Iterative Detection and Decoding Algorithms For MIMO Systems in Block-Fading Channels using LDPC Codes

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Abstract—We propose iterative detection and decoding (IDD) algorithms with Low-Density Parity-Check (LDPC) codes for Multiple Input Multiple Output (MIMO) systems operating in block-fading and fast Rayleigh fading channels. Soft-input soft-output minimum mean-square error receivers with successive interference cancellation are considered. In particular, we devise a novel strategy to improve the bit error rate (BER) performance of IDD schemes, which takes into account the soft *a posteriori* output of the decoder in a block-fading channel when Root-Check LDPC codes are used. A MIMO IDD receiver with soft information processing that exploits the code structure and the behavior of the log likelihood ratios is also developed. Moreover, we present a scheduling algorithm for decoding LDPC codes in block-fading channels. Simulations show that the proposed techniques result in significant gains in terms of BER for both block-fading and fast-fading channels.

Index Terms—LDPC codes, MIMO systems, IDD schemes, block fading channels.

I. INTRODUCTION

Modern wireless communication standards for cellular and local area networks advocate the use of Low-Density Parity-Check (LDPC) codes for high throughput applications [1]. Since multiple-input multiple-output (MIMO) systems are often subject to multi-path propagation and mobility, these systems are characterized by time-varying channels with fluctuating signal strength. In applications subject to delay constraints and slowly-varying channels, only limited independent fading realizations are experienced [2]. A simple and useful model that captures the essential characteristics of such scenarios is the block-fading channel [3]. A family of LDPC codes called Root-Check codes were proposed in [4] and can achieve the maximum diversity of a block-fading channel when decoded with the Belief Propagation (BP) algorithm. Recent LDPC techniques [5]–[11] that improve the coding gain and have low-complexity encoding and reduced storage requirements have been investigated.

MIMO systems can bring significant multiplexing [12]–[14] and diversity gains [15], [16] in wireless communication systems. In the block-fading channel the structure of the channel and the degrees of freedom introduced by multiple antennas must be exploited in order to appropriately design the receiver. Approaches to receiver design include MIMO detectors [**?**], [17]–[30], decoding strategies [31] and iterative detection and decoding (IDD) schemes [22], [32]. Among the

most cost-effective detectors are the successive interference cancellation (SIC) used in the Vertical Bell Laboratories Layered Space-Time (VBLAST) systems [18], [19] and decision feedback (DF) [20]–[25], [27] techniques. These suboptimal detectors can offer a good trade-off between performance and complexity. Prior contributions on IDD schemes include the seminal work of Wang and Poor with turbo concepts [22] and the LDPC-based scheme reported by Yue and Wang [32]. In IDD schemes, the decoder plays an important role in the overall performance and complexity. Vila Casado and et. al. in [31] have suggested that the use of appropriate scheduling mechanisms for LDPC decoding can significantly reduce the number of required iterations. Prior work on MIMO detectors and IDD schemes have dealt with quasi-static Rayleigh fading channels or fast Rayleigh fading channels. However, there are very few studies related to the case of block-fading channels with MIMO systems. To the best of our knowledge, the only study which discusses MIMO systems under block-fading channels is the work by Capirone and Tarable [33]. They have shown that using Root-Check LDPC codes with MIMO allows a system to achieve the desired channel diversity.

In contrast, in our work two key elements of an IDD system are considered. First, by properly manipulating the log-likelihood ratios (LLRs) at the output of the decoder and exploiting the code structure we can obtain significant gains over standard LLR processing for IDD schemes in block fading channels. Second, to improve the overall performance we introduce a new scheduling strategy for block-fading channels in IDD systems. The main contributions of our work are the development of a novel IDD scheme that exploits the code structure and a novel strategy for manipulation of LLRs that improves the performance of MIMO IDD systems in block-fading channels. In addition, we have also developed a method of sequential scheduling to further improve the performance of MIMO IDD systems in block-fading channels. The gains provided by the proposed IDD scheme and algorithms do not require significant extra computational effort or any extra memory storage.

The rest of this paper is organized as follows. In Section II we describe the system model. In Section III we discuss the proposed log-likelihood ratio (LLR) compensation strategy. In Section IV we introduce the proposed scheduling method. Section V analyzes some aspects of the proposed techniques. Section VI depicts and discusses the simulation results, while Section VII concludes the paper.

