Low-Complexity Variable Step-Size Mechanism for Code-Constrained Constant Modulus Stochastic Gradient Algorithms Applied to CDMA Interference Suppression

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***Abstract*—The code-constrained constant modulus algorithm (CCM) implemented with a stochastic gradient (SG) technique is a very effective and efficient blind approach for interference suppression when a communication channel is frequency-selective. In nonstationary wireless environments, users frequently enter and exit the system, making it very difficult for the receiver to compute a predetermined step-size. This suggests the deployment of mechanisms to automatically adjust the step-size in order to ensure good tracking of the interference and the channel. In this paper, the performance of blind CCM adaptive receivers for direct sequence code division multiple access (DS-CDMA) systems that employ stochastic gradient (SG) algorithms with variable step-size mechanisms is investigated. We propose a novel low-complexity variable step-size mechanism for blind CCM CDMA receivers. Convergence and tracking analyses of the proposed adaptation techniques are carried out for multipath channels. Finally, numerical experiments are presented for nonstationary environments, showing that the new mechanism achieves superior performance to previously reported methods at a reduced complexity.**

***Index Terms*—Adaptive receivers, blind multiuser detection, direct sequence code division multiple access (DS-CDMA), interference suppression, variable step-size mechanisms.**

I. Introduction

THE constant modulus algorithm (CMA) is based on a criterion that penalizes deviations of the modulus of the received signal away from a fixed value determined by the source alphabet [1], the original work on the CMA has been done independently by Godard [2] and by Treichler and Agee [3]. The code-constrained constant modulus algorithm (CCM) is based on the CMA and forced to satisfy one or a set of linear constraints such that signals from the desired user are detected in a flat fading channel [4], [5] or multipath environment [6], [7]. The CCM algorithm has been studied and implemented in many wireless communication applications including blind multiuser detection, blind equalization, source separation, interference suppression, and antenna beamforming. A linear receiver equipped with the CCM algorithm is a very effective blind approach for intersymbol interference (ISI) and multiaccess interference (MAI) suppression when a communication channel is frequency-selective [8], [9]. The CCM design approach has proven to be highly suitable to certain communications technologies such as spread spectrum systems. In particular, DS-CDMA spread spectrum signalling has become a highly popular multiple access technique which is widely used for personal communications, third-generation mobile telephony, and indoor wireless communications. The advantages of DS-CDMA include superior operation in multipath environments, flexibility in the allocation of channels, increased capacity in bursty and fading environments, and the ability to share bandwidth with narrowband communication systems without deterioration of either's systems performance [10], [11].

Detecting a desired user in a DS-CDMA system requires processing the received signal in order to mitigate different types of interference, namely, MAI, ISI, and the noise at the receiver. The major source of interference in most CDMA systems is MAI, which arises due to the fact that users communicate through the same physical channel with nonorthogonal signals. Multiuser detection has been proposed as a means to suppress MAI, increasing the capacity and the performance of CDMA systems [10], [12]. The optimal multiuser detector of Verdu [13] suffers from exponential complexity and requires: the knowledge of timing, amplitude and signature sequences. This fact has motivated the development of various suboptimal strategies with affordable complexity. The linear minimum mean squared error (MMSE) receiver implemented with an adaptive filter is one of the most prominent schemes for use in the downlink because it only requires the timing of the desired user and a training sequence. A blind adaptive linear receiver has been developed in [11], and operates without knowledge of the channel input. In [11] Honig *et al.* have shown that the minimum variance (MV) criterion leads to a solution identical to that obtained from the minimization of the mean squared error (MSE). A disadvantage of the original MV detector is that it suffers from the problem of signature mismatch and thus has to be modified for multipath environments.

When designing an adaptive receiver for a DS-CDMA system, we need to consider what kind of algorithm should be

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used. Despite the fast convergence of recursive least squares (RLS) algorithms, however, it is preferable to implement adaptive receivers with stochastic gradient (SG) algorithms (e.g., LMS) due to complexity and cost issues. For this reason the improvement of blind SG techniques is an important research and development topic. In this regard, the works in [9] and [11] employ standard SG algorithms with fixed step-size (FSS) that are not efficient with respect to convergence and steady-state performance. Indeed, the performance of adaptive SG receivers is strongly dependent on the choice of the step-size in [14]. In nonstationary wireless environments, users frequently enter and exit the system, making it very difficult for the receiver to compute a predetermined step-size. This suggests the deployment of mechanisms to automatically adjust the step-size of an SG algorithm in order to ensure good tracking of the interference and the channel. Previous works have shown significant gains in performance due to the use of averaging methods (AV) [15], [16] or adaptive gradient step-size (AGSS) mechanism [17], [18], where one SG algorithm adapts the parameter vector and another SG recursion adapts the step-size. The works in [15] and [16] have borrowed the idea of averaging originally developed by Polyak [19] and applied it to CDMA receivers with the MV criterion. The AGSS algorithms in [17], [18] can be considered MV and CCM extensions of the papers [20]–[22]. All these methods require an additional number of operations (i.e., additions and multiplications) proportional to the processing gain N and to the number of multipath components L_p.

Furthermore, there is a very little number of works employing variable step-size mechanisms with blind techniques using the constant modulus criterion. In this work, we propose a novel low-complexity variable step-size mechanism for blind CDMA receivers in multipath channels that are used for MAI and ISI suppression based on an SG algorithm and the CCM approach. The additional number of operations of the proposed techniques does not depend on the processing gain N and the number of paths of the channel L_p. Convergence and tracking analyses of the proposed adaptation techniques are carried out for a multipath scenario, and analytical results are derived for the computation of the excess MSE. We also generalize the CCM SG-AGSS in [18] for multipath scenarios. In addition, simulation experiments are presented for nonstationary environments, showing that the new mechanisms are superior to previously reported methods and exhibit a reduced complexity.

The paper is structured as follows. Section II briefly describes the DS-CDMA system model. The adaptive blind SG CCM receiver design and CCM SG-AGSS algorithm extension for multipath channel are described in Section III. Section IV is devoted to the novel variable step-size mechanism. Convergence and tracking analyses of the resulting algorithm are developed in Section V. Section VI presents and discusses the simulation results. Section VII draws the conclusions.

II. DS-CDMA System Model

Let us consider the downlink of an uncoded synchronous binary phase-shift keying (BPSK) DS-CDMA system with K users, N chips per symbol and L_p propagation paths. The signal broadcasted by the base station intended for user k has a baseband representation given by

$$x_k(t) = A_k \sum_{i=-\infty}^{\infty} b_k(i) s_k(t - iT) \tag{1}$$

where $b_k(i) \in \{\pm 1\}$ denotes the ith symbol for user k, the real valued spreading waveform and the amplitude associated with user k are $s_k(t)$ and A_k, respectively. The spreading waveforms are expressed by $s_k(t) = \sum_{i=1}^{N} a_k(i)\phi(t - iT_c)$, where $a_k(i) \in \{\pm 1/\sqrt{N}\}$, $\phi(t)$ is the chip waveform, T_c is the chip duration and $N = T/T_c$ is the processing gain. Assuming that the channel is constant during each symbol and the receiver is synchronized with the main path, the received signal is

$$r(t) = \sum_{k=1}^{K} \sum_{l=0}^{L_p - 1} h_l(t) x_k(t - \tau_l) + n(t) \tag{2}$$

where $h_l(t)$ and τ_l are, respectively, the channel coefficient and the delay associated with the lth path. Assuming that the delays are multiples of the chip rate, the spreading codes are repeated from symbol to symbol and the received signal $r(t)$ after filtering by a chip-pulse matched filter and sampled at chip rate yields the M-dimensional received vector

$$\begin{aligned} \mathbf{r}(i) &= \sum_{k=1}^{K} (A_k b_k(i) \mathbf{C}_k \mathbf{h}(i) + \boldsymbol{\eta}_k(i)) + \mathbf{n}(i) \\ &= \sum_{k=1}^{K} (A_k b_k(i) \mathbf{p}_k(i) + \boldsymbol{\eta}_k(i)) + \mathbf{n}(i) \end{aligned} \tag{3}$$

where $M = N + L_p - 1, \mathbf{n}(i) = [n_1(i) \ldots n_M(i)]^T$ is the complex Gaussian noise vector, and $E\{\mathbf{n}(i)\mathbf{n}^H(i)\} = \sigma^2 \mathbf{I}$, where $(.)^T$ and $(.)^H$ denotes transpose and Hermitian transpose, respectively, and $E\{.\}$ stands for expected value. The channel vector is $\mathbf{h}(i) = [h_0(i) \ldots h_{L_p-1}(i)]^T$ with $h_l(i) = h_l(iT_c)$ for $l = 0, \ldots, L_p - 1$, $\boldsymbol{\eta}_k(i)$ is the ISI, and assumes that the channel order is not greater than N, i.e. $L_p - 1 \leq N$, $\mathbf{s}_k = [a_k(1) \ldots a_k(N)]^T$ is the signature sequence for user k and $\mathbf{p}_k(i) = \mathbf{C}_k \mathbf{h}(i)$ is the effective signature sequence for user k, the $M \times L_p$ convolution matrix $\mathbf{C}_k$ contains one-chip shifted versions of $\mathbf{s}_k$.

