Low-Complexity Reduced-Rank Linear Interference Suppression Based on Set-Membership Joint Iterative Optimization for DS-CDMA Systems

Patrick Clarke, *Student Member, IEEE*, and Rodrigo C. de Lamare, *Senior Member, IEEE*

***Abstract*—This paper presents and analyzes a novel low-complexity reduced-rank linear interference suppression technique for direct-sequence code-division multiple access (DS-CDMA) systems based on the set-membership joint iterative optimization of receive parameters. Set-membership filtering is applied to the design and adaptation of the dimensionality-reducing projection matrix and the reduced-rank interference suppression filter. The specification of error bounds on the projection matrix and reduced-rank filter lead to the formation of two constraint sets from which estimates of the adaptive structures are selected at each time instant. The result is a low-complexity sparsely updating reduced-rank technique that does not require eigendecomposition or subspace tracking procedures. We develop least squares and stochastic gradient-type algorithms and a low-complexity rank-selection algorithm and also devise a time-varying adaptive error bound implementation. We present a stability and mean-square-error convergence analysis of the proposed algorithms along with a study of their complexity. The proposed schemes are applied to interference suppression in the uplink of a multiuser spread-spectrum DS-CDMA system, and the results confirm the validity of the analysis and the effective operation of the schemes. Performance comparisons are given against existing reduced-rank and full-rank algorithms, which act to highlight the improvements obtained by the proposed technique and algorithms.**

***Index Terms*—Adaptive techniques, direct-sequence code-division multiple access (DS-CDMA), interference suppression, reduced-rank methods, set-membership (SM) filtering.**

I. Introduction

REDUCED-RANK signal processing has been promoted in the last decade as a viable and attractive solution to a range of applications where the number of elements in adaptive filters has become prohibitively high [1]–[22]. Due to their performance in the presence of multiuser interference (MUI), narrowband interference, and fading channels, a resurgence of interest has also occurred in spread-spectrum systems such as direct-sequence code-division multiple access (DS-CDMA) and direct-sequence ultrawideband (DS-UWB). One key feature of these systems is their use of extended spreading codes, which act to suppress MUI and intercell interference. However, due to the problem of chip synchronization in the uplink of DS-CDMA systems, the use of orthogonal codes to suppress MUI is restricted to the downlink. Consequently, in the uplink, pseudorandom codes are utilized to randomize each user's signal to cosystem and cospectrum users; however, this approach leads to increased MUI compared to the downlink. Therefore, interference suppression techniques are required. Linear and nonlinear approaches, including direct equalization, successive interference cancellation, and decision feedback, have been proposed as interference suppression and reception techniques for DS-CDMA systems [23]–[26]. However, the significant dimensionality of the structures necessary for both linear and nonlinear reception and interference suppression of these spread signals result in a tradeoff between complexity, convergence, training sequence length, and tracking performance [27], whether optimally or iteratively implemented. These factors also have an impact on the power consumption and robustness of a system, both of which are critical in mobile systems and wireless sensor networks.

Reduced-rank signal processing offers an alternative to conventional interference suppression techniques and has the ability to combat a number of the aforementioned drawbacks. By introducing a layer of preliminary signal processing that reduces the dimensionality of the input signal, smaller receive and interference suppression filters can be used. However, this extra layer of processing comes at the cost of increased complexity, and consequently, there is a quest for low-complexity reduced-rank methods. In the communications theory, reduced-rank techniques originated in the eigendecomposition of the received signal's covariance matrix. Following decomposition, the largest eigenvalues and corresponding eigenvectors are then selected to form the reduced-rank signal subspace and the dimensionality/rank-reducing projection matrix, which transforms the full-rank signal [15], [16]. The principal components and cross-spectral metric are two early techniques based on the singular value decomposition (SVD) of an estimate of the covariance matrix. These schemes operate through optimization functions based on the optimum reduced-rank representation and secondary error criteria [28], [29], respectively. However, the inherent complexity of SVD fuelled the search for alternative reduced-rank methods. This condition led to the emergence of the following two approaches: 1) the multistage Wiener filter (MSWF) [13], [30] and 2) the auxiliary vector filter (AVF) [10], [11]. Both of these techniques possess the desirable

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The authors are with the Communications Research Group, Department of Electronics, University of York, YO10 5DD York, U.K. (e-mail: pjc500@ohm.york.ac.uk; rcdl500@ohm.york.ac.uk).