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II. System Model

Consider a Root-Check LDPC-coded MIMO point-to-point transmission system with n_{tx} transmit antennas and n_{rx} receive antennas, where $n_{tx} \geq n_{rx}$. The system encodes a block of $L = \frac{N}{m}$ symbols $\mathbf{s} = [s_1, s_2, \cdots, s_L]^T$ from a constellation $\mathcal{A} = \{a_1, a_2, \cdots, a_C\}$, where $(\cdot)^T$ denotes the transpose, $C = 2^m$ denotes the number of constellation points and m is the number of bits per symbol, with a Root-Check LDPC encoder with rate $\frac{1}{F}$ for each transmit antenna and obtains a block of N encoded symbols $\mathbf{x} = [x_1, x_2, \cdots, x_N]^T$. At each time instant t, the encoded symbols of the n_{tx} antennas are organized into a $n_{tx} \times 1$ vector $\mathbf{x}[t] = [x_1[t], x_2[t], \cdots, x_{n_{tx}}[t]]^T$ and transmitted over a block-fading channel with F independent fading blocks. The received signal is demodulated, matched filtered, sampled and organized in an $n_{rx} \times 1$ vector $\mathbf{r}[t] = [r_1[t], r_2[t], \cdots, r_{n_{rx}}[t]]^T$ with sufficient statistics for detection which is described by

$$\mathbf{r}[t] = \sum_{k=1}^{n_{rx}} \mathbf{h}_{k,f} \cdot x_k[t] + \mathbf{v}[t] = \mathbf{H}\mathbf{x}[t] + \mathbf{v}[t], \qquad (1)$$

where the $n_{rx} \times 1$ vector $\mathbf{v}[t]$ is a zero mean complex circular Gaussian noise with covariance matrix $E\left[\mathbf{v}[t]\mathbf{v}^H[t]\right] = \sigma_\mathbf{v}^2 \mathbf{I}$, where $E[\cdot]$ stands for the expected value, $(\cdot)^H$ denotes the Hermitian operator, $\sigma_\mathbf{v}^2$ is the noise variance, $\mathbf{I}$ is the identity matrix, $t = \{1, 2, \cdots, \frac{L}{n_{tx}}\}$ is the time index and $f = \{1, 2, \cdots, F\}$ is the index corresponding to the fading instants. Moreover, t and f are related by $f = \lceil F \cdot n_{rx} \cdot \frac{t}{L} \rceil$, where $\lceil \cdot \rceil$ is a ceiling function. In the case of fast fading we assume that each received symbol will experience a distinct fading coefficient, which means $F = L$. The uncoded symbol vector $\mathbf{s}$ has a covariance matrix $E\left[\mathbf{s}\mathbf{s}^H\right] = \sigma_\mathbf{s}^2 \mathbf{I}$, where $\sigma_\mathbf{s}^2$ is the signal power. The model (1) is used to represent the data transmission, where each block of symbols is associated with a fading coefficient. For a given block, the encoded symbol vector $\mathbf{x}$ is obtained by mapping $\mathbf{s}$ into coded bits and forming the vector $\mathbf{x} = [x_0, \cdots, x_j, \cdots, x_{n_{tx} \cdot m - 1}]^T$. The elements $\mathbf{h}_{n_{rx}, n_{tx}}$ of the $n_{rx} \times n_{tx}$ channel matrix $\mathbf{H}$ represent the complex channel gains from the n_{tx}-th transmit antenna to the n_{rx}-th receive antenna. In our paper, we define the signal-to-noise ratio (SNR) as $\text{SNR} = n_{tx} \cdot \frac{E_s}{R \cdot m \cdot N_0}$. An IDD scheme with a soft MIMO detector and LDPC decoding is used to assess the performance of the system. The soft MIMO detector incorporates extrinsic information provided by the LDPC decoder, and the LDPC decoder incorporates soft information provided by the MIMO detector. We call inner iterations the iterations done by the LDPC decoder, and outer iterations those between the decoder and the detector. In addition, in the decoder a novel scheduling method is used for block-fading channels. The proposed scheduling method combines the benefits of the Layered Belief Propagation (LBP) and the Residual Belief Propagation (RBP) [31] algorithms as will be discussed in Section IV. In the IDD scheme, for the j-th code bit x_j of the transmitted vector $\mathbf{x}$ of each antenna, the extrinsic LLR of the estimated bit of the soft MIMO detector is given by

$$l_E[x_j] = l_C[x_j] - l_A[x_j], \qquad (2)$$

where $l_A[x_j]$ is the *a priori* LLR ($l_A[x_j] = 0$ at the first iteration) of the bit x_j computed by the LDPC decoder in the previous iteration ($l_C[x_j] = 0$ at the first iteration) and $l_C[x_j]$ is the *a posteriori* LLR of the bit x_j computed by the soft MIMO detector. We have adopted in this work linear minimum mean square error receive filters with SIC (MMSE-SIC) [18] receivers even though other approaches to computing receive filters are possible [**?**], [34].