$$\mathbf{C}_k = \begin{pmatrix} a_k(1) & 0 & \ldots & 0 \\ \vdots & a_k(1) & \ddots & \vdots \\ a_k(N) & \vdots & \ddots & 0 \\ 0 & a_k(N) & \ddots & a_k(1) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ddots & a_k(N) \end{pmatrix}.$$

III. Blind Adaptive SG CCM Algorithms

The linear receiver design is equivalent to determining an FIR filter $\mathbf{w}_k(i)$ with M coefficients that provide an estimate of the desired symbol, as illustrated in Fig. 1 and given by

$$\hat{b}_k(i) = \text{sgn}\left(\Re\left[\mathbf{w}_k^H(i)\mathbf{r}(i)\right]\right) \tag{4}$$

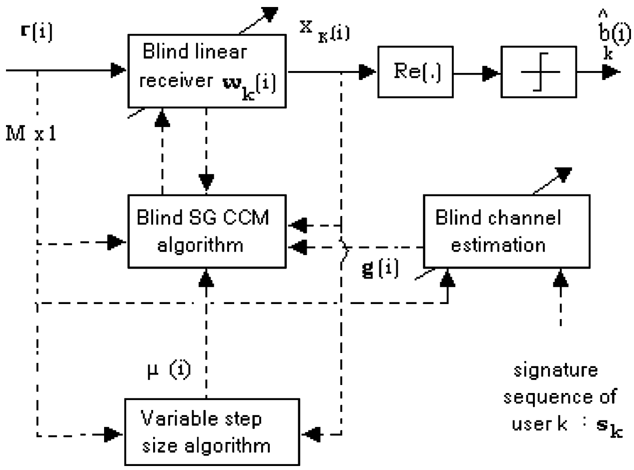

Fig. 1. Block diagram of the blind adaptive CCM receiver with variable step-size mechanisms.

where $\Re(.)$ selects the real part, $\mathrm{sgn}(.)$ is the signum function and the receiver parameter vector $\mathbf{w}_k$ is optimized according to the CM cost function subject to appropriate constraints.

In this section, we describe the multipath blind adaptive SG CCM algorithm for estimating the parameters of the linear receiver first, and then we generalize the blind CCM-AGSS [18] for multipath scenarios.

A. Multipath Blind Adaptive SG CCM Algorithm

First, let us describe the design of the blind adaptive SG CCM algorithm in multipath channel. Consider the cost function, $J = E\{e^2(i)\}$, where $e(i) = |\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1$ subject to the multipath constraint given by $\mathbf{C}_k^H\mathbf{w}_k(i) = \mathbf{g}(i)$, where the matrix $\mathbf{C}_k$ was introduced in Section II, and $\mathbf{g}(i)$ is the $L_p \times 1$ constraint channel vector to be determined. The blind channel estimation in [23] is employed in these algorithms. Thus, in order to derive an adaptive expression for the SG CCM linear receiver let us consider the unconstrained optimization problem given in the form of a Lagrangian cost function:

$$\mathcal{L} = \left(\mathbf{w}_k^H(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_k(i) - 1\right)^2 + \boldsymbol{\lambda}^H\left(\mathbf{C}_k^H\mathbf{w}_k(i) - \mathbf{g}(i)\right) + \left(\mathbf{w}_k^H(i)\mathbf{C}_k - \mathbf{g}^H(i)\right)\boldsymbol{\lambda} \quad (5)$$

where $\boldsymbol{\lambda}$ is a vector of Lagrange multipliers, we consider the following gradient search procedure:

$$\mathbf{w}_k(i+1) = \mathbf{w}_k(i) - \mu\nabla_{\mathbf{w}_k^*}\mathcal{L} \quad (6)$$

where μ is the SG algorithm step-size. The recursion in (6) may be obtained from (5) by taking the gradient with respect to $\mathbf{w}_k^*$ we obtain $\nabla_{\mathbf{w}_k^*}\mathcal{L} = e(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_k(i) + \mathbf{C}_k\boldsymbol{\lambda}$. Then, (6) becomes

$$\mathbf{w}_k(i+1) = \mathbf{w}_k(i) - \mu\left(e(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_k(i) + \mathbf{C}_k\boldsymbol{\lambda}\right) \quad (7)$$

where $\boldsymbol{\lambda}$ also needs to be determined. By using (6) and enforcing the constraints on $\mathbf{w}_k$ as $\mathbf{C}_k^H\mathbf{w}_k(i+1) = \mathbf{g}(i)$, $\boldsymbol{\lambda}$ can be solved

$$\boldsymbol{\lambda} = \frac{1}{\mu}\left(\mathbf{C}_k^H\mathbf{C}_k\right)^{-1} \times\left(\mathbf{C}_k^H\mathbf{w}_k(i) - \mu\mathbf{C}_k^H e(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_k(i) - \mathbf{g}(i)\right). \quad (8)$$

Substituting (8) in (7), we arrive at the update rule for the adaptive filter weight vector $\mathbf{w}_k$

$$\mathbf{w}_k(i+1) = \prod_{\mathbf{C}}\left[\mathbf{w}_k(i) - \mu e(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_k(i)\right] + \mathbf{C}_k\left(\mathbf{C}_k^H\mathbf{C}_k\right)^{-1}\mathbf{g}(i) \quad (9)$$

where $\prod_{\mathbf{C}} = \mathbf{I} - \mathbf{C}_k(\mathbf{C}_k^H\mathbf{C}_k)^{-1}\mathbf{C}_k^H$ as reported in [24], and $\mathbf{g}(i)$ is the blind channel estimation vector which has been proposed in [23].

B. Blind CCM SG-AGSS in Multipath Channel

The CCM SG-AGSS algorithm in single path channel has been proposed by Yuvapoositanon and Chambers in [18]. In this part we extend the algorithm to multipath channels.

We treat $\mathbf{w}_k$ as a function of μ, the step-size variation can change the filter weights, and define $\mathbf{Y}_k(i) = \partial\mathbf{w}_k(i)/\partial\mu$. We consider the gradient search procedures of variable step-size as follows

$$\mu(i+1) = \mu(i) - \alpha\nabla_{\boldsymbol{\mu}}J \quad (10)$$

where α denotes the adaptation rate of the step-size $\mu(i)$ with $\alpha > 0$. By taking the gradient of the cost function $J = E\{e^2(i)\}$ with respect to the step-size μ we have

$$\nabla_{\boldsymbol{\mu}}J = e(i)\left(\mathbf{Y}_k^H(i)\mathbf{r}(i)z_k^*(i) + \mathbf{r}^H(i)\mathbf{Y}_k(i)z_k(i)\right) \quad (11)$$

Based on (10) and (11), we can have another SG update equation which is

$$\mu(i+1) = \left[\mu(i) - \alpha e(i)Re\left(\mathbf{Y}_k^H(i)\mathbf{r}(i)z_k^*(i)\right)\right]_{\mu^-}^{\mu^+} \quad (12)$$

where $z_k(i) = \mathbf{w}_k^H(i)\mathbf{r}(i)$, and $[.]_{\mu^-}^{\mu^+}$ denotes the truncation to the limits of the range $[\mu^-, \mu^+]$, From (9) we can derive the update equation of $\mathbf{Y}_k(i)$

$$\mathbf{Y}_k(i+1) = \prod_{\mathbf{C}}\left\{\mathbf{Y}_k(i) - \mu(i)\mathbf{r}(i)\mathbf{r}^H(i)\left[\mathbf{Y}_k(i)e(i) + \mathbf{w}_k(i)\left(\mathbf{Y}_k^H(i)\mathbf{r}(i)z_k^*(i) + \mathbf{r}^H(i)\mathbf{Y}_k(i)z_k(i)\right)\right]\right\}. \quad (13)$$

By combining (9), (12), and (13) we obtain the multipath blind CCM SG-AGSS algorithm.

IV. Blind Time Averaged Variable Step-Size Algorithm

This section describes the proposed low-complexity time averaged variable step-size (TASS) mechanism for CDMA receivers that adjusts the step-size μ of the update equation of the receiver. A convergence analysis of the mechanism is carried out and approximate expressions relating the mean convergence factor $E\{\mu(i)\}$, the mean square convergence factor $E\{\mu^2(i)\}$ and the minimum variance are derived. It is worth noting that in the mechanism, $\mu(i)$ is truncated between $\{\mu_{max}, \mu_{\min}\}$. In addition, the computational complexity of the novel mechanism is presented in terms of additions and multiplications and compared to the CCM-AGSS one.

