Robust MMSE Precoding Based on Switched Relaying and Side Information for Multiuser MIMO Relay Systems

Yunlong Cai, Rodrigo C. de Lamare, Lie-Liang Yang, and Minjian Zhao

Abstract—This study proposes a novel precoding scheme for multiuser multiple-input multiple-output (MIMO) relay systems in the presence of imperfect channel state information (CSI). The base station (BS) and the MIMO relay station (RS) are both equipped with the same codebook of unitary matrices. According to each element of the codebook, we create a *latent* precoding matrix pair, namely a BS precoding matrix and an RS precoding matrix. The RS precoding matrix is formed by multiplying the appropriate unitary matrix from the codebook by a power scaling factor. Based on the given CSI and a block of transmit symbols, the optimum precoding matrix pair, within the class of all possible latent precoding matrix pairs derived from the various unitary matrices, is selected by a suitable selection mechanism for transmission, which is designed to minimize the squared Euclidean distance between the pre-estimated received vector and the true transmit symbol vector. We develop a minimum mean square error (MMSE) design algorithm for the construction of the latent precoding matrix pairs. In the proposed scheme, rather than sending the complete processing matrix, only the index of the unitary matrix and its power scaling factor are sent by the BS to the RS. This significantly reduces the overhead. Simulation results show that compared to other recently reported precoding algorithms the proposed precoding scheme is capable of providing improved robustness against the effects of CSI estimation errors and multiuser interference.

Index Terms—Robust precoding, MMSE, switched relaying, multiuser MIMO relay.

I. INTRODUCTION

Optimum non-regenerative relay station (RS) precoding matrices for single user MIMO relay systems have been well studied in the literature [2]-[7]. Guan and Luo employed the

constrained minimum mean squared error (MMSE) criterion to jointly design the RS precoding matrix and the receive filtering matrix at the destination [2]. In [3], Rong and Gao generalized the optimum RS precoding matrix by including the direct link. In the case of imperfect channel state information (CSI), Xing *et al.* [4] proposed a joint robust design algorithm for the linear RS precoding matrix and the receive filtering matrix. By taking base station (BS) precoding into account some researchers investigated the joint design algorithm of the BS and RS precoding matrices [5]-[7]. In particular, Zhang *et al.* [5] proposed a joint iterative optimization algorithm to design the BS precoding matrix, RS precoding matrix and receive filtering matrix. Tseng and Wu designed an iterative algorithm by minimizing the MMSE upper bound, instead of the original MMSE in [6], [7].

Of recent, the study of precoding techniques in multiuser MIMO relay systems is becoming more and more of importance [8]-[13]. Zhang *et al.* [8] minimized the weighted sum-power consumption under the minimum quality-of-service (QoS) constraints by jointly optimizing linear beamforming and power control at the BS and RS. Chae *et al.* [9] combined the BS nonlinear precoding matrix with a linear processing algorithm at the RS. They also relied on the fact that the CSI can be obtained via feedback or channel reciprocity. By using feedback quantized CSI, while assuming perfect CSI at the destination, two robust linear precoding schemes at the RS based on zero forcing (ZF) and the MMSE criteria were proposed for downlink multiuser MIMO relay systems to handle quantization errors, [10]. However, the authors did not consider BS precoding in their work. In [11], Xu *et al.* proposed singular value decomposition (SVD) based BS and RS precoding algorithms under the assumption of perfectly known CSI, where the BS precoding matrix is designed based on the SVD of the first time slot channel, and the ZF RS precoding matrix is obtained based on the second time slot channel. In the presence of imperfect CSI, the studies in [12] and [13] proposed extensions of the method reported in [11], where robust precoding algorithms were developed to deal with CSI quantization and estimation errors, respectively. Notice that the BS and RS precoding matrices in [11]-[13] are not designed to optimize a cost function regarding an overall system performance, therefore they may suffer from performance degradation.

In this work, we propose a novel precoding scheme based on switched relaying (SR) processing for multiuser MIMO relay systems. In practice, in cellular systems it is preferable

to implement most of the signal processing operations at the BS rather than at the RS, since the BS is more powerful and the RS is expected to have a simple structure and low energy consumption [16]-[18]. In this regard, the proposed technique is implemented at the BS. The BS and the MIMO RS are both equipped with the same codebook of unitary matrices. According to each element of the codebook, we create a *latent* precoding matrix pair, namely a BS precoding matrix and an RS precoding matrix. The RS precoding matrix is formed by multiplying the appropriate unitary matrix from the codebook by a power scaling factor. We develop a design algorithm for computing the BS precoding matrix and RS power scaling factor in order to construct the latent precoding matrix pairs. Based on the given CSI and a block of transmitted symbols, the optimum pair within the group of latent precoding matrix pairs is chosen by a suitable selection mechanism for transmission, which is designed to minimize the squared Euclidean distance between the noiseless pre-estimated received vector and the true transmit symbol vector. Prior to payload transmission, the BS transmits the index of the unitary matrix and the RS power scaling factor information corresponding to the optimum latent precoding matrix pair to the RS through signalling channels [19]-[22], where the optimum RS precoding matrix is formed[1]. In addition, we propose a method based on the most frequently selected candidates (MSC) for the codebook design. An analysis of the proposed algorithm in terms of computational complexity, probability of error and requirement of side information is carried out. Simulation results demonstrate that the proposed SR-based precoding scheme is capable of providing improved robustness against the effects of CSI estimation errors and interference compared to the existing precoding algorithms.

This paper is organized as follows. Section II briefly describes the system model. The proposed SR-based transmission scheme is introduced in Section III in terms of the latent precoding matrix design algorithm, the selection mechanism of the optimization latent precoding matrix pair and the codebook design. An analysis of the proposed algorithm is conducted in Section IV. Simulation results are presented in Section V and finally conclusions are drawn in Section VI.

In this paper, the superscripts $(.)^T$, $(.)^*$, $(.)^{-1}$, and $(.)^H$ denote transpose, element-wise conjugate, matrix inverse, and Hermitian transpose, respectively. Bold symbols denote matrices or vectors. The symbols $E[.]$, $|.|$, $||.||$, $Tr\{.\}$ and $\mathbf{I}$ represent the expectation operator, the norm of a scalar, the norm of a vector, the trace operation of a square matrix and an identity matrix of appropriate dimension, respectively. The operation $(x, :)$ denotes taking the x-th row vector from a matrix. The operation $(:, y)$ denotes taking the y-th column vector from a matrix. $\Re[.]$ selects the real part. $[y]^+ = \max[0, y]$. $||.||_F$ denotes the matrix Frobenius norm. The factor $\otimes$ denotes the operation of the Kronecker product.

II. SYSTEM MODEL

We consider the downlink of a multiuser MIMO cellular system consisting of one BS, one RS, and K mobile stations (MSs). In practice, this model is employed for the relay architectures of 3GPP LTE-Advanced [23]. We consider a case in which the BS and RS are equipped with N_t and N_r antennas, respectively, and the MS is equipped with a single antenna, where $K \leq \min\{N_t, N_r\}$. In addition, the BS and the RS are equipped with a finite codebook of unitary matrices, i.e. $\mathcal{T} = \{\mathbf{T}_1, \mathbf{T}_2, \ldots, \mathbf{T}_{2^B}\}$, 2^B is the codebook size, and consider half time-division duplex (TDD) non-regenerative relaying [24]. In the first phase, the received vector at the RS is given by

$$\mathbf{r}_R^{(l)} = \mathbf{H}_1 \mathbf{P}_l \mathbf{b} + \mathbf{n}_1. \tag{1}$$

In this expression, the elements of transmit symbol vector $\mathbf{b} = [b_1, \ldots, b_K]^T \in \mathcal{C}^{K \times 1}$ are independent and identically distributed (i.i.d.), $E[|b_k|^2] = 1$, $k \in \{1, \ldots, K\}$, and $\mathbf{P}_l \in \mathcal{C}^{N_t \times K}$ denotes the BS precoding matrix corresponding to the l-th latent precoding matrix pair, where $\forall l \in \{1, \ldots, 2^B\}$. The matrix $\mathbf{H}_1 \in \mathcal{C}^{N_r \times N_t}$ is the channel matrix between the BS and the RS, whose elements are i.i.d. complex circular Gaussian variables with zero mean and unit variance, which we indicate by the standard notation $\mathcal{CN}(0, 1)$, and $\mathbf{n}_1 \in \mathcal{C}^{N_r \times 1}$ is the additive complex Gaussian noise with covariance matrix $E[\mathbf{n}_1 \mathbf{n}_1^H] = \sigma_1^2 \mathbf{I}$, where σ_1^2 denotes the first phase noise variance.