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characteristic of the subspace rank or the number of auxiliary vectors not scaling with full-rank system dimensionality. However, complexity remained a major issue. The most recent method, reduced-rank signal processing based on the joint iterative optimization (JIO) of adaptive filters [15], [16], [31], combats the issue of complexity by interpreting the projection or transformation matrix as a bank of adaptive filters. These filters are then jointly adapted with the reduced-rank filter to arrive at an effective rank-reduction matrix and interference suppression filter [31]. The majority of existing reduced-rank algorithms for communications performs the dimensionality-reduction process and interference suppression as independent tasks and uses a conventional algorithm such as the least mean squares (LMS) only to perform the adaptation of the reduced-rank interference suppression filter. In contrast, JIO uses conventional algorithms to adapt both structures and introduces an exchange of information between the two processes, a combination which results in performance benefits. However, this method has a complexity that still exceeds the full-rank LMS by up to an order of magnitude.[1] Consequently, the formulation of a technique for reducing this complexity is of great interest and central to this paper.

Set-membership (SM) techniques are a low-complexity approach to established adaptive filtering and have been applied to linear receivers in code-division multiple access (CDMA) and channel estimation with promising results [32]–[34]. Consequently, the combination of these approaches and reduced-rank techniques for CDMA interference suppression form an attractive proposition with the potential to achieve the gains of reduced-ranks signal processing without the associated complexity. The basis of SM filtering lies in the set theory and the generation of a set of solutions to a bounded optimization problem instead of a single solution. First proposed for systems where a bound could be placed on the noise variance, it was later reformulated for a bounded error specification, which allowed it to be applied to channel equalization and interference suppression. The following two predominant error bounded SM implementations exist: 1) the normalized least mean square (NLMS) and 2) recursive least squares (RLS) algorithms. The latter approach is, in fact, rooted in optimal bounding ellipsoids (OBE) techniques but conveniently lends itself to a least squares (LS) interpretation [35]. Performance and complexity improvements over conventional adaptive methods result from SM filtering, because an "optimized" step size is utilized, and an element of the redundancy in the adaptation process is eliminated. This redundancy removal stems from the definition of a bounded set of valid estimates as opposed to a point estimate at each time instant such that it is possible that a previous solution to the optimization problem lies within the current set of solutions, thus removing the need to update the solution (filter coefficients) while not sacrificing performance. Further improvements in performance and complexity can be obtained by implementing a variable error bound that adapts the solution sets to suit the environment and assists in preventing the overbounding and underbounding [34], [36] of the solution set. However, compared to the complexity savings brought about by the removal of redundancy, the improvements in convergence brought about by SM techniques are less significant, and the overriding limiting factor remains the length of the filter. This shortfall of SM techniques can be addressed by introducing reduced-rank methods to alter the dimensionality of the signals under consideration. Consequently, investigation into a combination of SM and reduced-rank schemes has the potential to bear significant advances in low-complexity reduced-rank interference suppression [14], [18], [21].

This paper proposes the integration of SM filtering with the JIO reduced-rank method for linear MUI suppression in DS-CDMA systems [21]. A framework for the integration is set out, and the unique properties of JIO, which allow SM techniques to elegantly be applied to reduced-rank methods, where previously not possible, are highlighted. The generation of sets of solutions from which the subspace and the filter are chosen allows the selective updating capabilities and step-size optimization of SM schemes to be applied to the adaptation of these structures. This condition gives the JIO reduced-rank procedure an added element of adaptivity, which not only enables it to more reliably operate but also improves its convergence and steady-state performance. The overall result is a sparsely updating implementation of the JIO, whose complexity and performance can be controlled by manipulating the SM error bound and, hence, the bounded set of solutions. This condition makes the schemes particularly suited to mobile communications and wireless sensor networks, where battery life is a major consideration, and demands on the system are dynamic [37]. The derivation and implementation of two SM reduced-rank algorithms based on the SM-NLMS and bounding-ellipsoid adaptive constrained least squares (BEACON) algorithms are presented. An analysis that unifies and extends the currently available theory is then given for its stability, convergence properties, and steady-state error performance. In addition, a novel automatic SM rank-selection algorithm is presented, along with a variable error bound implementation, where the error bound is adaptively determined to arrive at an optimized bound, prevent overbounding and underbounding, and address the problem of bound selection when limited system knowledge is available. The performance of the proposed algorithms is evaluated and compared to existing methods for interference suppression in the uplink of a DS-CDMA system [37], [38].