III. Proposed LLR Compensation Scheme

We have investigated the performance of Root-Check LDPC codes in MIMO systems with IDD schemes using MMSE-SIC [18]. In particular, we have studied numerous scenarios where Root-Check LDPC codes lose in terms of bit error rate (BER) to the standard LDPC codes at high SNR. We have observed in simulations that the parity-check nodes from Root-Check LDPC codes do not converge. In particular, with Root-Check LDPC codes the LLRs exchanged between the decoder and the detector degrade the overall performance. To circumvent this, we have adopted the use of controlled doping via high-order Root-Checks in graph codes [35]. In our studies, the LLR magnitude of the parity check nodes connected to the deepest fading always presented lower magnitude level than the other parity check nodes. In contrast, for the case of standard LDPC codes this magnitude difference has not been verified. For the case of Root-Check LDPC codes, the difference in LLR magnitude (gaps) at the decoder output for the parity check nodes has lead us to devise an LLR compensation strategy to address these gaps. The gaps and the lower LLR magnitude for the parity check nodes place the LLR values close to the region associated with the non-reliable decision. In addition, in an IDD process such values can cause the detector to wrongly de-map the received symbols. Therefore, we have devised an LLR processing strategy for IDD schemes in block-fading channels (LLR-PS-BF). First, the *a posteriori* LLRs generated by the soft MIMO detector are organized in the N-dimensional vector $\mathbf{l}_C = [l_C[x_1], l_C[x_2], \cdots, l_C[x_N]]$. Assuming that the systematic symbols for a Root-Check LDPC code always converge to an LLR magnitude greater than zero, we proceed to the following calculations:

$$\alpha = \max_{1 \leq j \leq K}(|l_C[x_j]|) \text{ and } \beta = \max_{K+1 \leq j \leq N}(|l_C[x_j]|), \qquad (3)$$

where K is the length of the systematic bits. We then compute $\gamma = \alpha - \beta$, where $\gamma > 0$ due to the fact that the systematic nodes for a Root-Check LDPC code always converge to an LLR magnitude greater than zero. Once γ is computed, we can generate a vector $\mathbf{l}_{PA}$ described by

$$l_{PA}[j] = |l_C[x_j]|, \; j = K+1, \cdots, N, \qquad (4)$$

which represents the positive magnitude of all parity-check nodes. We then calculate the vector $\mathbf{l}_{PS}$ as described by

$$l_{PS}[j] = \text{sign}\left[l_C[x_j]\right], \; j = K+1, \cdots, N, \qquad (5)$$

which corresponds to the signals of all parity-check nodes. Furthermore, we obtain the vector $\mathbf{l}_{PT}$ as

$$\mathbf{l}_{PT} = (\mathbf{l}_{PA} + \gamma) \odot \mathbf{l}_{PS}, \qquad (6)$$

where $\odot$ is the Hadamard product. The final step in the proposed LLR-PS-BF algorithm is to generate the *a posteriori* LLRs to be used by the IDD scheme. Therefore, the optimized vector of the *a posteriori* LLRs is given by

$$\tilde{\mathbf{l}}_C = [l_C[x_1], \cdots, l_C[x_K], l_{PT}[x_{K+1}], \cdots, l_{PT}[x_N]]. \quad (7)$$

The aim of calculating $\mathbf{l}_{PT}$ is to ensure that the LLRs of the parity-check nodes do not get close to the region associated with non-reliable decisions. As a consequence, the LLRs fed back to the detector will not deteriorate the performance of the de-mapping operation. In the Appendix, we detail how the proposed LLR-PS-BF compensation scheme works.

We have carried out a preliminary study [36] where the LLR compensation is a particular case of the one presented in this work. In order to obtain the LLR-PS-BF scheme presented in [36] we should set some different values. In particular, $\beta = 0$ and $\mathbf{l}_{PA} = 0$ will lead to the same results presented in [36]. It must be noted that every time the soft MIMO detector generates an *a posteriori* LLR $\mathbf{l}_C$ the LLR-PS-BF compensation scheme must be applied when Root-Check LDPC codes are used. The main purpose of applying the proposed LLR-PS-BF compensation scheme is to enable convergence of the LLRs to suitable values and preserve useful information in the iterations. Therefore, the LLRs exchanged between the decoder and the detector will benefit from this operation. Consequently, a better performance in terms of BER will result.

