenario to the previous one in which the transmitter is equipped with $N_A = 128$ antenna elements, and there are $K = 8$ users with $N_U = 8$ antenna elements. The results in Fig. 5 show that the TMF with a single user has the best sum-rate performance, followed by the THP MMSE, the regularized BD (RBD), the linear MMSE and the TMF precoding algorithms. From the curves in Fig. 5, we can notice that the performance of TMF is much worse than that of THP and of RBD . This suggests that more sophisticated precoding techniques with lower complexity should be developed to maximize the capacity of massive MIMO systems.

VI. Future Trends and Emerging Topics

In this section, we discuss some future signal processing trends in the area of massive MIMO systems and point out some emerging topics that might attract the interest of researchers. The topics are structured as:

- Transmit processing:

  $\rightarrow$ Cost-effective scheduling algorithms: The development of methods that have low cost and are scalable such as greedy algorithms [33] and discrete optimization techniques [36] will play a crucial role in massive MIMO networks.

  $\rightarrow$ Calibration procedures: The transfer characteristics of the filters and amplifiers used for TDD operation will require designers to devise algorithms that can efficiently calibrate the links.

  $\rightarrow$ Precoders with scability in terms of complexity: The use of divide-and-conquer approaches, methods based on sensor array signal processing and sectorization will play an important role to reduce the dimensionality of the transmit processing problem. Moreover, the investigation and development of TMF strategies with non-linear cancellation strategies and low-cost decompositions for linear and non-linear precoders will be important to obtain efficient transmit methods.

- Receive processing:

  $\rightarrow$ Cost-effective detection algorithms: Techiques to perform dimensionality reduction [28]- [31] for detection problems will play an important role in massive MIMO devices. By reducing the number of effective processing elements, detection algorithms could be applied. In addition, the development of schemes based on RMF with non-linear interference cancellation capabilities might be a promising option that can close the gap between RMF and more costly detectors.

  $\rightarrow$ Decoding strategies with low delay: The development of decoding strategies with reduced delay will play a key role in applications such as audio and video streaming because of their delay sensitivity. Therefore, we argue that novel message passing algorithms with smarter strategies to exchange information should be investigated along with their application to IDD schemes.

  $\rightarrow$ Mitigation of impairments: The identification of impairments originated in the RF chains of massive MIMO systems will need mitigation by smart signal processing algorithms. For example, I/Q imbalance might be dealt with using widely-linear signal processing algorithms [88] and [89].

VII. Concluding Remarks

This article has presented a tutorial on massive MIMO systems and discussed signal processing challenges and future trends in this exciting reseach topic. Key application scenarios which include multibeam satellite, cellular and local area networks have been examined along with several operational requirements of massive MIMO networks. Transmit and receive processing tasks have been discussed and fundamental signal processing needs for future massive MIMO networks have been identified. Numerical results have illustrated some of the discussions on transmit and receive processing functions and future trends have been highlighted. Massive MIMO technology is likely to be incorporated into the applications detailed in this article on a gradual basis by the increase in the number of antenna elements and by the need for more sophistical signal processing tools to transmit and process a large amount of information.

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