In the second phase, the vector $\mathbf{r}_R^{(l)} \in \mathcal{C}^{N_r \times N_r}$ is operated by the RS precoding matrix $\mathbf{W}_l$ corresponding to the l-th latent precoding matrix pair, where $\mathbf{W}_l$ is formed by multiplying the appropriate unitary matrix from the codebook by a power scaling factor, i.e. $\mathbf{W}_l = \beta_l \mathbf{T}_l$, and $\mathbf{T}_l$ and β_l denote the selected unitary matrix from the codebook and the RS power scaling factor, respectively. The forwarded signal vector from the RS is given by

$$\mathbf{x}_R^{(l)} = \mathbf{W}_l \mathbf{r}_R^{(l)}. \tag{2}$$

The BS and RS power constraints are $E[||\mathbf{P}_l \mathbf{b}||^2] = Tr\{\mathbf{P}_l \mathbf{P}_l^H\} \leq P_t$ and $E[||\mathbf{x}_R^{(l)}||^2] = \beta_l^2 E\left[Tr\{\mathbf{T}_l(\mathbf{H}_1 \mathbf{P}_l \mathbf{P}_l^H \mathbf{H}_1^H + \sigma_1^2 \mathbf{I})\mathbf{T}_l^H\}\right] = \beta_l^2 E\left[Tr\{\mathbf{H}_1 \mathbf{P}_l \mathbf{P}_l^H \mathbf{H}_1^H + \sigma_1^2 \mathbf{I}\}\right] \leq P_r$, respectively. For the second phase, we model the transmission from the RS to the K MSs as a MIMO broadcast channel, we stack all the MSs' received data and obtain the received vector

$$\mathbf{y}^{(l)} = \mathbf{H}_2 \mathbf{x}_R^{(l)} + \mathbf{n}_2 = \mathbf{H}_2 \mathbf{W}_l \mathbf{H}_1 \mathbf{P}_l \mathbf{b} + \mathbf{H}_2 \mathbf{W}_l \mathbf{n}_1 + \mathbf{n}_2, \tag{3}$$

where $\mathbf{H}_2 \in \mathcal{C}^{K \times N_r}$ is the channel matrix between the RS and the MSs, its entries are i.i.d. zero mean complex circular Gaussian variables with unit variance, and $\mathbf{n}_2$ denotes the additive complex Gaussian noise, $E[\mathbf{n}_2 \mathbf{n}_2^H] = \sigma_2^2 \mathbf{I}$, where σ_2^2 denotes the second phase noise variance.

In order to obtain the required CSI, we simplify the two-hop channel estimation problem to two independent MIMO channel estimation procedures. The CSI can be estimated at the BS and RS, respectively, by using a specific channel estimation algorithm [25]-[33]. Since we consider the TDD

mode in this work, the downlink transmit CSI can be obtained due to reciprocity [19]. Note that the RS needs to feed the estimated second phase CSI back to the BS by using signalling channels [19]-[22]. In this work, we assume that the channel varies sufficiently slowly, and the BS can obtain the estimated second phase CSI. To model the statistical distribution of the estimation errors in the channel matrices, the well-known Kronecker model is adopted here for the covariance matrix of the CSI mismatch [4]. In particular, the true (but unknown) channel matrix is expressed as follows,

$$\mathbf{H}_j = \hat{\mathbf{H}}_j + \Delta\mathbf{H}_j, \quad j = 1, 2, \tag{4}$$

where $\hat{\mathbf{H}}_j$ denotes the estimated channel matrices, while $\Delta\mathbf{H}_j$ denotes the corresponding channel estimation error matrix. The latter can be expressed as

$$\Delta\mathbf{H}_j = \mathbf{\Sigma}_j^{\frac{1}{2}} \tilde{\mathbf{H}}_j \mathbf{\Psi}_j^{\frac{1}{2}}, \tag{5}$$

where the elements of $\tilde{\mathbf{H}}_j$ are i.i.d. Gaussian random variables with zero mean and unit variance, and $\mathbf{\Psi}_j$ and $\mathbf{\Sigma}_j$ denote the covariance matrices of the channel seen from the transmitter and receiver, respectively. Furthermore, the matrix $\Delta\mathbf{H}_j$ has the matrix-variate complex circular Gaussian distribution, which can be expressed as $\Delta\mathbf{H}_j \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{\Sigma}_j \otimes \mathbf{\Psi}_j^T\right)$ [4], [34], [35]. By using the estimation algorithm in [25], we have $\mathbf{\Psi}_j = \mathbf{R}_{T,j}$ and $\mathbf{\Sigma}_j = \sigma_{e,j}^2 \mathbf{R}_{R,j}$, where $\mathbf{R}_{T,j}$ and $\mathbf{R}_{R,j}$ are the transmit and receive antennas correlation matrices, respectively, and $\sigma_{e,j}^2$ is the channel estimation error variance. It is reasonable to assume that $\mathbf{\Psi}_j$ and $\mathbf{\Sigma}_j$ are slowly varying and can be known a priori by estimating long term channel statistics. It is important to note that the analysis to be presented in this paper can be applied in exactly the same way without assuming any specific form for the matrices $\mathbf{\Psi}_j$ and $\mathbf{\Sigma}_j$ as long as they are symmetric and full-rank [25], [26].

III. PROPOSED SR-BASED PRECODING SCHEME

As shown in Fig. 1, each unitary matrix in the codebook gives rise to a latent precoding matrix pair, namely a BS precoding matrix and an RS precoding matrix. The RS precoding matrix is formed by multiplying the appropriate codebook unitary matrix by a power scaling factor. Considering the size 2^B of the codebook, we can therefore design 2^B such latent precoding matrix pairs corresponding to each unitary matrix. The proposed SR-based relay transmission scheme works as follows.

- For the given first and second phase channel conditions (CSI), i.e. $\hat{\mathbf{H}}_1$ and $\hat{\mathbf{H}}_2$:
  - The BS computes all the 2^B latent precoding matrix pairs (BS and RS precoding matrices) based on all the entries in the codebook of unitary matrices[2] and the estimated CSI.
  - For each transmission data block:

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[2]We use unitary matrices as the codebook entries because each unitary matrix can generate an equivalent channel matrix $\bar{\mathbf{H}}_l = \mathbf{H}_2\mathbf{T}_l\mathbf{H}_1$ by performing rotations, and 2^B latent precoding matrix pairs produced by these equivalent channel matrices provide different capabilities of interference cancellation and symbol detection. The optimum latent precoding matrix pair can be selected from them.

1) Prior to transmission, the BS precodes the transmit data block with the BS precoding matrix taken from the computed optimum latent precoding matrix pairs.
2) The BS transmits the index of the unitary matrix and the RS power scaling factor information corresponding to the optimum latent precoding matrix pair to the RS through a signalling channel.
3) The RS determines the appropriate power scaling factor and then selects a unitary matrix from the codebook based on the feedforward side information, and form the optimum RS precoding matrix.
4) The block of transmit symbols is transmitted based on the BS precoding matrix and retransmitted by the RS using the RS precoding matrix corresponding to the optimum latent precoding matrix pair.

In this section, we firstly describe the design algorithm for the construction of the latent precoding matrix pairs. Secondly, in order to choose the optimum latent precoding matrix pair before transmission, we propose a selection mechanism based on the squared Euclidean distance. Finally, the design method for the codebook of unitary matrices is described.

A. Design for the Latent Precoding Matrix Pair

Each latent precoding matrix pair is designed based on the equivalent channel matrix $\bar{\mathbf{H}}_l = \mathbf{H}_2\mathbf{T}_l\mathbf{H}_1$ corresponding to the l-th unitary matrix within the codebook $\mathcal{T}$. In order to construct the 2^B latent precoding matrix pairs, we need to compute the BS precoding matrix $\mathbf{P}_l$ and the RS power scaling factor β_l for each latent precoding matrix pair. Remark that the RS precoding matrix $\mathbf{W}_l$ corresponding to the l-th latent precoding matrix pair consists of the l-th unitary matrix $\mathbf{T}_l$ and the RS power scaling factor β_l, namely $\mathbf{W}_l = \beta_l \mathbf{T}_l$. In the following, we propose a design method by minimizing the MSE in the scenario of imperfect CSI.