IV. PROPOSED IDD SCHEME BASED ON SCHEDULING

The structure of the proposed LLR-PS-BF with the IDD scheme is described in terms of iterations. In this work, we only consider the use of SIC which outperforms the parallel interference cancellation (PIC) detection scheme. When using SIC, the soft estimates of $\mathbf{r}[t]$ are used to calculate the LLRs of their constituent bits. We assume that the k-th layer MMSE filter output $u_k[t]$ is Gaussian and the soft output of the SISO detector for the k-th layer is written as [24]

$$u_k[t] = \mathbf{V}_k x_k[t] + \epsilon_k[t], \quad (8)$$

where $\mathbf{V}_k$ is a scalar variable which is equal to the k-th layer's signal amplitude and $\epsilon_k[t]$ is a Gaussian random variable with variance $\sigma^2_{\epsilon_k}$, since

$$\mathbf{V}_k[t] = E\left[x_k^*[t] u_k[t]\right] \quad (9)$$

and

$$\sigma^2_{\epsilon_k} = E\left[|u_k[t] - \mathbf{V}_k[t] x_k[t]|^2\right]. \quad (10)$$

The estimates of $\hat{\mathbf{V}}_k[t]$ and $\hat{\sigma}^2_{\epsilon_k}$ can be obtained by time averages of the corresponding samples over the transmitted packet. After the first iteration, the MMSE soft cancellation performs SIC by subtracting the soft replica of Multiple Access Interference (MAI) components from the received vector as

$$\hat{\mathbf{r}}_k[t] = \mathbf{r}[t] - \sum_{j=1}^{k-1} \mathbf{h}_j \hat{\mathbf{x}}_j[t]. \quad (11)$$

The soft estimation of the k-th layer is obtained as $u_k[t] = \boldsymbol{\omega}_k^H \hat{\mathbf{r}}_k[t]$, where the $n_{rx} \times 1$ MMSE filter vector is given

by $\boldsymbol{\omega}_k = \left(\mathbf{H}_k \mathbf{H}_k^H \sigma_v^2 \mathbf{I}\right)^{-1} \mathbf{h}_k$ and $\mathbf{h}_k$ denotes the matrix obtained by taking the columns $k, k+1, \cdots, n_{rx}$ of $\mathbf{H}$ and $\hat{\mathbf{r}}[t]$ is the received vector after the cancellation of previously detected $k-1$ layers. where the soft output of the filter is also assumed Gaussian. The first and the second-order statistics of the coded symbols $\hat{\mathbf{x}}[t]$ are also estimated via time averages of (9) and (10). We have developed our proposed IDD scheme by applying scheduling methods for decoding LDPC codes. Specifically, we have applied the Layered Belief Propagation (LBP) scheduling method as described in [31] to evaluate the overall performance versus the standard BP. We have observed a performance loss for the scheduling methods in the error floor region (high SNR region). To overcome this problem we have applied our proposed LLR-PS-BF scheme. As a result, the LBP has outperformed the standard BP as expected.

Based on the result obtained by LBP we have applied the Residual Belief Propagation (RBP) and the Node-Wise Belief Propagation (NWBP) to assess the overall performance. However, RBP and NWBP are outperformed by the standard BP. The reason is that the block-fading channel imposes some constraints in terms of LLRs received by the variable nodes. For practical purposes, let us assume a block-fading channel with $F = 2$ fadings and that half of the variable nodes have no channel information as the example given by Boutros [4, pp. 4, Fig. 10]. Furthermore, the idea of RBP and NWBP is to prioritize the update of a specific message or check node with the largest residual and then keep doing this in an iterative way. However, as soon as the block fading channel affects the messages sent by $\frac{N}{2}$ variable nodes to the check nodes, prioritizing such messages or nodes with no channel information leads to a performance degradation. Moreover, Gong and et.al. in [37] have reported that all dynamic scheduling strategies only concentrate on the largest residual when performing new residual computations. Nonetheless, the existence of smaller residuals do not mean the algorithm in the sub-graph of the Tanner graph has converged.

The NWBP strategy has certain advantages over RBP because it reinforces the root connections of a check node. It updates and propagates simultaneously all the check-to-variable messages $\mathbb{M}_{c_i \to v_b}$ that correspond to the same check node c_i as

$$\mathbb{M}_{c_i \to v_b} \; : \; \forall v_b \in \mathcal{N}(c_i), \quad (12)$$

where $\forall v_b \in \mathcal{N}(c_i)$ refers to all variable nodes v_b that belong to the set of check nodes $\mathcal{N}(c_i)$ that are connected to v_b. Then, it proceeds to calculate all the variable-to-check messages $\mathbb{M}_{v_b \to c_a}$ that correspond to the same variable node v_b as

$$\mathbb{M}_{v_b \to c_a} \; : \; \forall c_a \in \mathcal{N}(v_b) \setminus c_i, \quad (13)$$

where $\mathcal{N}(v_b) \setminus c_i$ is the set of variable nodes v_b that are connected to c_a except c_i. As a result, NWBP will individually treat per iteration the check node c_i with the largest residual, which in the case of a block-fading channel is not enough to gather all information required by the root connections. However, we can address this if at the beginning of each decoding iteration we calculate for each check node the metric given by

$$\varphi_{c_i} = \max \; r\left(\mathbb{M}_{c_i \to v_b}\right) \; : \; \forall v_b \in \mathcal{N}(c_i), \quad (14)$$