A. TASS Mechanism

The proposed TASS mechanism employs the instantaneous cost function $e^2(i) = \left(|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1\right)^2$, is denoted TASS and uses the update rule

$$\mu(i+1) = a\mu(i) + b\left(\left|\mathbf{w}_k^H(i)\mathbf{r}(i)\right|^2 - 1\right)^2 \quad (14)$$

where $0 < a < 1$, $b > 0$ and $\mathbf{w}_k$ is the parameter vector of the receiver. In the proposed TASS algorithm the step-size adjustment is controlled by the instantaneous constant modulus cost function. The motivation is that a large prediction error will cause the step-size to increase and provide faster tracking while a small prediction error will result in a decrease in the step-size to yield smaller misadjustment [25]. The step-size $\mu(i)$ is always positive and is controlled by the size of the prediction error and the parameters a and b. Furthermore, it is worth pointing out that other rules have been experimented and the TASS is a result of several attempts to devise a simple and yet effective mechanism. Indeed, the mechanism is simple to implement and a detailed analysis of the algorithm is possible under a few assumptions.

Assumption 1: Let us consider that for the algorithms in (14) when $i \rightarrow \infty$

$$E\left\{\mu(i)e^2(i)\right\} = E\left\{\mu(i)\right\}E\left\{e^2(i)\right\}$$

This assumption holds if μ is a constant, and we claim that it is approximately true if b is small and also because a should be close to one (as will be shown in the simulations), because $\mu(i)$ will vary slowly around its mean value. By writing

$$E\left\{\mu(i)e^2(i)\right\} = E\left\{\mu(i)\right\}E\left\{e^2(i)\right\} + E\left\{\left(\mu(i) - E\left\{\mu(i)\right\}\right)e^2(i)\right\} \quad (15)$$

we note that for b sufficiently small, the second term on the right-hand side (RHS) of (15) will be small compared to the first one. Assumption 1 helps us to proceed with the analysis.

Let us define the first-($E\{\mu(i)\}$) and second-order ($E\{\mu^2(i)\}$) statistics of the proposed TASS mechanism

$$E\left\{\mu(i+1)\right\} = aE\left\{\mu(i)\right\} + bE\left\{e^2(i)\right\}. \quad (16)$$

By computing the square of $\mu(i+1)$, we obtain $\mu^2(i+1) = a^2\mu^2(i) + 2ab\mu(i)e^2(i) + b^2e^4(i)$. Since b^4 is small, the last term of the previous equation is negligible as compared to the other terms, thus, with the help of Assumption 1 we assume that the expected value of $E\{\mu^2(i+1)\}$ is approximately

$$E\left\{\mu^2(i+1)\right\} \approx a^2E\left\{\mu^2(i)\right\} + 2abE\left\{\mu(i)\right\}E\left\{e^2(i)\right\}. \quad (17)$$

If we consider the steady-state values of $E\{\mu(i+1)\}$ and $E\{\mu^2(i+1)\}$ by making $\lim_{i\rightarrow\infty} E\{\mu(i+1)\} = \lim_{i\rightarrow\infty} E\{\mu(i)\} = E\{\mu(\infty)\}$ and $\lim_{i\rightarrow\infty} E\{\mu^2(i+1)\} = \lim_{i\rightarrow\infty} E\{\mu^2(i)\} = E\{\mu^2(\infty)\}$, and using $\lim_{i\rightarrow\infty} E\{\left(|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1\right)^2\} = \xi_{\min} + \xi_{\text{ex}}(\infty)$ [11], [14] we have the following:

$$E\left\{\mu(\infty)\right\} \approx \frac{b\left(\xi_{\min} + \xi_{\text{ex}}(\infty)\right)}{1-a} \quad (18)$$

$$E\left\{\mu^2(\infty)\right\} \approx \frac{2ab^2\left(\xi_{\min} + \xi_{\text{ex}}(\infty)\right)^2}{(1-a)^2(1+a)} \quad (19)$$

where the steady-state minimum value $\xi_{\min}$ is provided by [26], the blind CCM receiver is assuming convergence to the MMSE receiver. The quantity ξ_{ex} is the steady-state excess error of the CM cost function. To further simplify those expressions, let us consider another assumption.

Assumption 2: Let us consider that for (18) and (19), $(\xi_{\min} + \xi_{\text{ex}}(\infty)) \approx \xi_{\min}$ and $(\xi_{\min} + \xi_{\text{ex}}(\infty))^2 \approx \xi_{\min}^2$, respectively.

This assumption holds if $\xi_{\min} \gg \xi_{\text{ex}}(\infty)$ and we claim that it is approximately true when the SG adaptive algorithm is close to the optimum solution and $\xi_{\text{ex}}(\infty)$ is a small fraction of $\xi_{\min}$.

By using Assumption 2 we have the following:

$$E\left\{\mu(\infty)\right\} \approx \frac{b\xi_{\min}}{1-a} \quad (20)$$

$$E\left\{\mu^2(\infty)\right\} \approx \frac{2ab^2\xi_{\min}^2}{(1-a)^2(1+a)} \quad (21)$$

Note that (20) and (21) will be used for the computational of the excess MSE of the algorithm. Our studies reveal that (20) and (21) have proven to be valid and useful for predicting the steady-state performance of the TASS mechanism.

B. Computational Complexity

In this section, we focus on the additional computational complexity of the proposed TASS mechanism and AGSS mechanism. We compute the number of additions and multiplications to compare the different parts of those two variable step-size mechanisms. In Table I, we show the additional computational complexity of the algorithms for multipath channels. An important advantage of the proposed adaptation rule is that it requires only a few fixed number of operations while the other existing technique has additional complexity proportional to the processing gain N and to the number of propagation paths L_p. Note that we estimated the number of arithmetic operations by taking into account the number of complex additions and multiplications required by the mechanisms.

TABLE I
ADDITIONAL COMPUTATIONAL COMPLEXITY IN MULTIPATH CHANNELS

	Number of operations per symbol	
Mechanism	Additions	Multiplications
AGSS	$2M^2 + 3M$	$3M^2 + 7M + 5$
TASS	1	3

V. ANALYSES OF THE PROPOSED ALGORITHM

In this section, we investigate the convergence behavior and tracking analysis of our mechanism when used in the CCM-based algorithm in terms of the steady-state excess MSE

(EMSE). The CCM blind receivers are inherently nonlinear and time-variant systems. The nonlinearities in the update equations of these receivers usually lead to significant difficulties in the study of their performance. A very efficient approach named energy conservation principle has been proposed by Sayed and Rupp in [27] and [28], and it was extended by Mai and Sayed in [29] and Yousef and Sayed in [30] to the steady-state and tracking analyses of CMA that bypasses many of these difficulties. This approach has been proposed with CCM algorithms for analyzing adaptive multiuser receivers by Whitehead and Takawira in [31]. Our work makes two contributions, the first of which is the derivation of the steady-state and tracking performance of the blind CCM receiver in multipath channels. The second contribution is that we focus on the analysis of the novel variable step-size mechanism and incorporate them in the derived expressions.

A. The Modification of the CCM Update Equation

In order to use the energy conservation principle to do the steady-state and tracking analyses, we write the multipath channel CCM filter weights update equation in another way.

We consider an equivalent Lagrangian cost function

$$\mathcal{L} = \left(\left|\mathbf{w}_k^H(i)\mathbf{r}(i)\right|^2 - 1\right)^2 + 2Re\left[\lambda^*\left(\mathbf{w}_k^H\bar{\mathbf{s}}_k - 1\right)\right] \quad (22)$$

where $\bar{\mathbf{s}}_k$ is the effective signature waveform of user k, $\bar{\mathbf{s}}_k = \mathbf{C}_k\mathbf{g}$. By taking the gradient with respect to $\mathbf{w}_k^*$, we get the new filter weight vector update equation

$$\mathbf{w}_k(i+1) = \mathbf{w}_k(i) - \mu(i)e(i)z_k^*(i)\left(\mathbf{I} - \frac{\bar{\mathbf{s}}_k\bar{\mathbf{s}}_k^H}{||\bar{\mathbf{s}}_k||^2}\right)\mathbf{r}(i) \quad (23)$$

where $e(i) = |\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1$, $z_k(i) = \mathbf{w}_k^H(i)\mathbf{r}(i)$. These two (9) and (23) are equivalent with different forms. We will drop the index k for notation simplicity in what follows.