We aim to design the precoding matrix $\mathbf{P}_l$ and the RS power scaling factor β_l, which minimize the total MSE under the BS and RS power transmit power constraints. The optimization problem is given by

$$\min_{\mathbf{P}_l, \beta_l} \zeta \tag{6}$$

$$\text{s.t.} \quad Tr\{\mathbf{P}_l\mathbf{P}_l^H\} = P_t, \quad \beta_l^2 E\left[Tr\{\mathbf{H}_1\mathbf{P}_l\mathbf{P}_l^H\mathbf{H}_1^H + \sigma_1^2\mathbf{I}\}\right] \leq P_r, \tag{7}$$

where[3]

$$\begin{aligned}\zeta &= E[\|\mathbf{b} - \mathbf{y}^{(l)}\|^2] \\ &= Tr\Big\{E\Big[\mathbf{I} - \beta_l\mathbf{P}_l^H\bar{\mathbf{H}}_l^H - \beta_l\bar{\mathbf{H}}_l\mathbf{P}_l + \beta_l^2\bar{\mathbf{H}}_l\mathbf{P}_l\mathbf{P}_l^H\bar{\mathbf{H}}_l^H \\ &\quad + \frac{Tr\{\mathbf{P}_l\mathbf{P}_l^H\}}{P_t}\beta_l^2(\sigma_1^2\hat{\mathbf{H}}_2\hat{\mathbf{H}}_2^H + \sigma_1^2\Delta\mathbf{H}_2\Delta\mathbf{H}_2^H) \\ &\quad + \sigma_2^2\frac{Tr\{\mathbf{P}_l\mathbf{P}_l^H\}}{P_t}\mathbf{I}\Big]\Big\}. \end{aligned} \tag{8}$$

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[3]In this work, a pre-fixed receiver is used at the destination to reduce complexity. In this case, the design of receiver only depends on the channel and is oblivious to transmitter [36]. Since we focus on the multiuser scenario with a single antenna, the pre-fixed receiver of each user should be a scalar and the virtual multi-antenna receive filtering matrix is an identity matrix [36]. As a result, the attenuation and the phase shift from the BS to the mobile station can be pre-equalized by the proposed BS and RS precoding matrices.
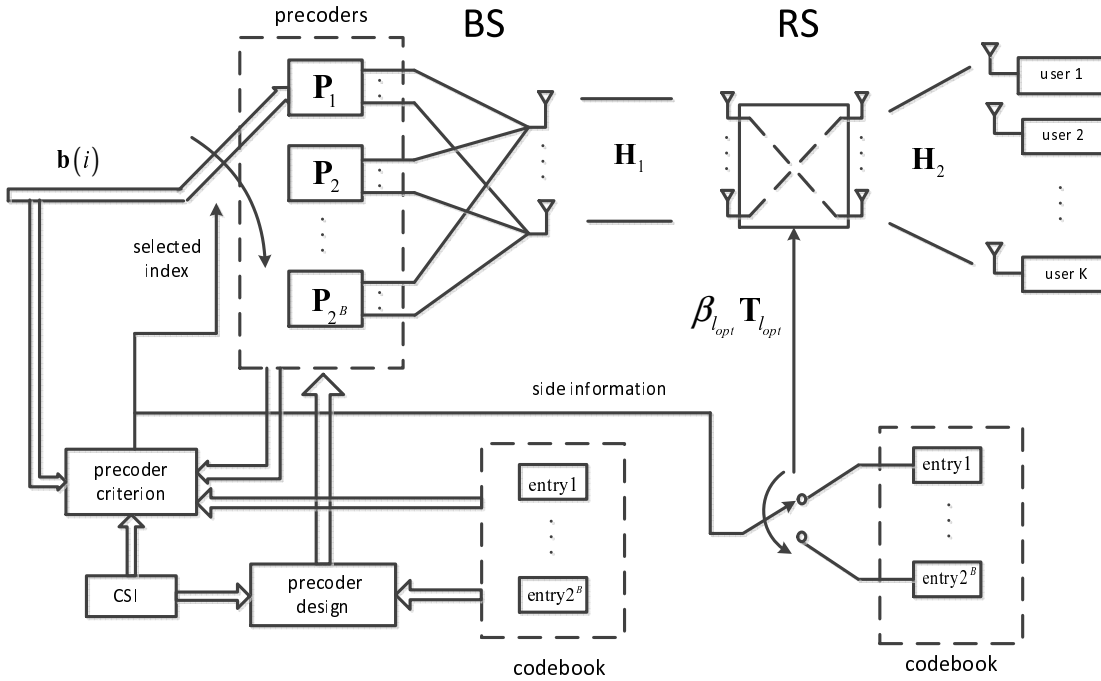
Fig. 1. Proposed precoding scheme using switched relaying processing for multiuser MIMO relay systems.

Note that $\mathbf{b}$, $\mathbf{n}_1$, $\mathbf{n}_2$, $\Delta\mathbf{H}_1$ and $\Delta\mathbf{H}_2$ are uncorrelated and we take expectation over them individually. We used the fact that $\mathbf{T}_l$ is a unitary matrix and assume that $Tr\{\mathbf{P}_l\mathbf{P}_l^H\} = P_t$ without loss of generality [37]. By employing the statistical property of the CSI estimation error (4) and applying the Karush-Kuhn-Tucker (KKT) conditions [38], we obtain the expressions for the robust BS precoding matrix and the RS power scaling factor corresponding to the l-th latent precoding matrix pair as follows:

$$
\mathbf{P}_l = \beta_l \left(\mathbf{M} + \left(\frac{\beta_l^2 \sigma_1^2 Tr\{\hat{\mathbf{H}}_2 \hat{\mathbf{H}}_2^H\}}{P_t} + \frac{K\sigma_2^2}{P_t} \right)\mathbf{I} \right.
$$
$$
\left. + \frac{\beta_l^2 \sigma_1^2 Tr\{\boldsymbol{\Psi}_2\}\boldsymbol{\Sigma}_2}{P_t} + \lambda\beta_l^2\hat{\mathbf{H}}_1^H\hat{\mathbf{H}}_1 + \lambda\beta_l^2 Tr\{\boldsymbol{\Sigma}_1\}\boldsymbol{\Psi}_1 \right)^{-1}
$$
$$
\times \hat{\mathbf{H}}_1^H\mathbf{T}_l^H\hat{\mathbf{H}}_2^H,
$$

$$(9)$$

$$
\beta_l = \frac{Tr\{\Re[\mathbf{P}_l^H\hat{\mathbf{H}}_1^H\mathbf{T}_l^H\hat{\mathbf{H}}_2^H]\}}{\Omega_1 + \lambda\Omega_2}
$$

$$(10)$$

where

$$
\mathbf{M} = \beta_l^2 \Big(E\big[\hat{\mathbf{H}}_1^H\mathbf{T}_l^H\hat{\mathbf{H}}_2^H\hat{\mathbf{H}}_2\mathbf{T}_l\hat{\mathbf{H}}_1\big]
$$
$$
+ Tr\{\hat{\mathbf{H}}_2\mathbf{T}_l\boldsymbol{\Sigma}_1\mathbf{T}_l^H\hat{\mathbf{H}}_2^H\}\boldsymbol{\Psi}_1
$$
$$
+ Tr\{\boldsymbol{\Sigma}_2\}\hat{\mathbf{H}}_1^H\mathbf{T}_l^H\boldsymbol{\Psi}_2\mathbf{T}_l\hat{\mathbf{H}}_1
$$
$$
+ Tr\{\boldsymbol{\Sigma}_2\} Tr\{\mathbf{T}_l^H\boldsymbol{\Psi}_2\mathbf{T}_l\boldsymbol{\Sigma}_1\}\boldsymbol{\Psi}_1 \Big),
$$