Following the example graph given in [4, pp. 4, Fig. 10], we consider that the first half of the variable nodes are under fading with $h_1 = 1$ and the second half has no channel information, i.e., $h_2 = 0$, and $M_{CH} = \frac{N}{2}$ check nodes. Therefore, after 20 inner iterations we can have the following values:

$$\varphi_{c_{1,\cdots,\frac{M_{CH}}{2}}} = 0,$$
$$\varphi_{c_{\frac{M_{CH}+1}{2},\cdots,M_{CH}}} \geq 1. \tag{15}$$

Then, we can solve the block-fading problem by generating a queue Q of all φ_{c_i} in a descending order from the largest to the smallest to obtain the corresponding indexes of the check nodes as

$$Q = [i_1, i_{M_{CH}}] \therefore \{\varphi_{c_a} \in \mathbb{N} : \varphi_{c_{i_1}} > \varphi_{c_a} > \varphi_{c_{i_{M_{CH}}}}\}. \tag{16}$$

Therefore, in a pre-defined order based on the queue Q, an iteration consists of the sequential update of all variable to check messages $\mathbb{M}_{v \to c}$ as well as all the check to variable messages $\mathbb{M}_{c \to v}$. This approach is called Residual LBP (RLBP).

Therefore, if we adopt a strategy like RLBP it will lead to a prioritization, at each iteration, of the largest to the smallest check-to-variable residual being updated and propagated. As a result, we still have a performance degradation compared to the standard LBP. It turns out that, as discussed in [37], the smaller residuals of the sub-graph on the Tanner graph do not necessarily indicate convergence. We have then devised a dynamic scheduling strategy which overcomes the problems caused by a block-fading channel. The proposed scheduling strategy, called Residual Ordered LBP (ROLBP), alternates at each decoding iteration between two different strategies. For every other iteration the ROLBP strategy requires the computation of the check nodes metric (14) and ordering (16) while RLBP requires this for every iteration. The ROLBP technique can be described by the following calculations:

First, initialize all $\mathbb{M}_{c \to v} = 0$ and all $\mathbb{M}_{v_j \to c_i} = C_{v_j}$, where C_{v_j} is the channel information LLR of the variable node v_j. Then, compute all the residuals of the messages as

$$r(\mathbb{M}_{c \to v}), \quad \text{generate } Q, \tag{17}$$

where Q is the list of residuals in descending order. We then proceed to the calculation of Ξ as

$$\Xi = \begin{cases} Q(1), \cdots, Q(M_{CH}), & \text{if the iteration is odd} \\ 1, \cdots, M_{CH}, & \text{if the iteration is even} \end{cases}. \tag{18}$$

For each $i \in \Xi(1), \cdots, \Xi(M_{CH})$ calculate:

$$\forall c_i \in \mathcal{N}(v_j) \to \text{generate and propagate } \mathbb{M}_{v_k \to c_i} \tag{19}$$

$$\forall v_k \in \mathcal{N}(c_i) \to \text{generate and propagate } \mathbb{M}_{c_i \to v_k} \tag{20}$$

$$\text{Update and compute} \to \text{All } r(\mathbb{M}_{c \to v}) \text{ regenerate } Q \tag{21}$$

Finally, if the decoding stopping rule is not satisfied then recalculate all the equations from (17) up to (21). Again returning to the example given in [4, pp. 4, Fig. 10], the values of φ_{c_i} for ROLBP throughout the iterations are:

$$\varphi_{c_{1,\cdots,M_{CH}}} \geq 0, \tag{22}$$

which results in a scheduling method that decreases the prioritization as seen in (15). By adopting this strategy we ensure that ROLBP outperforms both the standard BP and RLBP algorithms. The reason is that we give enough information to the root connections and avoid the values for φ_{c_i} as in (15) which cause a degradation in performance of Root-Check based LDPC codes. The pseudo-code is described in Algorithm 1.

Algorithm 1 Proposed LLR-SP-BF Scheduling IDD Scheme

1. **Require**: $\mathbf{r}[t]$, $\mathbf{H}$, σ_v^2, $\mathbf{l}_A$ *a priori* information, TI.