This paper is organized as follows. Section II introduces the system model and linear reception of DS-CDMA signals, and Section III gives the integration of SM filtering with the JIO of adaptive filters and the formulation of a JIO-SM framework. Section IV derives and presents two algorithms based on the minimum-mean-square-error (MMSE) and LS error criteria, followed by an analysis of their complexity, automatic rank-selection, and adaptive error bound variants of the proposed algorithms. Section V presents the stability and mean-square-error (MSE) analysis of the proposed algorithms along with limitations of the analysis that result from the complex interdependent relationship between the adaptive structures of the schemes, followed in Section VI by the application and simulation of the proposed and existing algorithms to the

[1]Depending on the rank of the scheme.

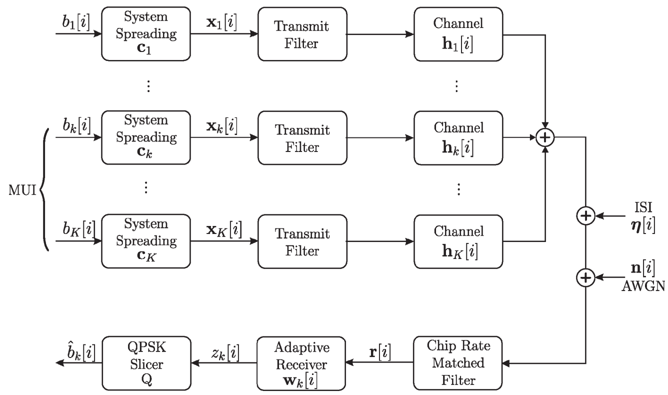

Fig. 1. DS-CDMA uplink system model.

DS-CDMA system and the evaluation of their performance. Finally, Section VII gives the conclusions.

Notation: Throughout this paper, bold uppercase and lowercase letters represent matrices and vectors, respectively. The complex conjugate, complex conjugate transpose (Hermitian), inverse, and transpose operations are denoted by $(.)^*$, $(.)^H$, $(.)^{-1}$, and $(.)^T$, respectively. The trace of a matrix is represented by $\mathrm{tr}(.)$, and $\mathbf{I}_m$ represents an $m \times m$ identity matrix. Block structures made up of 1s and 0s will be represented by $\mathbf{1}_{M\times D}$ and $\mathbf{0}_{M\times D}$, respectively, where M and D specify the dimensions of the structures. Reduced-rank vectors and matrices are given with the addition of a tilde $(\tilde{.})$, and estimated values are denoted by the addition of a hat $(\hat{.})$.

II. DS-CDMA System Model and Linear Receivers

In this paper, we consider a discrete-time model of the uplink of a symbol synchronous Universal Mobile Telecommunications System (UMTS) DS-CDMA system, as given in Fig. 1, where there are K users and N chips per symbol [39]. The system has a chip rate of 3.84 Mchips/s and an assumed bandwidth of 5 MHz and uses quaternary phase-shift keying (QPSK) modulation. The multipath channel of each user is modeled in accordance with the UMTS Vehicular A channel model [40], and each path delay is assumed to be a multiple of the chip rate. For every user in the uplink, an independent L-path time-varying channel is generated. Each user's channel realization is assumed to be constant over each symbol period and has a maximum delay spread of $T_{D_{max}} = (L-1)T_c$, where $T_c = (1/3.84 \times 10^6)s$ is the chip duration. The channel for user k is given by

$$\begin{aligned}\mathbf{h}_k[i] &= [h_{k,1}[i]\ h_{k,2}[i]\ \cdots\ h_{k,L}[i]]\\ &= [\alpha_{k,1}[i]p_{k,1}\ \alpha_{k,2}[i]p_{k,2}\ \cdots\ \alpha_{k,L}[i]p_{k,L}]\end{aligned} \tag{1}$$