$$(11)$$

$\Omega_1 = Tr\{\mathbf{P}_l^H\hat{\mathbf{H}}_1^H\mathbf{D}\hat{\mathbf{H}}_1\mathbf{P}_l\} + Tr\{\mathbf{D}\boldsymbol{\Sigma}_1\} Tr\{\mathbf{P}_l^H\boldsymbol{\Psi}_1\mathbf{P}_l\} + \sigma_1^2 Tr\{\boldsymbol{\Psi}_2\}\boldsymbol{\Sigma}_2 + \sigma_1^2 Tr\{\hat{\mathbf{H}}_2\hat{\mathbf{H}}_2^H\}$, $\Omega_2 = Tr\{\hat{\mathbf{H}}_1\mathbf{P}_l\mathbf{P}_l^H\hat{\mathbf{H}}_1^H\} + Tr\{\boldsymbol{\Sigma}_1\} Tr\{\mathbf{P}_l^H\boldsymbol{\Psi}_1\mathbf{P}_l\} + \sigma_1^2 N_r$ and $\mathbf{D} = \mathbf{T}_l^H(\hat{\mathbf{H}}_2^H\hat{\mathbf{H}}_2 + Tr\{\boldsymbol{\Sigma}_2\}\boldsymbol{\Psi}_2)\mathbf{T}_l$. The Lagrange multiplier λ in (9) and (10)

is given by

$$
\lambda = \left[\frac{\Re[\mathbf{P}_l^H\hat{\mathbf{H}}_1^H\mathbf{T}_l^H\hat{\mathbf{H}}_2^H]\sqrt{\frac{\Omega_2}{P_r}} - \Omega_1}{\Omega_2} \right]^+ .
$$

$$(12)$$

The detailed derivation is shown in Appendix A.

The solutions for the robust BS precoding matrix and the RS power scaling factor corresponding to the l-th latent precoding matrix pair can be obtained by implementing (9), (10) and (12) iteratively with an initial value of $\mathbf{P}_l$. The iterative optimization algorithm is summarized in Table I.

Note that after we have obtained all the 2^B latent precoding matrix pairs, the optimum latent precoding matrix pair should be chosen according to a selection mechanism to provide the best performance. In the following, we will focus on the description of the proposed selection mechanism.

B. Selection Mechanism

Having tried various optimization rules, the squared Euclidean distance seems to be the best candidate for a simple and yet effective selection mechanism. Ideally, the optimum latent precoding matrix pair can be chosen to minimize the accumulated squared Euclidean distance between the true transmit symbol and the received soft information in one transmission data block. Note that the proposed algorithm implemented at the BS cannot obtain the exact received signal at the MS, but it has full information about the transmitted symbols. To overcome this limitation, we propose to use the noiseless information to estimate the received signal. The simulation results in Section V verify the effectiveness of the approximation. Let $\mathbf{s}_j$ denote a $KM \times 1$ vector corresponding to the j-th transmission data block, which is given by $\mathbf{s}_j = [\mathbf{b}_{(j-1)M+1}^T, \ldots, \mathbf{b}_{jM}^T]^T$, where $j \in \{1, 2, \ldots\}$ and M is the

TABLE I
THE ITERATIVE OPTIMIZATION ALGORITHM FOR THE LATENT PRECODING MATRIX PAIR

1	**for** the l-th latent precoding matrix pair.
2	Initialization: $\mathbf{P}_l$.
3	Compute the Lagrange multiplier λ based on (12).
4	Compute the RS power scaling factor β_l by using (10).
5	Compute the robust BS precoding matrix $\mathbf{P}_l$ based on (9) and $\mathbf{P}_l \leftarrow \sqrt{\frac{P_t}{Tr\{\mathbf{P}_l\mathbf{P}_l^H\}}}\mathbf{P}_l$.
6	Repeat step 3, step 4 and step 5 until $||\mathbf{P}_l^i - \mathbf{P}_l^{i-1}||^2 \leq \epsilon$ and $|\beta_l^i - \beta_l^{i-1}|^2 \leq \epsilon$, where ϵ is a predefined threshold value (e.g. ϵ=0.0001). $\mathbf{P}_l^i$ and β_l^i denote $\mathbf{P}_l$ and β_l in the i-th iteration, respectively, while $\mathbf{P}_l^{i-1}$ and β_l^{i-1} denote $\mathbf{P}_l$ and β_l in the $(i-1)$-th iteration, respectively.
7	Obtain the l-th latent precoding matrix pair $\{\mathbf{P}_l, \mathbf{W}_l\}$, where $\mathbf{W}_l = \beta_l \mathbf{T}_l$.
8	Repeat step 2-7 until all 2^B latent precoding matrix pairs are obtained.

block length. The $K \times 1$ vector $\mathbf{b}_{(j-1)M+m} = [b_1, \ldots, b_K]^T$ denotes the m-th transmit vector of the j-th block, $m \in \{1, \ldots, M\}$. Let $\mathbf{u}_j^{(l)}$ denote the $KM \times 1$ pre-estimated vector, which is given by $\mathbf{u}_j^{(l)} = [\hat{\mathbf{y}}_{(j-1)M+1}^{T(l)}, \ldots, \hat{\mathbf{y}}_{jM}^{T(l)}]^T$, where $\hat{\mathbf{y}}_{(j-1)M+m}^{(l)}$ denotes the $K \times 1$ noiseless BS pre-estimated received vector based on the l-th unitary matrix for the m-th transmit vector of the j-th block, and it is given by

$$\hat{\mathbf{y}}_{(j-1)M+m}^{(l)} = \beta_l \hat{\mathbf{H}}_2 \mathbf{T}_l \hat{\mathbf{H}}_1 \mathbf{P}_l \mathbf{b}_{(j-1)M+m}. \tag{13}$$

The optimum latent precoding matrix pair is chosen based on minimizing the summation of the squared Euclidean distance values in one transmission data block. Hence, we have the following selection rule:

$$l_{opt} = \arg \min_{1 \leq l \leq 2^B} \left\{ ||\mathbf{s}_j - \mathbf{u}_j^{(l)}||^2 \right\}. \tag{14}$$

It is worth mentioning that the selection operation takes place once per block.

C. Codebook Design

In the following, we introduce a design method for the codebook of unitary matrices[4], referred to as most frequently selected candidates (MSC), the basic principle of which is to build a codebook which contains the unitary matrices for the most likely selected elements. To build the codebook, we need to perform an extensive set of experiments and compute the frequency of the indices of the selected unitary matrices. Finally, we create the codebook based on the statistics of the indices and choose the 2^B candidates which are most frequently selected as entries of the codebook. The algorithm is summarized in Table II, where $\mathbf{d}$ denotes the vector of squared Euclidean distances for α possible unitary matrices. We generate the α unitary matrices randomly, where α should

[4]According to the discussion of the Grassmannian subspace packing and Lloyd algorithms in [39]-[41], the codebook $\mathcal{T} = \{\mathbf{T}_1, \mathbf{T}_2, \ldots, \mathbf{T}_{2^B}\}$ should be designed such that $\delta = \min_{1 \leq l < m \leq 2^B} d(\mathbf{T}_l, \mathbf{T}_m)$ is as large as possible, where $d(\mathbf{T}_l, \mathbf{T}_m) = \sqrt{N_r - ||\mathbf{T}_l^H \mathbf{T}_m||_F^2}$. Note that our proposed algorithm is based on employing unitary matrices as the entries of the codebook. In this respect, due to $||\mathbf{T}_l^H \mathbf{T}_m||_F^2 = N_r$ for every two given unitary matrices, we have $d(\mathbf{T}_l, \mathbf{T}_m) = 0$. Therefore, the Grassmannian or Lloyd algorithm becomes the general method that groups 2^B randomly generated unitary matrices to create the codebook and may not be suitable for the proposed SR-based precoding scheme.

be a large integer but practical for the experiment, $2^B < \alpha$. The quantity N_e denotes the total number of experiments, $\mathbf{d}_{idx}$ is defined for the storage of the selected candidates for every experiment. The vector $\mathbf{d}_0$ contains the list of all α unitary matrices. The vectors $\{\mathbf{b}_1, \ldots, \mathbf{b}_\Phi\}$ denote all the possibilities of the $K \times 1$ transmit vector. For the case of quadrature phase shift keying (QPSK) modulation, we have $\Phi = 4^K$ possibilities. The vector $\hat{\mathbf{y}}_j^{(l)}$ denotes the noiseless BS pre-estimated received vector with respect to the l-th unitary matrix and the transmit vector $\mathbf{b}_j$, and it is given by $\hat{\mathbf{y}}_j^{(l)} = \beta_l \hat{\mathbf{H}}_2 \mathbf{F}_l \hat{\mathbf{H}}_1 \mathbf{P}_l \mathbf{b}_j$, $j \in \{1, \ldots, \Phi\}$. We highlight that in each run, after we have computed the squared Euclidean distance for all the unitary matrices, the one which yields the minimum squared Euclidean distance is stored in $\mathbf{d}_{idx}$ at step 9. Finally, the MSC codebook $\mathcal{T}_{MSC}$ is created by selecting the most frequently selected 2^B unitary matrices according to the histogram of $\mathbf{d}_{idx}$. Note that the MSC codebook design method is implemented offline.