B. The Range of Step-Size Values for Convergence

Before the convergence analysis of the proposed variable step-size algorithm, we discuss the range of the step-size for convergence. Here, let us consider the blind CCM filter weight update equation:

$$\begin{aligned}\mathbf{w}(i+1) &= \mathbf{w}(i) - \mu(i)e(i)z^*(i)\left(\mathbf{I} - \frac{\bar{\mathbf{s}}\bar{\mathbf{s}}^H}{||\bar{\mathbf{s}}||^2}\right)\mathbf{r}(i)\\ &= \mathbf{w}(i) - \mu(i)e(i)\mathbf{r}^H(i)\mathbf{w}(i)\left(\mathbf{I} - \frac{\bar{\mathbf{s}}\bar{\mathbf{s}}^H}{||\bar{\mathbf{s}}||^2}\right)\mathbf{r}(i)\\ &= \left[\mathbf{I} - \mu(i)e(i)\mathbf{v}(i)\mathbf{r}^H(i)\right]\mathbf{w}(i) \quad (24)\\ \tilde{\mathbf{w}}(i+1) &= \mathbf{w}_{\text{opt}} - \mathbf{w}(i+1)\\ &= \left(\mathbf{I} - \mu(i)e(i)\mathbf{v}(i)\mathbf{r}^H(i)\right)\tilde{\mathbf{w}}(i)\\ &\quad + \mu(i)e(i)\mathbf{v}(i)\mathbf{r}^H(i)\mathbf{w}_{\text{opt}} \quad (25)\end{aligned}$$

where $\mathbf{v}(i) = (\mathbf{I} - (\bar{\mathbf{s}}\bar{\mathbf{s}}^H/||\bar{\mathbf{s}}||^2))\mathbf{r}(i)$. By taking expectations on both sides of (25) and using Assumption 1 we have

$$E\{\tilde{\mathbf{w}}(i+1)\} = (\mathbf{I} - E\{\mu(i)\}\mathbf{R}_{vr}(i))E\{\tilde{\mathbf{w}}(i)\} \quad (26)$$

where $\mathbf{R}_{vr}(i) = E\{e(i)\mathbf{v}(i)\mathbf{r}^H(i)\}$ and $\mathbf{R}_{vr}\mathbf{w}_{\text{opt}} = \mathbf{0}$ [11]. Therefore, it can be concluded that $\mathbf{w}$ converges to $\mathbf{w}_{\text{opt}}$ and (26) is stable if and only if $\prod_{i=0}^{\infty}(\mathbf{I} - E\{\mu(i)\}\mathbf{R}_{vr}) \rightarrow 0$, which is necessary and sufficient condition for $\lim_{i\rightarrow\infty} E\{\tilde{\mathbf{w}}(i)\} = \mathbf{0}$ and $E\{\mathbf{w}(i)\} \rightarrow \mathbf{w}_{\text{opt}}$. For stability, a sufficient condition for (26) to hold implies that

$$0 \le E\{\mu(\infty)\} < \min_k \frac{2}{|\lambda_k^{vr}|}. \quad (27)$$

[14] where λ_k^{vr} is the kth eigenvalue of $\mathbf{R}_{vr}$ that is not real since $\mathbf{R}_{vr}$ is not symmetric.

C. Steady-State Analysis

The EMSE arises and depends on the presence of MAI, ISI, AWGN in multipath channels and the nature of the SG algorithm. It is related to the error in the filter coefficients $\tilde{\mathbf{w}}(i)$ via the *a priori* estimation error, which is defined as

$$e_a(i) \triangleq \tilde{\mathbf{w}}^H(i)\mathbf{r}(i) \quad (28)$$

where $\tilde{\mathbf{w}}(i) = \mathbf{w}_{\text{opt}} - \mathbf{w}(i)$, and $\mathbf{w}_{\text{opt}}$ is the optimum filter in terms of the blind algorithm. Let us define the MSE at time i using the fact that $\tilde{\mathbf{w}}(i)$

$$\begin{aligned}\epsilon(i) &= E\left\{\left|b(i) - \mathbf{w}^H(i)\mathbf{r}(i)\right|^2\right\}\\ &= \epsilon_{\min} + E\left\{|e_a(i)|^2\right\} + \mathbf{s}^H E\{\tilde{\mathbf{w}}(i)\} + E\left\{\tilde{\mathbf{w}}^H(i)\right\}\mathbf{s}\\ &\quad - E\left\{\mathbf{w}_{\text{opt}}^H\mathbf{r}(i)\mathbf{r}^H(i)\tilde{\mathbf{w}}(i)\right\}\\ &\quad - E\left\{\tilde{\mathbf{w}}^H(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_{\text{opt}}\right\} \quad (29)\end{aligned}$$

where

$$\epsilon_{\min} = E\left\{\left|b(i) - \mathbf{w}_{\text{opt}}^H\mathbf{r}(i)\right|^2\right\}. \quad (30)$$

When $i \rightarrow \infty$, since $\mathbf{w}(i) \rightarrow \mathbf{w}_{\text{opt}}$ and $E\{\tilde{\mathbf{w}}(i)\} \rightarrow 0$ we have the steady-state MSE

$$\lim_{i\rightarrow\infty} \epsilon(i) = \epsilon_{\min} + \lim_{i\rightarrow\infty} E\left\{|e_a(i)|^2\right\}. \quad (31)$$

The steady-state EMSE is then defined as [29]

$$\zeta \triangleq \lim_{i\rightarrow\infty} E\left\{|e_a(i)|^2\right\}. \quad (32)$$

The feedback approach was derived in [29], [30] and [32], which is based on the energy conservation principle made in those papers. By following the idea of Sayed [29], we provide a unified approach to quantifying the EMSE of our adaptive blind receiver that can be made to fit the general class of adaptive SG algorithms given by

$$\mathbf{w}(i+1) = \mathbf{w}(i) + \mu\mathbf{u}(i)F_e(i) \quad (33)$$

where $F_e(i)$ is a generic scalar function determined by the adaptive algorithm. The major result of the feedback approach is the energy-preserving equation which relates the *a priori* estimation error to the error function $F_e(i)$ and the vector $\mathbf{u}$ in

(33) once the algorithm has reached the steady state. In our case $\mathbf{u}(i) = (\mathbf{I} - (\bar{\mathbf{s}}\bar{\mathbf{s}}^H/\|\bar{\mathbf{s}}\|^2))\mathbf{r}(i)$, and $F_e(i) = -e(i)z^*(i)$.

Now, subtract both sides of (33) from some vector $\mathbf{w}_{\mathrm{opt}}$ to get the weight error equation

$$\tilde{\mathbf{w}}(i+1) = \tilde{\mathbf{w}}(i) - \mu\mathbf{u}(i)F_e(i) \tag{34}$$

where $\tilde{\mathbf{w}}(i) = \mathbf{w}_{\mathrm{opt}} - \mathbf{w}(i)$. Define *a priori* and *a posteriori* estimation errors $e_a(i) = \tilde{\mathbf{w}}^H(i)\mathbf{r}(i)$ and $e_p(i) = \tilde{\mathbf{w}}^H(i+1)\mathbf{r}(i)$. We now show how to rewrite (34) in terms of the error measures $\{\tilde{\mathbf{w}}(i), \tilde{\mathbf{w}}(i+1), e_a(i), e_p(i)\}$ alone. For this purpose, we note that if we multiply the Hermitian of (34) by $\mathbf{r}(i)$ from the right, we obtain

$$\begin{aligned} e_a(i) &= e_p(i) + \mu\mathbf{u}^H(i)\mathbf{r}(i)F_e^*(i) \\ &= e_p(i) + \mu\mathbf{r}^H(i)\left(\mathbf{I} - \frac{\bar{\mathbf{s}}\bar{\mathbf{s}}^H}{\|\bar{\mathbf{s}}\|^2}\right)\mathbf{r}(i)F_e^*(i). \end{aligned} \tag{35}$$

Since $\mathbf{r}^H(i)(\mathbf{I} - (\bar{\mathbf{s}}\bar{\mathbf{s}}^H/\|\bar{\mathbf{s}}\|^2))\mathbf{r}(i) = \|\mathbf{u}(i)\|^2$, we can obtain

$$e_p(i) = e_a(i) - \mu\|\mathbf{u}(i)\|^2 F_e^*(i). \tag{36}$$

Solving for $F_e^*(i)$ gives

$$F_e^*(i) = \frac{e_a(i) - e_p(i)}{\mu\|\mathbf{u}(i)\|^2} \tag{37}$$

so that we can rewrite (34) as

$$\tilde{\mathbf{w}}(i+1) = \tilde{\mathbf{w}}(i) - \frac{\mathbf{u}(i)}{\|\mathbf{u}(i)\|^2}\left[e_a^*(i) - e_p^*(i)\right]. \tag{38}$$

Rearranging (38) leads to

$$\tilde{\mathbf{w}}(i) + \frac{\mathbf{u}(i)}{\|\mathbf{u}(i)\|^2}e_p^*(i) = \tilde{\mathbf{w}}(i+1) + \frac{\mathbf{u}(i)}{\|\mathbf{u}(i)\|^2}e_a^*(i). \tag{39}$$

If we define

$$\bar{\mu}(i) = 1/\|\mathbf{u}(i)\|^2 \tag{40}$$

then by squaring (39), we observe that the following energy relation is obtained:

$$\begin{aligned} &\|\tilde{\mathbf{w}}(i)\|^2 + \bar{\mu}(i)|e_p(i)|^2 + \bar{\mu}(i)\mathbf{u}^H(i)\tilde{\mathbf{w}}(i)e_p(i) \\ &\quad + \bar{\mu}(i)\tilde{\mathbf{w}}^H(i)\mathbf{u}(i)e_p^*(i) \\ &\qquad = \|\tilde{\mathbf{w}}(i+1)\|^2 + \bar{\mu}(i)|e_a(i)|^2 + \bar{\mu}(i)\mathbf{u}^H(i)\tilde{\mathbf{w}}(i+1)e_a(i) \\ &\qquad\quad + \bar{\mu}(i)\tilde{\mathbf{w}}^H(i+1)\mathbf{u}(i)e_a^*(i). \end{aligned} \tag{41}$$