2. **for** $\mathbf{l}_0 = 1 \to TI$ {Turbo Iteration} **do**

3.   Calculate MMSE filter $\boldsymbol{w}_k = \left(\mathbf{H}_{k,f}\mathbf{H}_{k,f}^H + \frac{\sigma_v^2}{\sigma_s^2}\mathbf{I}\right)^{-1}\mathbf{h}_{k,f}$

4.   **Detection Scheme - SIC**
    $\hat{\mathbf{r}}_k[t] = \text{Perform} - \text{SIC}(\mathbf{r}[t], \mathbf{H}, \sigma_v^2, \mathbf{w}_k)$, perform the MMSE SIC detection scheme for each k-th layer.

5.   **Obtain The Extrinsic Bit LLR**

6.   **First**: Determine $\sigma_{\epsilon_k}^2$ based on the best channel realization by means of calculating: $\delta_f = \underset{1 \leq f \leq F}{\arg\max} |\det(\mathbf{h}_{k,f})|$, where δ_f is the index of f which $|\det(\mathbf{h}_{k,f})|$ has the maximum value.

7.   Therefore, $\mathbf{V}_k[t]$ and $\sigma_{\epsilon_k}^2$ must be calculated where the fading happens at index δ_f. This is unique for block-fading channels, other types of channels do not require these additional steps. Then, the extrinsic LLR is obtained as:
    $l_E[x_j] = l_C[x_j] - l_A[x_j]$

8.   **LDPC Decoding**

9.   **if** Using Scheduling **then**

10.   Do the decoding with equations from (17) up to (21);

11.   **else**

12.   Decode using standard belief propagation;

13.   **end if**
14.   Obtain the a posteriori LLR $\mathbf{l}_C$ of the soft MIMO detector.

15.   **if** LDPC = RootCheck **then**

16.   **Apply the proposed LLR-PS-BF scheme equations (3) up to (7)**

17.   Calculate the extrinsic information $l_E[x_j]$ based on $l_C[x_j]$ to be sent to the decoder.

18.   **else**

19.   Calculate the extrinsic information $l_E[x_j]$ based on $l_C[x_j]$ to be sent to the detector.

20.   **end if**

21. **end for**

The computational complexity of the decoding algorithms depends on the variable node degree d_v and the check node degree d_c. The number of edges in the Tanner graph is $\epsilon = d_v N_{VN} = d_c N_{CN}$, where N_{VN} is the number of variable nodes and N_{CN} is the number of check nodes. In terms of complex multiplications, one ϵ update of BP corresponds to $d_c N_{CN}/4$ operations, $d_c N_{CN}(1 + (d_v - 1)(d_c - 1))/4$ operations for NWBP, $d_c N_{CN}/4$ operations for LBP, $d_c N_{CN}/2$ operations for RLBP, and $1.5 d_c N_{CN}/2$ operations for ROLBP.

The most complex decoding algorithm is NWBP, which is followed by RLBP, the proposed ROLBP algorithm, BP and LBP.

V. SIMULATIONS

The bit error rate (BER) performance of the proposed LLR-PS-BF with a SIC IDD scheme is compared with Root-Check LDPC codes and LDPC codes using a different number of antennas. It must be remarked that our proposed LLR-PS-BF scheme can be applied to other types of IDD schemes [27]. Both LDPC codes used in the simulations have block length $N = 1024$ for all rates. The maximum number of inner iterations was set to 20 and a maximum of 5 outer iterations were used. The Root-Check LDPC codes require less iterations than standard LDPC codes for convergence of the decoder (inner iterations) [5], [7]. Using Root-Check LDPC codes in IDD schemes reduces the need for inner iterations, whereas the number of outer iterations remains at five. We have used codes with rates $1/2$ and $1/4$ for the sake of transmission efficiency and because they can be of practical relevance. Rates lower than $1/4$ are not attractive in terms of efficiency. We considered the proposed algorithms and all their counterparts in the independent and identically-distributed (i.i.d) block fading channel model. The coefficients are taken from complex circular Gaussian random variables with zero mean and unit variance. The modulation used is QPSK. The SNR at the receiver is calculated as $SNR_{RCV} = \frac{1}{2 \cdot \sigma_{\epsilon_k}^2}$ which is based on equation (10).
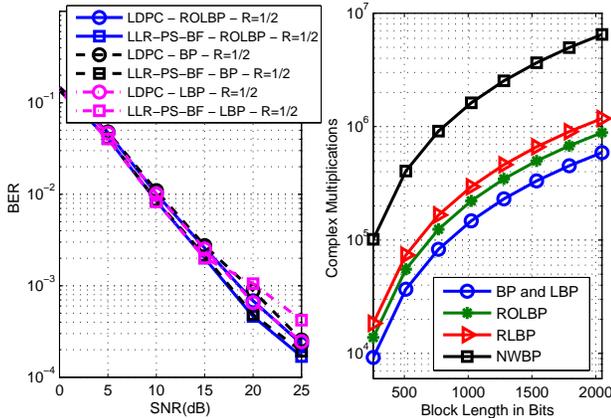

Figure 1. BER performance of LLR-PS-BF with Root-Check LDPC versus LDPC code both codes are rate $R = \frac{1}{2}$ and block length $N = 1024$. The decoding strategies considered are BP, LBP and ROLBP and the computational complexity is expressed in complex multiplications. A point-to-point MIMO system with 2×2 configuration in a block-fading channel with $F = 2$, QPSK modulation, 5 outer iterations and 20 inner iterations is used.