IV. ANALYSIS OF THE PROPOSED ALGORITHM

In this section, we carry out an analysis of the proposed algorithm in terms of downlink transmission efficiency, the error probability performance and computational complexity.

A. Downlink Transmission Efficiency

From the aforementioned discussion about the proposed scheme, we know that for every block prior to payload transmission, there is a preamble transmission from the BS to the RS which contains the index of the selected unitary matrix and the RS power scaling factor. We insert the limited feedforward bits at the beginning of the corresponding data block. The block of the multiantenna scheme comprises M symbol periods each one consisting of K spatial streams, and the feedforward rate of the optimum index is one per data block. We use B bits to represent 2^B unitary matrices and C bits to represent the RS power scaling factor, and assume that Q-ary modulation is used for the proposed SR-based precoding scheme, thus a number of $B + C$ signalling bits has to be sent for every $KM \log_2(Q)$ transmitted bits in the block. Note that we rely on a TDD system, transmit CSI can be obtained by exploiting channel reciprocity. Therefore, the

TABLE II
MSC CODEBOOK DESIGN ALGORITHM

1	Initialize the vectors $\mathbf{d}$ and $\mathbf{d}_{idx}$, generate null vectors for them. $\mathbf{d} \leftarrow \mathbf{0}$, $\mathbf{d}_{idx} \leftarrow \mathbf{0}$. Decide the number of experiments N_e and the size of the codebook 2^B.
2	Choose an appropriate value for α.
3	Generate α unitary matrices randomly as the candidates, store them in the set $\{\mathbf{F}_1, \mathbf{F}_2, \ldots, \mathbf{F}_\alpha\}$ and assign the list of the unitary matrices to the vector $\mathbf{d}_0$.
4	**for** $n_e = 1$ to N_e **do**
5	Generate the testing matrices $\hat{\mathbf{H}}_1$ and $\hat{\mathbf{H}}_2$.
6	**for** $l = 1$ to α **do**
7	Compute the precoding matrix $\mathbf{P}_l$ and β_l based on $\mathbf{F}_l$, $\hat{\mathbf{H}}_1$ and $\hat{\mathbf{H}}_2$. Compute the squared Euclidean distance for all the possibilities of the transmit symbol vector, and assign it to the l-th element of the vector $\mathbf{d}$. $\mathbf{d}(l) \leftarrow \sum_{j=1}^{\Phi} \|\mathbf{b}_j - \hat{\mathbf{y}}_j^{(l)}\|^2$.
8	**end**
9	Select the entry corresponding to the minimum squared Euclidean distance from the vector $\mathbf{d}_0$ in the n_e-th experiment, assign it to the n_e-th element of the vector $\mathbf{d}_{idx}$. $\mathbf{d}_{idx}(n_e) \leftarrow \mathbf{MINIndex}(\mathbf{d})$.
10	**end**
11	Based on the vector $\mathbf{d}_{idx}$, a histogram $\mathbf{HIST}(\mathbf{d}_{idx})$ is generated. The codebook $\mathcal{T}_{MSC}$ is created by selecting the most frequently selected 2^B candidates according to $\mathbf{HIST}(\mathbf{d}_{idx})$. $\mathcal{T}_{MSC} \leftarrow \mathbf{SELECT}(\mathbf{HIST}(\mathbf{d}_{idx}))$.

downlink transmission efficiency is given by

$$\eta = \frac{KM \log_2(Q)}{KM \log_2(Q) + B + C}. \quad (15)$$

Let us focus on the QPSK modulation and employ a data block of $M = 10$ symbols. For a configuration with $N_t = N_r = K = 6$, by using $B = C = 6$ feedforward bits we achieve a downlink transmission efficiency of 91%.

B. Discussion of Error Probability Performance

In this part, an error probability performance analysis for our proposed algorithm is carried out. We divide the problem into two circumstances based on the side information fed forward from the BS to the RS, and discuss the performance based on the total probability theorem.

It is easy to show that the average error probability over all the destination MSs can be derived as

$$\bar{\mathcal{P}}_e = \frac{1}{K} \sum_{k=1}^{K} \mathcal{P}_{e_k}, \quad (16)$$

where $\mathcal{P}_{e_k}$ denotes the probability of making an error in the symbol detection for the k-th MS. We will rely here on presenting a simple approach to estimate these probabilities. By using the total probability theorem, we can write

$$\mathcal{P}_{e_k} = \mathcal{P}\{e_k|\mathbf{E}_1^{(k)}\}\mathcal{P}\{\mathbf{E}_1^{(k)}\} + \mathcal{P}\{e_k|\mathbf{E}_2^{(k)}\}\mathcal{P}\{\mathbf{E}_2^{(k)}\}, \quad (17)$$

where the events $\mathbf{E}_j^{(k)}$, $j = 1, 2$, are associated with the perfect feedforward transmission of side information and the imperfect feedforward transmission of side information, respectively. They are two mutually exclusive events, with $\mathcal{P}\{\cup_{j=1}^{2} \mathbf{E}_j^{(k)}\} = 1$.

For the event of perfect side information, we assume that the residual multiuser interference can be approximated as a Gaussian random variable. In the case with QPSK modulation,

the error probability $\mathcal{P}\{e_k|\mathbf{E}_1^{(k)}\}$ on the event $\mathbf{E}_1^{(k)}$ can be expressed by

$$\mathcal{P}\{e_k|\mathbf{E}_1^{(k)}\} = \mathcal{Q}\left(\sqrt{\gamma_k^{(l_{opt})}}\right) \quad (18)$$

where $\gamma_k^{(l_{opt})}$ denotes the k-th MS's signal-to-interference-plus-noise ratio (SINR) of the optimum latent precoding matrix pair caused by the unitary matrix $\mathbf{T}_{l_{opt}}$. The structure of $\gamma_k^{(l_{opt})}$ is given as (19), where $k \in \{1, \ldots, K\}$. The function $\mathcal{Q}(.)$ is defined as the Gaussian error function $\mathcal{Q}(x) = (1/2)\mathrm{erfc}(x/\sqrt{2})$.

The probability $\mathcal{P}\{\mathbf{E}_1^{(k)}\}$ relies on the feedforward transmission scheme of side information. For the case where binary PSK modulation is used in a frequency-nonselective, slow Rayleigh fading channel, the error probability for each side information bit is given by $\mathcal{P}_b = \frac{1}{2}\left(1 - \sqrt{\frac{\Gamma}{1+\Gamma}}\right)$ [42], where $\Gamma = \frac{E_b}{N_0}E[\varphi^2]$, φ represents the Rayleigh-distributed amplitude of the channel coefficient, E_b denotes the energy per bit and N_0 is the noise power spectral density. In the event that we transmit B side information bits, the probability $\mathcal{P}\{\mathbf{E}_1^{(k)}\}$ is expressed as

$$\mathcal{P}\{\mathbf{E}_1^{(k)}\} = (1 - \mathcal{P}_b)^B. \quad (20)$$

In the case of imperfect side information, the error probability expression of $\mathcal{P}\{e_k|\mathbf{E}_2^{(k)}\}$ on the event $\mathbf{E}_2^{(k)}$ cannot be derived due to misadjustment in the latent precoding matrix pair selection at the RS. However, in the case that the detection of side information is significantly affected by errors, the selected index of the latent precoding matrix pair at the BS is not in accordance with the one at the RS. The decision on the preprocessing data becomes random and the error probability $\mathcal{P}\{e_k|\mathbf{E}_2^{(k)}\}$ is 0.5. Following the above example, the error probability $\mathcal{P}_{e_k}$ for the k-th MS is given by $\mathcal{P}_{e_k} = 0.5(1 - (1 - \mathcal{P}_b)^B) + \mathcal{Q}\left(\sqrt{\gamma_k^{(l_{opt})}}\right)(1 - \mathcal{P}_b)^B$, where the probability of imperfect side information transmission $\mathcal{P}\{\mathbf{E}_2^{(k)}\}$ is $1 - (1 - \mathcal{P}_b)^B$. We remark that an accurate error