By taking expectations of both sides of (41), when the filter operation is in steady state, namely $i \to \infty$, we can obtain

$$\begin{aligned} E\left\{\|\tilde{\mathbf{w}}(i)\|^2\right\} + E\left\{\bar{\mu}(i)|e_p(i)|^2\right\} &= E\left\{\|\tilde{\mathbf{w}}(i+1)\|^2\right\} \\ &\quad + E\left\{\bar{\mu}(i)|e_a(i)|^2\right\}. \end{aligned} \tag{42}$$

When the filter operation is in steady state for $i \to \infty$, we also can write

$$E\left\{\|\tilde{\mathbf{w}}(i)\|^2\right\} = E\left\{\|\tilde{\mathbf{w}}(i+1)\|^2\right\}. \tag{43}$$

Now, with (33), the effect of the weight error vector is canceled out from (42), and we are reduced to studying only the equality $E\{\bar{\mu}(i)|e_a(i)|^2\} = E\{\bar{\mu}(i)|e_p(i)|^2\}$. Substituting (36) into the equation we can have the energy preserving equation.

The energy preserving equation in the steady state is used to solve for the EMSE and is given as

$$E\left\{\bar{\mu}(i)|e_a(i)|^2\right\} = E\left\{\bar{\mu}(i)\left|e_a(i) - \frac{\mu(i)}{\bar{\mu}(i)}F_e^*(i)\right|^2\right\} \tag{44}$$

where $F_e(i) = -e(i)z^*(i) = (1 - |y(i)|^2)y^*(i)$, and $y(i) = (\mathbf{w}_{\mathrm{opt}} - \tilde{\mathbf{w}}(i))^H\mathbf{r}(i) = \mathbf{w}_{\mathrm{opt}}^H\mathbf{r}(i) - e_a(i) = Ab(i) + M(i) + \eta(i) + v(i) - e_a(i)$, where $Ab(i)$ is the desired user's signal, M is the residual MAI as the output of the optimum filter, η is the filtered ISI, and v is the filtered AWGN.

By expanding the RHS of (44), the equation can be simplified to $D = F$, where $D = E\{\mu(i)\}E\{e_a^*y(i)(1 - |y(i)|^2)\} + E\{\mu(i)\}E\{e_a y^*(i)(1 - |y(i)|^2)\}$ and $F = E\{\mu^2(i)\}E\{\|\mathbf{u}\|^2|y(i)|^2(1 - |y(i)|^2)^2\}$. Based on the analytical results in [30] and [31], we can make several assumptions.

Assumption 3: In the steady state, $\|\mathbf{u}\|^2$ and $|F_e|^2$ are uncorrelated. The quantities $\{b, M, \eta, v, e_a\}$ are zero-mean random variables, and are mutually independent. We also have $E\{b_k^{2m}\} = 1$ for any positive integer m. The user's power A_k is equal to 1. The residual MAI and ISI are Gaussian random variables.

By using Assumption 3 and substituting $y(i)$ into equation $D = F$, we have

$$\begin{aligned} &E\{\mu^2\}E\left\{\|\mathbf{u}\|^2\right\}G_1E\left\{|e_a|^2\right\} \\ &\quad + 3E\{\mu^2\}E\left\{\|\mathbf{u}\|^2\right\}E\{M^2\}E\left\{|e_a|^4\right\} \\ &\quad + 3E\{\mu^2\}E\left\{\|\mathbf{u}\|^2\right\}E\{v^2\}E\left\{|e_a|^4\right\} \\ &\quad + E\{\mu^2\}E\left\{\|\mathbf{u}\|^2\right\}\left(3E\{\eta^2\} + 1\right)E\left\{|e_a|^4\right\} \\ &\quad + E\{\mu^2\}E\left\{\|\mathbf{u}\|^2\right\}G_2 + E\{\mu^2\}E\left\{\|\mathbf{u}\|^2\right\}E\left\{|e_a|^6\right\} \\ &\qquad = 2E\{\mu\}\left(E\{M^2\}E\left\{|e_a|^2\right\} + E\{v^2\}E\left\{|e_a|^2\right\}\right. \\ &\qquad\qquad \left. + E\{\eta^2\}E\left\{|e_a|^2\right\} + E\left\{|e_a|^4\right\}\right). \end{aligned} \tag{45}$$

$$G_1 = 3 + 3\sigma_M^4 + 6\sigma_M^2\sigma_v^2 + 6\sigma_M^2\sigma_\eta^2 + 3\sigma_v^4 + 6\sigma_v^2\sigma_\eta^2 + 3\sigma_\eta^4. \tag{46}$$

$$\begin{aligned} G_2 = &\sigma_\eta^6 + 3\sigma_v^2\sigma_\eta^4 + 3\sigma_v^4\sigma_\eta^2 + 3\sigma_M^2\sigma_\eta^4 + \sigma_v^6 + 6\sigma_v^2\sigma_\eta^2\sigma_M^2 \\ &+ 3\sigma_M^2\sigma_v^4 + 3\sigma_M^4\sigma_\eta^2 + 3\sigma_M^4\sigma_v^2 + \sigma_M^6 + \sigma_\eta^4 + 2\sigma_v^2\sigma_\eta^2 \\ &+ \sigma_v^4 + 2\sigma_M^2\sigma_\eta^2 + 2\sigma_M^2\sigma_v^2 + \sigma_M^4 + 4\sigma_v^2 \\ &+ 2\sigma_\eta^2 + 2\sigma_M^2 + 2. \end{aligned} \tag{47}$$

It is the convergence state $i \to \infty$, so we can assume $E\{M^{2m}\} = (E\{M^2\})^m = \sigma_M^{2m}$, $E\{\eta^{2m}\} = (E\{\eta^2\})^m = \sigma_\eta^{2m}$, and $E\{v^{2m}\} = (E\{v^2\})^m = \sigma_v^{2m}$, where σ_M, σ_η and σ_v are the variances of the Gaussian distribution.

In this circumstance, the high power terms $E\{|e_a|^4\}$ and $E\{|e_a|^6\}$ may be neglected. So we obtain the EMSE

$$\begin{aligned} \zeta &= E\left\{|e_a|^2\right\} \\ &= \frac{E\left\{\mu^2(\infty)\right\}E\left\{\|\mathbf{u}\|^2\right\}G_2}{2E\left\{\mu(\infty)\right\}\left(\sigma_M^2 + \sigma_v^2 + \sigma_\eta^2\right) - E\left\{\mu^2(\infty)\right\}E\left\{\|\mathbf{u}\|^2\right\}G_1} \end{aligned} \tag{48}$$

where $E\{\mu(\infty)\} \approx b\xi_{\min}/(1-a)$, $E\{\mu^2(\infty)\} \approx 2ab^2\xi_{\min}^2/(1-a)^2(1+a)$ and, the details of G_1 and G_2 are given by (46) and (47), respectively.

Assumption 4: The residual MAI and ISI powers σ_M^2 and σ_η^2 at the output of the optimum filter are significantly lower than the output noise power σ_v^2, namely, $\sigma_v^2 \gg \sigma_M^2$ and $\sigma_v^2 \gg \sigma_\eta^2$.

Thus, a simplified expression can be derived if all the terms that contain M and η are removed. The simplified expression is then given by

$$\begin{aligned}\zeta &= E\left\{|e_a|^2\right\} \\ &\approx \frac{ab\xi_{\min}E\left\{\|\mathbf{u}\|^2\right\}\left(\sigma_v^6+\sigma_v^4+4\sigma_v^2+2\right)}{(1+a)(1-a)\sigma_v^2 - ab\xi_{\min}E\left\{\|\mathbf{u}\|^2\right\}(3+3\sigma_v^4)}. \end{aligned} \tag{49}$$

D. Tracking Analysis

Here, we examine the operation of these novel step-size algorithms in a nonstationary environment, for which the optimum solution takes on a time-varying form. The minimum point of the error-performance surface is no longer fixed. Consequently, the adaptive blind algorithm now has the added task of tracking the minimum point of the error-performance surface. In other words, the algorithm is required to continuously track the statistical variations of the input, the occurrence of which is assumed to be "slow" enough for tracking to be feasible. We shall continue to rely on the energy-conservation framework [30] and use it to derive expressions for the excess MSE of an adaptive filter when the input signal properties vary with time. The presentation will reveal that there are actually minor differences between mean-square analysis and tracking analysis.

EMSE expressions for the tracking performance of the CCM algorithm were published in [30]. The derivation of tracking performance EMSE for the blind MUD was proposed in [31]. Here we focus on the analysis of the novel variable step-size mechanism incorporated in the parameter estimation of the CCM-SG algorithm.