In Fig. 1 the results for a point-to-point 2×2 MIMO system, block-fading channel with $F = 2$ fadings and code rate $R = \frac{1}{2}$ are presented along with an illustration of the computational complexity of the decoding algorithms in complex multiplications. The proposed LLR-PS-BF scheme with Root-Check LDPC codes using the ROLBP strategy outperforms BP by about 1 dB in terms of SNR for the same BER performance. When we compared the LLR-PS-BF with

a Root-Check LDPC scheme with both using ROLBP, LLR-PS-BF has a gain of up to 2 dB in terms of SNR for the same BER performance. The gain of the ROLBP algorithm alone is also up to 2 dB in SNR for the same BER performance. The complexity of the ROLBP algorithm is higher than that of the standard BP and the LBP algorithms but lower than the RLPB and NWBP algorithms.
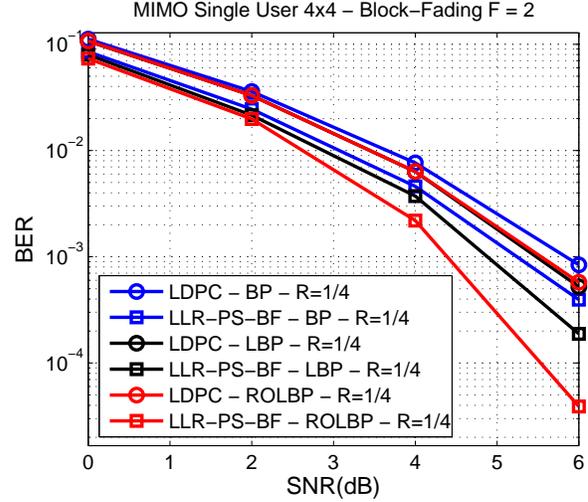

Figure 2. BER performance of LLR-PS-BF with Root-Check LDPC versus LDPC code. The codes have rate $R = \frac{1}{4}$ and block length $N = 1024$. The decoding strategies considered are BP, LBP and ROLBP. A point-to-point MIMO system in a 4×4 configuration in a block-fading channel with $F = 2$, QPSK modulation, 5 outer iterations and 20 inner iterations is employed.

Fig. 2 presents the results for a point-to-point 4×4 MIMO system, block-fading channel with $F = 2$ fadings and code rate $R = \frac{1}{4}$. On average, all Root-Check based codes using LLR-PS-BF compensation outperform the standard LDPC codes for all decoding strategies. In addition, ROLBP outperforms BP by about 1.25 dB. ROLBP with LLR-PS-BF outperforms standard LDPC codes with BP by up to 1.5 dB in terms of SNR for the same BER performance.

Fig. 3 shows the outcomes for a point-to-point 2×2 MIMO system, fast-fading channel and code rate $R = \frac{1}{2}$. As the BER performance for standard LDPC codes using different decoding strategies has lead to the same performance, we have plotted only one curve to represent BP, LBP and ROLBP. The gains of the proposed LLR-PS-BF IDD scheme using ROLBP are about 1 dB with respect to standard LDPC codes. Furthermore, at low SNR the LLR-PS-BF scheme with ROLBP has outperformed LBP by about 1.5 dB in terms of SNR. The scenarios with $F = L/2$ or $F = L/4$ cases can be addressed by using Root-Check LDPC codes with $F = 2$ and the proposed LLR compensation scheme. In particular, the design of Root-Check LDPC codes for $F = L/2$, $F = L/4$ or other F is unnecessary as the Root-Check LDPC code with $F = 2$ is able to capture the advantages for a wide range of F.