$$\gamma_k^{(l_{opt})} = \frac{\beta_{l_{opt}}^2 \bar{\mathbf{H}}_{l_{opt}}(k,:)\mathbf{P}_{l_{opt}}(:,k)\mathbf{P}_{l_{opt}}^H(:,k)\bar{\mathbf{H}}_{l_{opt}}^H(k,:)}{\sum_{k'\neq k}^K \left(\beta_{l_{opt}}^2 \bar{\mathbf{H}}_{l_{opt}}(k,:)\mathbf{P}_{l_{opt}}(:,k)\mathbf{P}_{l_{opt}}^H(:,k)\bar{\mathbf{H}}_{l_{opt}}^H(k,:)\right) + \beta_{l_{opt}}^2\sigma_1^2\mathbf{H}_2(k,:)\mathbf{H}_2^H(k,:) + \sigma_2^2}, \tag{19}$$

probability expression of $\mathcal{P}_{e_k}$ cannot be obtained as a result of the specific nature of the proposed scheme. It remains an open problem. In section V, we will illustrate the error probability performance in the presence of side information errors.

C. Computational Complexity

We measure the complexity in terms of the number of floating point (FLOP). From [43], we know that a complex addition and multiplication has 2 and 6 FLOPs, respectively. We note that the complexity of the matrix inversion is cubic in the number of BS or RS antennas [44]. In Table III, we show the complexity of the conventional precoding algorithm, the proposed latent precoding matrix pair design algorithm and the selection mechanism of the proposed scheme. The overall complexity of the proposed algorithm includes the complexity of the selection mechanism and the design complexity of each latent precoding matrix pair multiplied by the codebook size 2^B. The complexity of the proposed algorithm increases with the codebook size. In the simulation section, we will show that for a limited increase in complexity the performance of the proposed SR-based robust precoding design algorithm outperforms the performance of the conventional precoding algorithms significantly. In practice, the codebook size should be chosen to achieve a suitable trade-off between performance requirements and implementation complexity, based on a given channel environment.

V. SIMULATION RESULTS

In this section, we conduct simulations to evaluate the proposed SR-based robust precoding scheme and compare it with existing precoding algorithms for multiuser MIMO relaying systems [10]-[13]. In the simulations, we assume that both the first phase MIMO channel and the second phase MIMO broadcast channel are quasi-static flat fading channels with a Rayleigh distribution. 10000 channel realizations are employed for each simulation. The configuration of the system is $N_t = N_r = K = 6$. By using the exponential model [4], [45] and [46], the channel estimation error covariance matrices can be expressed as

$$\mathbf{\Psi}_1 = \mathbf{\Psi}_2 = \begin{bmatrix} 1 & \theta & \theta^2 & \theta^3 & \theta^4 & \theta^5 \\ \theta & 1 & \theta & \theta^2 & \theta^3 & \theta^4 \\ \theta^2 & \theta & 1 & \theta & \theta^2 & \theta^3 \\ \theta^3 & \theta^2 & \theta & 1 & \theta & \theta^2 \\ \theta^4 & \theta^3 & \theta^2 & \theta & 1 & \theta \\ \theta^5 & \theta^4 & \theta^3 & \theta^2 & \theta & 1 \end{bmatrix}, \tag{21}$$

$$\mathbf{\Sigma}_1 = \sigma_e^2 \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 & \rho^4 & \rho^5 \\ \rho & 1 & \rho & \rho^2 & \rho^3 & \rho^4 \\ \rho^2 & \rho & 1 & \rho & \rho^2 & \rho^3 \\ \rho^3 & \rho^2 & \rho & 1 & \rho & \rho^2 \\ \rho^4 & \rho^3 & \rho^2 & \rho & 1 & \rho \\ \rho^5 & \rho^4 & \rho^3 & \rho^2 & \rho & 1 \end{bmatrix}, \quad \mathbf{\Sigma}_2 = \sigma_e^2 \mathbf{I}, \tag{22}$$

where θ and ρ denote the correlation coefficients, and σ_e^2 is the estimation error variance. Since the destination MSs are far apart spaced and uncorrelated, the correlation coefficient in the covariance matrix of the second phase channel seen from the receiver is zero. The estimated channels, $\hat{\mathbf{H}}_j$, $j = 1, 2$, are therefore generated based on the following distribution:

$$\hat{\mathbf{H}}_j \sim \mathcal{CN}\left(\mathbf{0}, \frac{(1-\sigma_e^2)}{\sigma_e^2}\mathbf{\Sigma}_j \otimes \mathbf{\Psi}_j^T\right) \tag{23}$$

such that the channel realizations $\mathbf{H}_j = \hat{\mathbf{H}}_j + \Delta\mathbf{H}_j$ have unit variance. We set $P_t = P_r = K$, and define the input SNR= $P_t/\bar{\sigma}^2$, where $\bar{\sigma} = \sigma_1 = \sigma_2$. In the simulations, the BS employs $C = 6$ bits to quantize the computed RS power scaling parameter. With regard to the scalar information, we use a nonuniform scalar quantizer [47]. This information is fed forward to the RS, together with the B bits corresponding to the index of the selected latent precoding matrix pair. The iterative optimization algorithm for each latent precoding matrix pair uses the identity matrix as the initial value of the precoding matrix. QPSK modulation is used as the modulation scheme. Among the analyzed techniques in this paper, we consider the following:

- SR precoding: the proposed SR-based robust precoding algorithm.
  1) B-bit: the limited feedforward schemes employ B bits corresponding to the index of the selected latent precoding matrix pair, namely 2^B is the codebook size.
  2) MSC: the proposed SR-based precoding scheme with the codebook generated by the MSC method.
  3) Random: the proposed SR-based precoding scheme with 2^B randomly generated unitary matrices in the codebook.
- Robust Identity: The BS precoding matrix is designed based on the robust MMSE technique with the conventional relay scheme, which amplifies the energy of the received data at the RS and forwards the signal directly [24]. That is to say, the RS precoding matrix is an identity matrix.
- Robust Relay MMSE: the MMSE-based robust MIMO RS precoding algorithm proposed in [10][5].
- SVD-ZF: the SVD-based joint BS and RS ZF precoding algorithm proposed in [11], which is only based on the estimated CSI.
- SVD-RZF: the SVD-based robust joint BS and RS ZF precoding algorithm proposed in [13].

Fig. 2 shows the average SER performance versus input SNR for the proposed SR-based precoding scheme, i.e. 1-, 2-, 3-, 4-, and 6-bit for the index of the selected latent precoding matrix pair, respectively. We apply the MSC method for the unitary matrix codebook design. We set $N_e = 10000$ and $\alpha =$

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[5]Although it is developed based on the feedback quantized channel errors, we have extended the algorithm straightforwardly to the case with channel estimation errors for the comparison.