In the time-varying channel, the optimum filter coefficients are assumed to vary according to the model $\mathbf{w}_{\text{opt}}(i+1) = \mathbf{w}_{\text{opt}}(i)+\mathbf{q}(i)$, where $\mathbf{q}(i)$ denotes a random perturbation. This is typical in the context of tracking analyses of adaptive filters [14], [33], and [34]. Based on these works, we make an assumption.

Assumption 5: The sequence $\mathbf{q}(i)$ is a stationary sequence of independent zero-mean vectors and positive definite autocorrelation matrix $\mathbf{Q} = E(\mathbf{q}(i)\mathbf{q}^H(i))$, which is mutually independent of the sequences $\{\mathbf{u}(i)\}$, $\{v(i)\}$, $\{M(i)\}$ and $\{\eta(i)\}$.

Now, we first redefine the weight error vector as $\tilde{\mathbf{w}}(i) = \mathbf{w}_{\text{opt}}(i) - \mathbf{w}(i)$ and then, $\tilde{\mathbf{w}}(i)$ satisfies

$$\tilde{\mathbf{w}}(i+1) = \tilde{\mathbf{w}}(i) - \mu\mathbf{u}(i)F_e(i) + \mathbf{q}(i). \tag{50}$$

We define $e_a(i) = (\mathbf{w}_{\text{opt}}(i) - \mathbf{w}(i))^H\mathbf{r}(i)$ and $e_p(i) = (\mathbf{w}_{\text{opt}}(i) - \mathbf{w}(i+1))^H\mathbf{r}(i)$, so from (33) we have

$$e_a(i) = e_p(i) + \mu\|\mathbf{u}(i)\|^2 F_e^*(i). \tag{51}$$

We obtain that (36) and (38) still hold for the nonstationary case, from (50) and (51) we obtain

$$\tilde{\mathbf{w}}(i+1)+\bar{\mu}(i)\mathbf{u}(i)e_a^*(i) = \tilde{\mathbf{w}}(i)+\mathbf{q}(i)+\bar{\mu}(i)\mathbf{u}(i)e_p^*(i). \tag{52}$$

As aforementioned, by squaring (52) and taking the expected value, when the filter is operating in steady state we have

$$\begin{aligned} &E\left\{\|\tilde{\mathbf{w}}(i+1)\|^2\right\} + E\left\{\bar{\mu}(i)\,|e_a(i)|^2\right\} \\ &\quad = E\left\{\|\tilde{\mathbf{w}}(i)+\mathbf{q}(i)\|^2\right\} + E\left\{\bar{\mu}(i)\,|e_p(i)|^2\right\}. \end{aligned} \tag{53}$$

$$\begin{aligned} &E\left\{\|\tilde{\mathbf{w}}(i)+\mathbf{q}(i)\|^2\right\} \\ &\quad = E\left\{\left(\tilde{\mathbf{w}}^H(i)+\mathbf{q}^H(i)\right)\left(\tilde{\mathbf{w}}(i)+\mathbf{q}(i)\right)\right\} \\ &\quad = E\left\{\|\tilde{\mathbf{w}}(i)\|^2\right\} + E\left\{\tilde{\mathbf{w}}^H(i)\mathbf{q}(i)\right\} + E\left\{\mathbf{q}^H(i)\tilde{\mathbf{w}}(i)\right\} \\ &\qquad + E\left\{\mathbf{q}^H(i)\mathbf{q}(i)\right\} \end{aligned} \tag{54}$$

by using Assumption 5, we have $E\{\tilde{\mathbf{w}}^H(i)\mathbf{q}(i)\} = E\{\mathbf{q}^H(i)\tilde{\mathbf{w}}(i)\} = 0$.

When $i \to \infty$, $E\{\|\tilde{\mathbf{w}}(i+1)\|^2\} = E\{\|\tilde{\mathbf{w}}(i)\|^2\}$, so based on Assumption 5 the energy preserving equation of tracking performance is given as

$$E\left\{\bar{\mu}(i)\,|e_a(i)|^2\right\} = Tr(\mathbf{Q}) + E\left\{\bar{\mu}(i)\left|e_a(i) - \frac{\mu}{\bar{\mu}(i)}F_e^*(i)\right|^2\right\}. \tag{55}$$

Expanding the equation, it can be simplified to

$$D = Tr(\mathbf{Q}) + F \tag{56}$$

where D and F were described before, $Tr(\mathbf{Q}) = E(\mathbf{q}^H(i)\mathbf{q}(i))$. By using the previous assumptions, the high power terms $E\{|e_a|^4\}$ and $E\{|e_a|^6\}$ may be neglected. Finally, we obtain

$$\begin{aligned}\zeta &= E\left\{|e_a|^2\right\} \\ &= \frac{E\left\{\mu^2(\infty)\right\}E\left\{\|\mathbf{u}\|^2\right\}G_2 + Tr(\mathbf{Q})}{2E\left\{\mu(\infty)\right\}\left(\sigma_M^2+\sigma_v^2+\sigma_\eta^2\right) - E\left\{\mu^2(\infty)\right\}E\{\|\mathbf{u}\|^2\}G_1} \end{aligned} \tag{57}$$

where $E\{\mu(\infty)\} \approx b\xi_{\min}/(1-a)$, $E\{\mu^2(\infty)\} \approx 2ab^2\xi_{\min}^2/(1-a)^2(1+a)$.

By using assumption 4, a simplified expression can be derived if all the terms that contain M and η are removed. The simplified solution is given by (58) shown at the bottom of the page.

$$\begin{aligned}\zeta &= E\left\{|e_a|^2\right\} \\ &\approx \frac{ab\xi_{\min}^2E\left\{\|\mathbf{u}\|^2\right\}\left(\sigma_v^6+\sigma_v^4+4\sigma_v^2+2\right) + (1-a)^2(1+a)Tr(\mathbf{Q})}{(1+a)(1-a)\xi_{\min}\sigma_v^2 - ab\xi_{\min}^2E\left\{\|\mathbf{u}\|^2\right\}(3+3\sigma_v^4)}. \end{aligned} \tag{58}$$

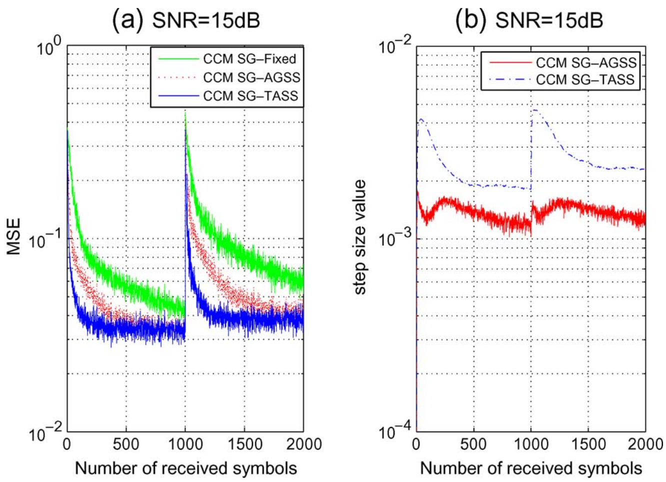

Fig. 2. (a) MSE performance in nonstationary environment of AWGN channel, start with four users including a 5-dB high power level interferer, after 1000 received symbols four new users including a 5-dB high power level interferer enter in the system. (b) Step-size variation in the nonstationary environment.

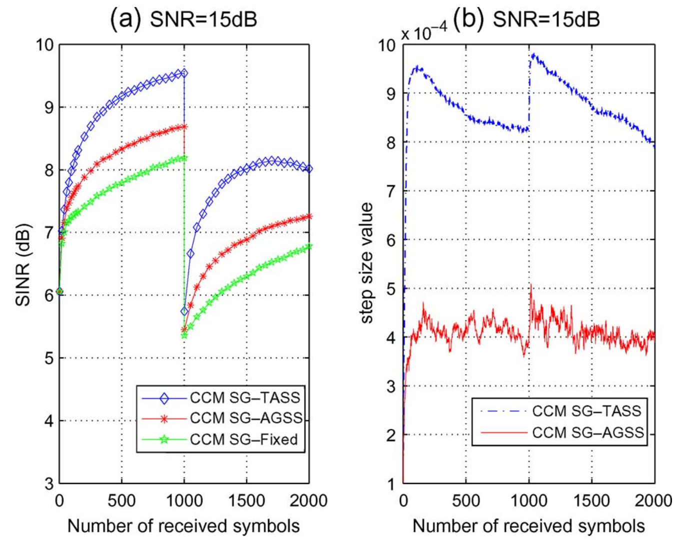

Fig. 3. (a) SINR performance in nonstationary environment of multipath time-varying channel. (b) Step-size values for the variable step-size mechanisms in the nonstationary environment, $\mathrm{SNR} = 15$ dB, $f_d T = 5 \times 10^{-6}$, FSS is 10^{-4}.

