**Stochastic Processes**

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List 1

1. Consider the sample space given by $Ω=\{1,2,3,4,5,6\}$ and the subsets $A=\left\{1,2\right\}$, $B=\{1,3,5\}$ and $C=\{2,4,6\}$. Determine the following:

a) $A ∪ B$

b) $A ∪ C$

c) $A ∩ B ∩C$

d) $\overbar{A} ∩ B $

e) $(A-B)∪ C$

2. Show that the operations of union and intersection over sets are associative, that is

$$\left(A ∪ B \right)∪C=A ∪ \left(B ∪C\right)$$

and

$$\left(A ∩ B \right)∩C=A ∩ \left(B ∩C\right)$$

3. Show that the intersection operation is distributive over the union operation, that is

$$A∩\left(B ∪ C \right)=\left(A ∩B\right)∪ \left(A ∩C\right)$$

Verify is the union operation is distributive over the intersection operation. Explain

4. Consider a communications network with 3 devices (a, b, c) and 3 links (ab, bc e ac) as depicted in the figure below.

Each elemento of the network can be found in 2 possible states: on or off. Now suppose that there is communication between 2 devices when both are on and there is at least one link on. Consider the experiment that consists of observing the states of the links and the devices at each time instant and determine the following:

a) a compact representation of the sample points

b) the number of sample points in the sample space

c) the number of sample points associated with the events given by

 A ={ ω ϵ Ω: a and c can communicate}

 B ={ ω ϵ Ω: a communicates with b and with c}

 C ={ ω ϵ Ω: a and c do not communicate over link ac}

 D = A U B

 E = A ∩ B

 F = A ∩ C

5. Consider the toss of a die with faces numbered from 1 to 6. Determine the sample space Ω, associated with this experiment. Consider the events A ={1} and B={3,4} . These subsets of Ω constitute a class of events that is denoted as C. Determine the σ-Algebra generated by this class C = {A,B}.

6. A telephone company subscriber a in country A can communicate with subscriber b of another telephone company in country B using 2 companies E1 and E2. The probability of congestion in E1 is 0.05 whereas the probability of congestion in E2 is 0.02. Moreover, we know that if E1 is congested then the probability of E2 being congested is 0.15. Determine the probability that subscriber a can communicate with subscriber b.

7. The control panel of a device has 2 indicators, A and B. The device is composed of 2 modules, 1 and 2, that are subject to failures. When there is a device failure, the probability that the failure is from module 2 is 0.7. If there is a failure then one of the indicators will be on. We also know that if the failure comes from module 1 then A is on with probability 0.6 and B is on with probability 0.4. However, if the failure comes from module 2 then A is on with probability 0.3 and and B is on with probability 0.7. Determine the following:

a) The probability of failure of module 1 given that A is on.

b) The probability that B is on given that a failure happened.

8. Two football players throw in alternating stakes a pair of dice of 6 faces each. The first player to obtain 7 points in the two faces observed is the winner. Determine the probability that the player who starts the game being the winner.