TABLE III
COMPUTATIONAL COMPLEXITY

Algorithms	Complexity
Robust Relay Precoding [10]	$\mathcal{O}[N_r^3 + KN_r^2 + N_r^2 + N_r]$
SR-based Precoding (each iteration):	
precoding matrix (9)	$\mathcal{O}[N_t^3 + (N_r + K)N_t^2 + (N_t + K)N_r^2 + K^2 + N_r N_t K + KN_t^2 + N_t K]$
β_l	$\mathcal{O}[N_r^3 + KN_r^2 + N_r K^2 + N_t K^2]$
Selection Mechanism	$\mathcal{O}[((K + N_r)N_t + N_r^2 + KN_r + K)M + KM]$

1000. The channel estimation error variance is given by $\sigma_e^2 = 0.002$, and the correlation coefficients are given by $\theta = \rho = 0$. The channel varies per transmission data block, each block contains $M = 10$ symbols. From the results, we can see that the best performance is achieved with the proposed scheme with $B = 6$ bits, and the average SER decreases as the number of feedforward bits increases. In the simulation, we assume that perfect side information is fed forward.
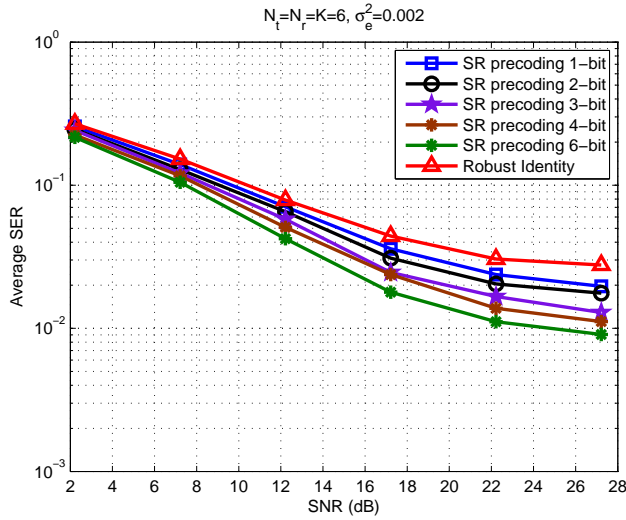

Fig. 2. Average SER performance versus SNR for the proposed SR-based robust precoding scheme. $\sigma_e^2 = 0.002$, $\theta = \rho = 0$.

We compare the codebooks of the unitary matrices which are created by two methods, namely the randomly generated method and the proposed MSC method. In particular, we show average SER performance curves versus input SNR for different values of estimation error variance. Note that the codebooks are designed offline. For the MSC algorithm we set the number of simulation $N_e = 10000$ and the number of candidates $\alpha = 1000$. The channel coefficients are generated independently. The results which are illustrated in Fig. 3 show that the performance of the proposed precoding scheme with different codebooks of unitary matrices, where we use $B = 6$ bits. We can see that the MSC method outperforms the random method. Compared to the random method, the proposed MSC codebook design method can have a gain of 2 dB.

Fig. 4 and Fig. 5 compare the average SER versus the SNR of the proposed SR-based precoding scheme with some existing relay precoding algorithms. The MSC method is used for the codebook design. The same system configuration and channel model are employed here. In Fig. 4, the channel error variance is given by $\sigma_e^2 = 0.002$, and the correlation

coefficients are given by $\theta = \rho = 0$. The performance of the proposed SR-based robust MMSE precoding scheme is much better than the others. In particular, the proposed SR-based robust precoding scheme with $B = 6$ bits can save over 3 dB in transmit power in comparison with the robust relay MMSE precoding algorithm, at an average SER level of 2×10^{-2}. The SER performance of the SR-based precoding scheme with $B = 6$ bits under perfect CSI is given as a reference. In Fig. 5, the channel error variance is given by $\sigma_e^2 = 0.006$, and the correlation coefficients are given by $\theta = \rho = 0$. We can see that the best performance is achieved with the proposed SR-based robust precoding scheme with $B = 6$ bits, followed by the robust relay MMSE precoding algorithm, the robust identity technique, the SVD-RZF precoding algorithm, and the SVD-ZF precoding algorithm. Specifically, at an average SER level of 5×10^{-2} the proposed SR-based robust precoding scheme can save 5 dB in comparison with the robust relay MMSE precoding algorithm. The results show the ability of the proposed SR-based precoding algorithm to handle channel uncertainties and multiuser interference.
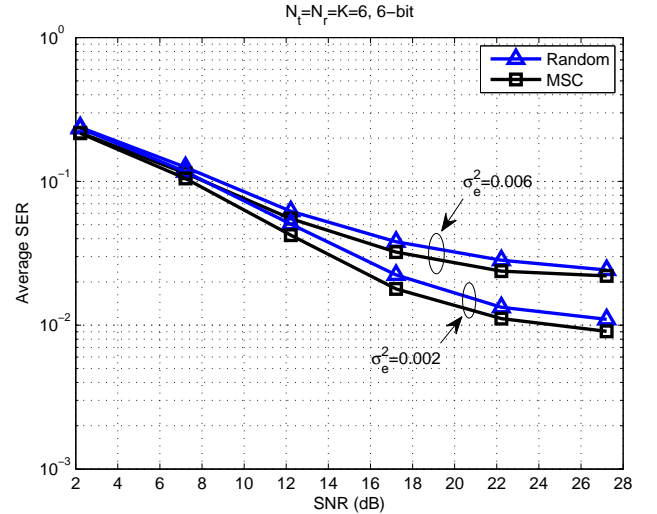

Fig. 3. Average SER performance versus SNR for the proposed SR-based robust precoding scheme. $B = 6$, $N_e = 10000$, $\alpha = 1000$. $\theta = \rho = 0$.

Fig. 6 shows the SER performance comparison for the proposed SR-based precoding scheme and the existing robust identity technique with different values of θ. In this simulation, we let $\rho = 0$, $\sigma_e^2 = 0.002$ and θ was varied. From Fig. 6, it can be seen that smaller correlation coefficients lead to a better performance. When the value of θ decreases, the performance of both algorithms improve. The performance of the proposed SR precoding algorithm with $B = 6$ bits is always superior to
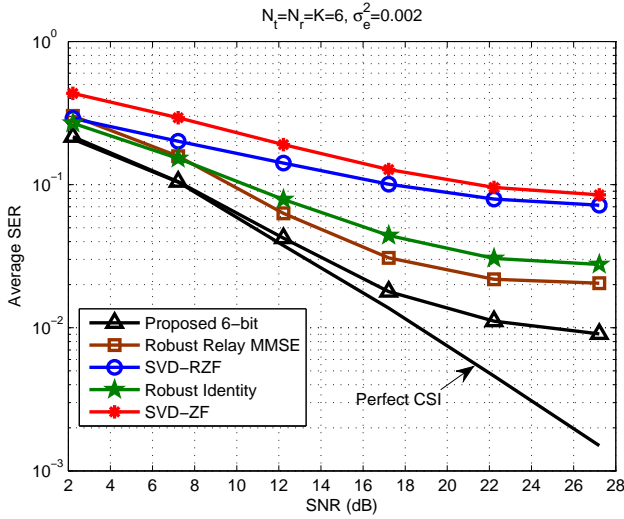
Fig. 4. Average SER performance versus SNR for the proposed SR-based robust precoding scheme and the existing relay precoding schemes. $\sigma_e^2 = 0.002$, $\theta = \rho = 0$, $B = 6$.
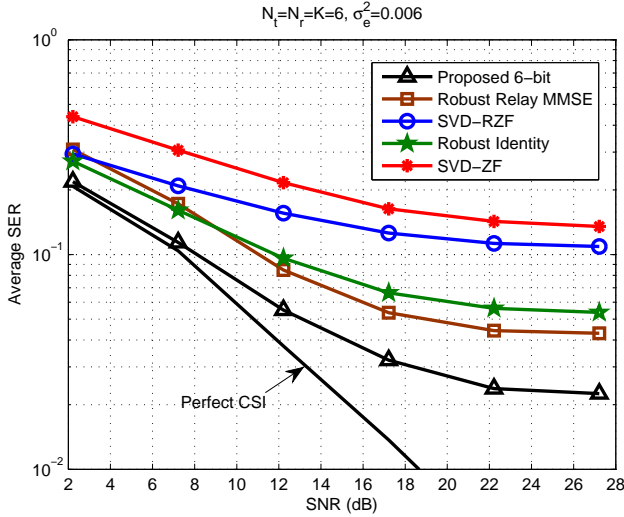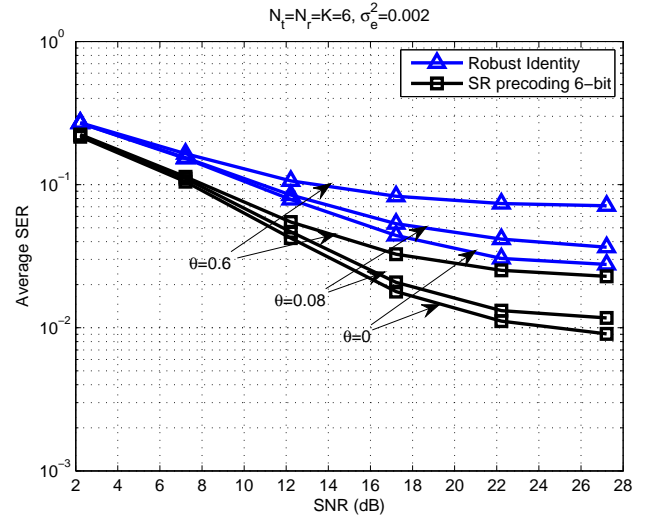

Fig. 6. Average SER performance versus SNR for the proposed SR-based robust precoding and robust identity schemes. $B = 6$, $\sigma_e^2 = 0.002$, $\rho = 0$.