VI. Simulations

In this section, we evaluate the performance of the proposed variable step-size mechanism and compare it to the existing algorithms. First, the MSE performance of nonstationary scenario is compared in AWGN single-path channel to evaluate the mechanisms, and then we carry out simulations to assess the signal-to-interference-plus-noise ratio (SINR) performance of the algorithms in nonstationary environments for multipath time-varying channel. The bit error rate (BER) performance is also taken into account, and at last we focus on the convergence and tracking analytical works to compare the simulation and analysis results. The DS-CDMA system employs spreading codes with spreading gain $N = 31$, the users in the system are assumed to have perfect power control. Our simulation results are based on the downlink of an uncoded system.

The first experiment studies the performance of the proposed CCM SG-TASS algorithm, the existing CCM SG-AGSS and the CCM SG fixed step-size algorithms in an AWGN channel. The DS-CDMA system employs random sequences as the spreading codes. Fig. 2(a) shows the MSE performance of the algorithms in a nonstationary environment of AWGN channel, SNR is 15 dB, where the SNR is defined as the received desired user's signal to noise power ratio. We show the convergence of the receivers in terms of MSE. For the nonstationary case the system starts with four users including one high power level interferer with 5 dB and after 1000 symbols, four new users including a 5-dB high power level user enter in the system. These results in Fig. 2(a) indicate that the convergence of the proposed CCM SG-TASS outperforms the convergence of the CCM SG-AGSS and the fixed step-size algorithms in AWGN nonstationary environment. Fig. 2(b) shows the variation of the step-size values in the nonstationary environment. In this experiment, the parameters of the TASS mechanism have been optimized with $\mu_- = 10^{-6}$, $\mu_+ = 5 \times 10^{-3}$, $\mu_0 = 10^{-4}$, $a = 0.98$, and $b = 5 \times 10^{-4}$. The optimized parameters of AGSS mechanism are $\mu_- = 10^{-6}$, $\mu_+ = 2 \times 10^{-3}$, $\mu_0 = 10^{-4}$, $\alpha = 0.0003$, and fixed step-size is 5×10^{-4}. The optimized parameters are chosen based on simulations results, to make the system work in a stable way and obtain good performance.

The second experiment considers the algorithms in multipath time-varying channel. In order to avoid the ambiguity, we only considered real channel models. Thus, the algorithm deals with the amplitude variation of the channel. The channel has a profile with three paths, and it is normalized. The channel parameters for these experiments are $p_0 = 0.8367$, $p_1 = 0.4472$, $p_2 = 0.3162$. The sequence of channel coefficients is $h_l(i) = p_l|\alpha_l(i)|$ $(l = 0, 1, 2)$, where $\alpha_l(i)$ is computed according to the Jakes' model. We optimized the limits of the parameters of the variable step-size mechanisms with $\mu_+ = 10^{-3}$, $\mu_- = 10^{-5}$, and $\mu_0 = 10^{-4}$, 2000 symbols are transmitted. The channel estimation algorithm in [23] is employed in the simulation.

First, we assess the SINR performance of the proposed TASS mechanism, the AGSS and fixed step-size (FSS) mechanisms, all based on the SG CCM blind algorithm. In this case, the random sequence is employed for the spreading code. Fig. 3(a) shows the convergence of the receivers in terms of SINR, in a scenario where the power levels of three interferers are 5 dB above the desired user, whilst the remaining interferers work at the same power level of the desired signal. In order to test the nonstationary scenario, the system starts with five users and three new users enter after 1000 symbols. We optimized the parameters of the mechanisms with $a = 0.98$, $b = 6 \times 10^{-5}$ for the TASS and $\alpha = 0.06$ for the AGSS. These results indicate that the proposed TASS mechanism converges to a higher SINR than the other methods. Fig. 3(b) shows the variation of the step-size values. Finally, we can see that the novel variable step-size algorithm can work very well in the nonstationary environment of the multipath time-varying channel.

The BER performance is studied next. In particular, we show the BER performance versus the received desired user's signal to noise power ratio and number of users (K) for the analyzed algorithms. Here, we use Gold sequences with $N = 31$, and

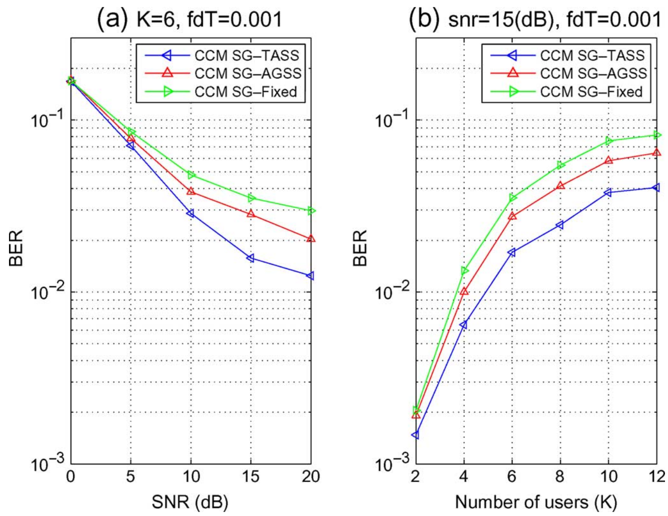

Fig. 4. (a) BER versus the SNR with multipath channels. (b) BER versus number of users with multipath channels. $f_dT = 1 \times 10^{-3}$, FSS is 10^{-4}.

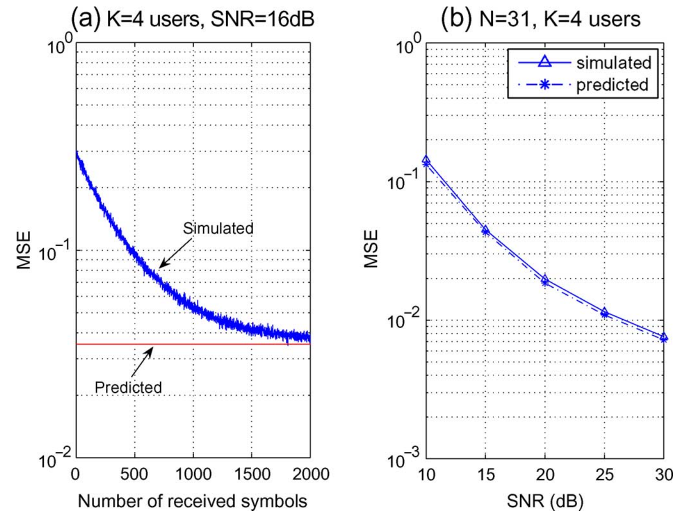

Fig. 5. MSE analytical versus simulated performance for the proposed TASS mechanism convergence analysis. (a) Number of users is 4, the SNR is 16 dB. (b) Number of users is 4.

assume every user's power is equal to 1, the fading rate f_dT is 1×10^{-3}, and 2000 symbols are transmitted. The results in Fig. 4(a) indicate that the best performance is achieved with the proposed CCM SG-TASS algorithm, followed by the CCM SG-AGSS and the CCM SG fixed step-size algorithms. Fig. 4(b) shows us that with an increase in the number of users in the system, our proposed algorithm still has the best performance. Specifically, CCM SG-TASS algorithm can save up to 4 dB and support up to three more users in comparison with the CCM SG-AGSS algorithm for the same performance.

In the third experiment, we consider the convergence and tracking analyses. The multipath channel model is the same as before. In order to simplify the simulations we employ the normalized fixed vector $[p_0, p_1, p_2]^T$ as the vector $\mathbf{g}$ to calculate the effective signature waveform $\bar{\mathbf{s}}_k$, where p_0, p_1, and p_2 are the values in the second experiment. The steady-state MSE between the desired and the estimated symbol obtained through simulation is compared with the steady-state MSE computed via the expressions derived in Section V. Before using (49) and (58) we have to calculate several values. From the conclusion in [26], we know that the optimal CCM minimum $\xi_{\min}$ roughly corresponds to the minimum mean square error. So, $\xi_{\min} \approx 1 - \bar{\mathbf{s}}_k^H \mathbf{R}^{-1} \bar{\mathbf{s}}_k$, where $\bar{\mathbf{s}}_k = \mathbf{C}_k \mathbf{g}$, and $\mathbf{R} = (\mathbf{P}\mathbf{P}^H + \sigma_n^2 \mathbf{I})$, $\mathbf{P} = [\bar{\mathbf{s}}_1, \ldots, \bar{\mathbf{s}}_K]$, the σ_n^2 here is the additive noise power in the receiver. The residual noise power σ_v^2 in (49) is equal to $\mathbf{w}_{\text{opt}}^H \sigma_n^2 \mathbf{w}_{\text{opt}}$, where $\mathbf{w}_{\text{opt}} = \mathbf{R}^{-1} \bar{\mathbf{s}}_k$. $E\{\|\mathbf{u}\|^2\} = (M-1)\sigma_n^2 + \sum_{k=2}^{K} A_k^2 \rho_{kk} - \sum_{k=2}^{K} A_k^2 \rho_{k1}^2 / \rho_{11}$, where $\rho_{ij} = \bar{\mathbf{s}}_i^H \bar{\mathbf{s}}_j$. The results can be derived by using a similar approach to [31], and they are shown in the Appendix.