VI. CONCLUSION

In this paper, we have presented an IDD scheme for MIMO systems in block-fading channels. Furthermore, we have proposed the ROLBP scheduling algorithm for the proposed
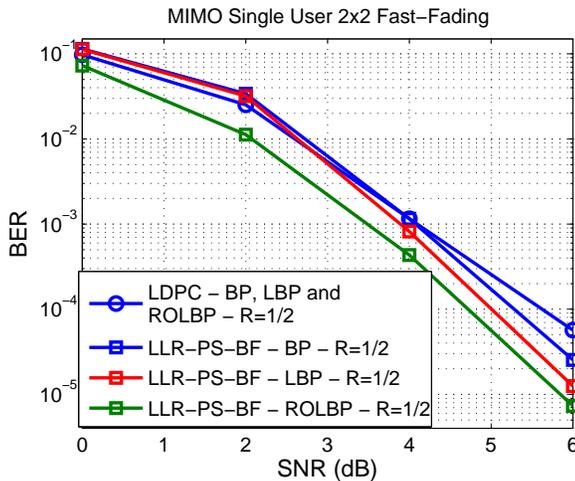
Figure 3. BER performance of LLR-PS-BF with Root-Check LDPC versus LDPC code. The codes have rate $R = \frac{1}{2}$ and block length $N = 1024$. The decoding strategies considered are BP, LBP and ROLBP. A point-to-point MIMO system with a 2×2 configuration in a fast-fading channel is considered, QPSK modulation, 5 outer iterations and 20 inner iterations is used.

to the region associated with non-reliable decisions while the compensated version $\tilde{\mathbf{l}}_C$ places the parity-check LLRs farther from such region.
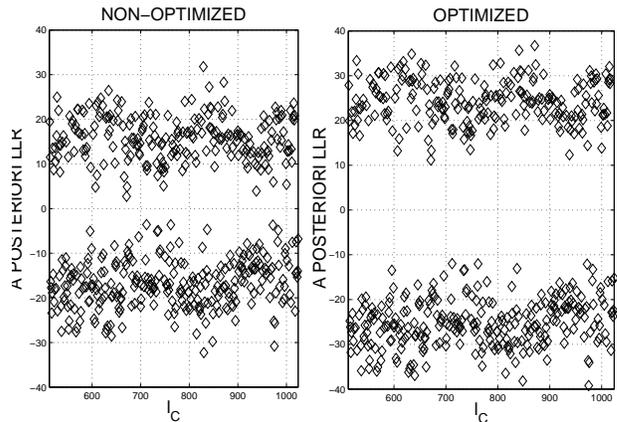

Figure 4. An example of the optimization of $\mathbf{l}_C$ made by the proposed LLR-PS-BF compensation scheme. For the case of length $N = 1024$, $K = 512$ and code rate $R = \frac{1}{2}$.

IDD scheme and studied different scheduling strategies. The proposed algorithms have resulted in a gain of up to 2 dB for a point-to-point 2×2 MIMO system and up to 1.5 dB for a 4×4 MIMO system in a block-fading channel with $F = 2$. For the case of a 2×2 MIMO system over fast-fading the proposed LLR-SP-BF IDD scheme has obtained a gain of up to 1.5 dB. The proposed algorithms are suitable for MIMO systems with users that experience high throughput rate and slow changes in the propagation channel. In such scenarios, the symbol period is much smaller than the coherence time.

APPENDIX
LLR-PS-BF MATHEMATICAL ANALYSIS

Mathematically speaking, we can interpret the LLR-PS-BF compensation scheme as a modification made by two functions $f[\mathbf{l}_C]$ and $g[\mathbf{l}_C]$. Given $\mathbf{l}_C$, an input vector of length N, we consider $K = \frac{N}{2}$ which is true for code rate $R = \frac{1}{2}$. First, the aim of $f[\mathbf{l}_C]$ is to obtain a real value $\Delta \in \Re^+$. Therefore, we have

$$\Delta = f[\mathbf{l}_C] = \left[\max(l_C) \quad , l_C[1], \cdots, l_C[K] \right].$$

Finally, the discrete signal $\mathbf{l}_C$ is processed by $g[\mathbf{l}_C]$ to generate the compensated version of $\mathbf{l}_C$ called $\tilde{\mathbf{l}}_C$. Therefore, $g[\mathbf{l}_C]$ is defined as

$$g[\mathbf{l}_C] = \begin{cases} l_C & , l_C[1], \cdots, l_C[K] \\ l_C + \frac{l_C}{|l_C|} \cdot \Delta & , l_C[K+1], \cdots, l_C[N] \end{cases},$$

where $\frac{l_C}{|l_C|}$ is the sign of l_C and $\tilde{\mathbf{l}}_C \Leftarrow g[\mathbf{l}_C]$. To further understand how the functions $f[\mathbf{l}_C]$ and $g[\mathbf{l}_C]$ act in the input vector $\mathbf{l}_C$ we provide an example in Fig. 4 for a vector $\mathbf{l}_C$ with $N = 1024$ and $K = 512$. We only show the parity-check LLRs ($K > 512$). On the left had side of Fig. 4 we have the non-optimized version of $\mathbf{l}_C$ while on the right hand side we depict the compensated $\tilde{\mathbf{l}}_C$. As we can see from Fig. 4, for the non-optimized vector $\mathbf{l}_C$ some of the parity-check LLRs tend

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