Fig. 5. Average SER performance versus SNR for the proposed SR-based robust precoding scheme and the existing relay precoding schemes. $\sigma_e^2 = 0.006$, $\theta = \rho = 0$, $B = 6$.
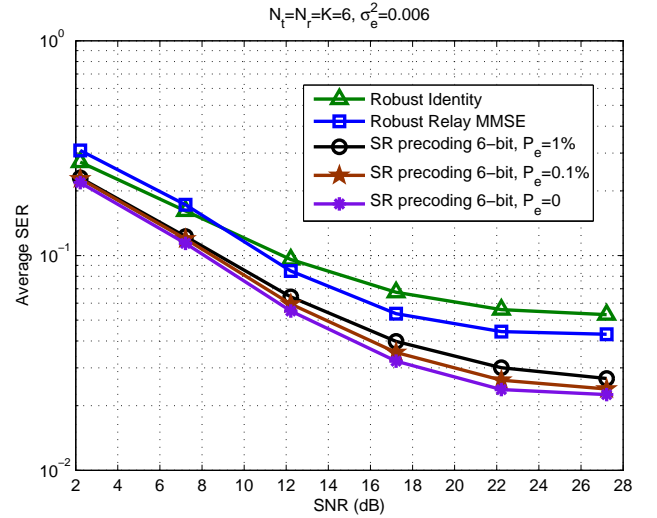

Fig. 7. Average SER performance versus SNR for the proposed SR-based robust precoding and conventional schemes. $B = 6$, $\sigma_e^2 = 0.006$, $\theta = \rho = 0$.

the performance of the conventional robust identity algorithm. In particular, the proposed SR precoding algorithm can save up to 10 dB in transmit power in comparison with the robust identity algorithm, at an SER level of 4×10^{-2} in the case with $\theta = 0.08$. This demonstrates the ability of the proposed algorithm to properly handle CSI uncertainty as well as channel correlation.

In the next simulation, we focus on examining the performance of the proposed algorithm in the presence of feedforward side information errors. The last results, shown in Fig. 7 illustrate the averaged SER performance with different levels of side information errors for the proposed SR-based precoding scheme. We use a structure based on a frame format where the indices are converted to 0s and 1s. This frame of 1s and 0s with the feedforward information is transmitted over a binary symmetric channel associated with a probability of error P_e. The burst error scenario in the limited feedforward channel can be easily transferred to the case of the binary symmetric channel by employing a conventional bit interleaver. In particular, we use $B = 6$ bits for the index of the selected unitary matrix and $C = 6$ bits to represent the computed RS power scaling factor. We let $\rho = \theta = 0$ and $\sigma_e^2 = 0.006$. The MSC method is used for the codebook design. The same system configuration and channel model are employed here. As we increase the feedforward side information errors, the performance of the proposed limited feedforward scheme decreases, since the unitary matrices at the BS and RS are not equal to each other due to feedforward errors. Associated with a side information error level of $P_e = 0.1\%$, the performance has 1dB degradation, compared with the perfect side information case at a BER level of 3×10^{-2}. In order to guarantee that the errors are controlled,

channel coding techniques should be used for the signalling feedforward channels with large errors.

VI. CONCLUSION

In this paper, we have proposed a robust MMSE BS precoding strategy based on SR processing for multiuser MIMO relaying systems. We have also developed a selection mechanism, which was used for symbol detection. A method based on the most selected candidates for the unitary matrix codebook design has been proposed. We have discussed the error probability, the computational complexity and the transmission efficiency of the proposed scheme and algorithms. The results have shown that the proposed SR-based scheme significantly outperforms the existing relay precoding algorithms in the presence of imperfect CSI. Our future work will extend our proposed algorithms to take into account systems with other precoding schemes.

APPENDIX A
DERIVATION FOR (9), (10) AND (12)

By focusing on the RS transmit power constraint, we obtain the following Lagrangian objective function[6]:

$$J(\mathbf{P}_l, \beta_l, \lambda) = \zeta + \lambda\Big(\beta_l^2 E\Big[Tr\{\mathbf{H}_1\mathbf{P}_l\mathbf{P}_l^H\mathbf{H}_1^H + \sigma_1^2\mathbf{I}\}\Big] - P_r\Big) \tag{24}$$

where λ denotes the Lagrange multiplier for the RS transmit power constraint. Based on the KKT conditions, we have:

$$\beta_l^2 E\Big[Tr\{\mathbf{H}_1\mathbf{P}_l\mathbf{P}_l^H\mathbf{H}_1^H + \sigma_1^2\mathbf{I}\}\Big] - P_r \le 0, \tag{25}$$

$$\lambda\Big(\beta_l^2 E\Big[Tr\{\mathbf{H}_1\mathbf{P}_l\mathbf{P}_l^H\mathbf{H}_1^H + \sigma_1^2\mathbf{I}\}\Big] - P_r\Big) = 0, \quad \lambda \ge 0 \tag{26}$$

$$\bigtriangledown J(\mathbf{P}_l, \beta_l, \lambda)_{\mathbf{P}_l^*} = \mathbf{0}, \quad \bigtriangledown J(\mathbf{P}_l, \beta_l, \lambda)_{\beta_l} = 0. \tag{27}$$

The RS transmit power is given by

$$\begin{aligned}
\beta_l^2 E\Big[Tr\{\mathbf{H}_1\mathbf{P}_l\mathbf{P}_l^H\mathbf{H}_1^H + \sigma_1^2\mathbf{I}\}\Big] &= \beta_l^2 Tr\{\hat{\mathbf{H}}_1\mathbf{P}_l\mathbf{P}_l^H\hat{\mathbf{H}}_1^H \\
&\quad + E\big[\Delta\mathbf{H}_1\mathbf{P}_l\mathbf{P}_l^H\Delta\mathbf{H}_1^H\big] \\
&\quad + \sigma_1^2\mathbf{I}\}.
\end{aligned} \tag{28}$$

By taking the gradient terms of (24) with respect to $\mathbf{P}_l^*$ and equating them to zero, we can obtain (9).

By taking the gradient terms of (24) with respect to β_l and equating them to zero, we obtain

$$\begin{aligned}
\bigtriangledown J(\mathbf{P}_l, \beta_l, \lambda)_{\beta_l} &= 2\beta_l E\Big[Tr\{\bar{\mathbf{H}}_l\mathbf{P}_l\mathbf{P}_l^H\bar{\mathbf{H}}_l^H\}\Big] \\
&\quad + 2\sigma_1^2\beta_l E\Big[Tr\{\Delta\mathbf{H}_2\Delta\mathbf{H}_2^H\}\Big] \\
&\quad + 2\sigma_1^2\beta_l Tr\{\hat{\mathbf{H}}_2\hat{\mathbf{H}}_2^H\} \\
&\quad + 2\lambda\beta_l E\Big[Tr\{\mathbf{H}_1\mathbf{P}_l\mathbf{P}_l^H\mathbf{H}_1^H + \sigma_1^2\mathbf{I}\}\Big] \\
&\quad - Tr\{\mathbf{P}_l^H\bar{\mathbf{H}}_l^H + \bar{\mathbf{H}}_l\mathbf{P}_l\} = 0.
\end{aligned} \tag{29}$$

By solving (29) we have (10).

---

[6]In this work, we simply scale the computed BS precoding expression to meet the BS transmit power constraint [37].

By substituting (10) into (26) and solving the equation, we have (12). In order to meet the transmit power constraint $Tr\{\mathbf{P}_l\mathbf{P}_l^H\} = P_t$, the proposed BS precoding matrix for the l-th latent precoding matrix pair is given by $\mathbf{P}_l \leftarrow \sqrt{\frac{P_t}{Tr\{\mathbf{P}_l\mathbf{P}_l^H\}}}\mathbf{P}_l$, where the arrow denotes an overwrite operation.

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