First, let us verify that the results (20), (21), and (49) of the section on convergence analysis of the mechanism can provide a means of estimating the excess MSE. In this simulation of convergence analysis, we employ random sequences of length $N = 31$, and assume that four users operate in the system and they have the same power level. The time-invariant multipath scenario with AWGN is considered. The results are shown in Fig. 5(a), for the multipath case. By comparing the curves, it can be seen that as the number of received symbols increases and the simulated MSE converges to the analytical result, showing the usefulness of our analysis and assumptions, where $a = 0.98$, $b = 0.00005$. The Fig. 5(b) shows the effect that the desired user's signal-to-noise power ratio has on the MSE, and a comparison between the steady-state analysis and simulation results. The results confirm that the MSE decreases monotonically with SNR. For each input SNR we can find suitable values of parameters a and b to let the simulation and analysis results agree well with each other.

The tracking analysis of the CCM SG-TASS algorithm in a fading channel has been discussed in Section V-D. Here, we verify that the results (20), (21), and (58) of the section on tracking analysis of the mechanism can provide a means of estimating the MSE. The tracking analysis has been evaluated by using random sequences with spreading gain 31, and we assume that six users operate with the same power level in the system. A time-varying multipath channel has been taken into account, the 5×10^{-5} fading rate Jakes' model is employed. The value of $\text{Tr}(\mathbf{Q})$ was computed with the aid of $J_0(2\pi f_d T)$ [35], which is the autocorrelation function of Jakes' model, where J_0 is the zero-order Bessel function of the first kind. f_d is the maximum Doppler shift, and T is the symbol interval [36]. $\text{Tr}(\mathbf{Q})$ is equal to 1.6×10^{-6}. Fig. 6 indicates that as the number of received symbols increases, the simulated MSE converges to the analytical result, showing the usefulness of our analysis and assumptions, where $a = 0.98$, $b = 0.00016$.

VII. Conclusion

In this paper, we have investigated blind adaptive CCM receivers for DS-CDMA systems that employ SG algorithms with variable step-size mechanisms. A low-complexity variable step-size mechanism has been proposed and analyzed for estimating the parameters of linear CDMA receiver that operate with SG algorithms in multipath channels. We compared the computational complexity of the new algorithm with the existent methods and further investigated the characteristics of

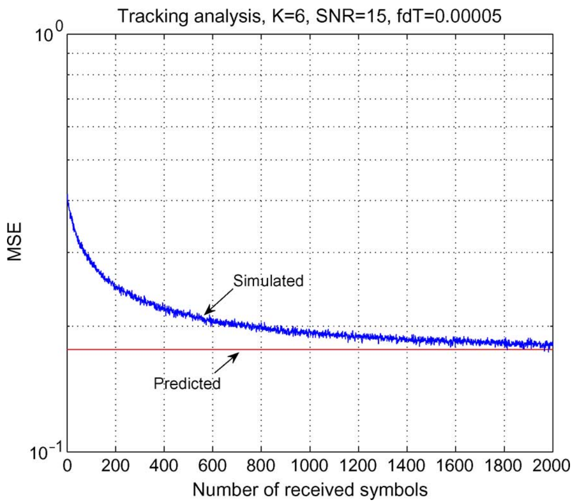

Fig. 6. MSE analytical versus simulated performance for the proposed TASS mechanism tracking analysis. Number of users is 6, the SNR is 15 dB, $f_d T = 5 \times 10^{-5}$.

the new mechanism via derived analytical expressions using the energy-preserving approach to predict the EMSE for convergence and tracking analyses. Simulation experiments were conducted to verify the analytical results and illustrate that the new blind adaptation mechanism significantly outperforms the conventional variable step-size mechanisms for blind CCM receivers at a lower complexity in both stationary and nonstationary scenarios.

APPENDIX

Derivation of $\xi_{\min}$, $\mathbf{w}_{\text{opt}}$, and $E\{\|\mathbf{u}\|^2\}$ in the third experiment.

Let us consider the MSE function as

$$\begin{aligned} E&\left\{\left\|b_k(i) - \mathbf{w}_k^H(i)\mathbf{r}(i)\right\|^2\right\} \\ &= E\left\{\left(b_k^*(i) - \mathbf{r}^H(i)\mathbf{w}_k(i)\right)\left(b_k(i) - \mathbf{w}_k^H(i)\mathbf{r}(i)\right)\right\} \\ &= E\left\{b_k^2(i)\right\} - E\left\{b_k^*(i)\mathbf{w}_k^H(i)\mathbf{r}(i)\right\} \\ &\quad - E\left\{b_k(i)\mathbf{r}^H(i)\mathbf{w}_k(i)\right\} \\ &\quad + E\left\{\mathbf{w}_k^H(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_k(i)\right\}. \end{aligned} \quad (59)$$

Because BPSK signal b_k and received noise $\mathbf{n}$ are generated independently with zero mean, where $b_k^2 = 1$, $k = 1 \ldots K$, so the (59) can be simplified as

$$\begin{aligned} E\left\{\left\|b_k(i) - \mathbf{w}_k^H(i)\mathbf{r}(i)\right\|^2\right\} = 1 - \mathbf{w}_k^H(i)\bar{\mathbf{s}}_k - \bar{\mathbf{s}}_k^H\mathbf{w}_k(i) \\ + \mathbf{w}_k^H(i)\mathbf{R}(i)\mathbf{w}_k(i) \end{aligned} \quad (60)$$

where we assume that every user's power is equal to 1, and $\mathbf{R} = E\{\mathbf{r}(i)\mathbf{r}^H(i)\}$, By taking the gradient of (60) with respect to $\mathbf{w}^*$ and letting it to zero, we have the $\mathbf{w}_{\text{opt}}$

$$\mathbf{w}_{\text{opt}} = \mathbf{R}^{-1}\bar{\mathbf{s}}_k \quad (61)$$

substituting $\mathbf{w}_{\text{opt}}$ in (60), we obtain the MMSE value, because the optimal CCM minimum $\xi_{\min}$ is roughly correspondent to the MMSE so

$$\xi_{\min} \approx 1 - \bar{\mathbf{s}}_k^H\mathbf{R}^{-1}\bar{\mathbf{s}}_k \quad (62)$$

$\mathbf{r}(i) = \sum_{k=1}^{K} A_k b_k(i)\bar{\mathbf{s}}_k + \mathbf{n}(i)$, $\mathbf{R} = E\{\mathbf{r}(i)\mathbf{r}^H(i)\} = \sum_{k=1}^{K} A_k^2\bar{\mathbf{s}}_k\bar{\mathbf{s}}_k^H + \sigma^2\mathbf{I}$.

We know that $\mathbf{u}(i) = (\mathbf{I} - (\bar{\mathbf{s}}_k\bar{\mathbf{s}}_k^H/\|\bar{\mathbf{s}}_k\|^2))\mathbf{r}(i)$, and we assume user 1 is the desired user. Hence

$$\begin{aligned} E\left\{\|\mathbf{u}\|^2\right\} &= E\left\{\mathbf{u}^H(i)\mathbf{u}(i)\right\} \\ &= E\left\{\mathbf{r}^H(i)\left(\mathbf{I} - \frac{\bar{\mathbf{s}}_1\bar{\mathbf{s}}_1^H}{\|\bar{\mathbf{s}}_1\|^2}\right)\mathbf{r}(i)\right\} \\ &= \left(\sum_{k=1}^{K} A_k^2\rho_{kk} + M\sigma_n^2\right) \\ &\quad - \left(\sum_{k=1}^{K} A_k^2\frac{\bar{\mathbf{s}}_k^H\bar{\mathbf{s}}_1\bar{\mathbf{s}}_1^H\bar{\mathbf{s}}_k}{\|\bar{\mathbf{s}}_1\|^2} + \frac{\mathbf{n}^H\bar{\mathbf{s}}_1\bar{\mathbf{s}}_1^H\mathbf{n}}{\|\bar{\mathbf{s}}_1\|^2}\right) \\ &= \left(\sum_{k=1}^{K} A_k^2\rho_{kk} + M\sigma_n^2\right) \\ &\quad - \left(\sum_{k=1}^{K} A_k^2\frac{\rho_{k1}^2}{\rho_{11}} + \sigma_n^2\right) \\ &= \sum_{k=2}^{K} A_k^2\rho_{kk} - \sum_{k=2}^{K} A_k^2\frac{\rho_{k1}^2}{\rho_{11}} + (M-1)\sigma_n^2 \end{aligned} \quad (63)$$

where $\rho_{ij} = \bar{\mathbf{s}}_i^H\bar{\mathbf{s}}_j$.

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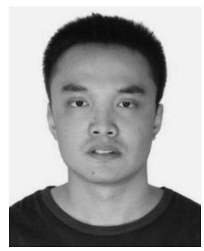

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