where $\mathbf{p}_k = [p_{k,1}\ p_{k,2}\ \cdots\ p_{k,L}]$ is the average power profile of the channel, and $\alpha_k[i] = [\alpha_{k,1}[i]\ \alpha_{k,2}[i]\ \cdots\ \alpha_{k,L}[i]]$ are independent Raleigh-distributed intersymbol complex-fading coefficients generated in accordance with the Clarke model, where 20 scatters are assumed. These complex coefficients include the Doppler effect, where the Doppler shift and symbol period are denoted by f_d and $T_s = NT_c$, respectively [41] and are specified for each simulation.

Pseudorandom spreading codes are repeated from symbol to symbol, and the M-dimensional received signal $\mathbf{r}[i]$ at the base station after chip-pulse matched filtering and sampling at the chip rate is given by

$$\mathbf{r}[i] = A_1 b_1[i]\mathbf{H}_1[i]\mathbf{c}_1[i] + \underbrace{\sum_{k=2}^{K} A_k b_k[i]\mathbf{H}_k[i]\mathbf{c}_k[i]}_{\text{MUI}} + \boldsymbol{\eta}[i] + \mathbf{n}[i] \tag{2}$$

where $M = N + L - 1$, and $\mathbf{n}[i] = [n_1[i]\ \ldots\ n_M[i]]^T$ is the complex Gaussian noise vector with zero mean and covariance matrix $E[\mathbf{n}[i]\mathbf{n}^H[i]] = \sigma_n^2\mathbf{I}$. The notation $E[\cdot]$ stands for expected value, and the kth user's symbol is $b_k[i]$ and assumed to have been drawn from a general QPSK constellation normalized to unit power. The amplitude of user k is A_k, and $\boldsymbol{\eta}[i]$ is the intersymbol interference (ISI) that results from the multipath channel. The $M \times N$ convolution channel matrix $\mathbf{H}_k[i]$ contains one-chip shifted versions of the zero-padded channel vector $\mathbf{h}_k[i]$, and the $N \times 1$ vector $\mathbf{c}_k[i]$ is the spreading code of user k. The structures can be described by

$$\mathbf{H}_k[i] = \begin{bmatrix} h_{k,1}[i] & & \mathbf{0} \\ \vdots & \ddots & h_{k,1}[i] \\ h_{k,L}[i] & & \vdots \\ \mathbf{0} & \ddots & h_{k,L}[i] \end{bmatrix}, \quad \mathbf{c}_k[i] = \begin{bmatrix} c_k^1[i] \\ \vdots \\ c_k^N[i] \end{bmatrix}. \tag{3}$$

In this model, the ISI span and contribution $\boldsymbol{\eta}_k[i]$ are functions of the processing gain N and channel length L. If $1 < L \le N$, the following three symbols would interfere in total: 1) the current symbol; 2) the previous symbol; and 3) the successive symbol. In the case of $N < L \le 2N$, the five symbols would interfere: the current symbol, the two previous symbols, and the two successive symbols. In most practical DS-CDMA systems, $1 < L \le N$; therefore, only three symbols are usually affected [40].

Training-based adaptive multiuser linear receivers of the sort considered in this paper are tasked with the suppression of the interference in (2). The design of such receivers corresponds to determining a finite impulse response (FIR) filter $\mathbf{w}_k[i] = [w_{k,1}[i]\ w_{k,2}[i]\ \ldots\ w_{k,M}]^T$ with M coefficients, which provides an estimate of the desired symbol as given by

$$\begin{aligned}\hat{b}_k[i] &= \mathrm{Q}(z_k[i])\\ &= \frac{1}{\sqrt{2}}\mathrm{sgn}(\Re[z_k[i]]) + \frac{1}{\sqrt{2}}\mathrm{sgn}(\Im[z_k[i]])\, j\\ &= \frac{1}{\sqrt{2}}\mathrm{sgn}\left(\Re\left[\mathbf{w}_k^H[i]\mathbf{r}[i]\right]\right) + \frac{1}{\sqrt{2}}\mathrm{sgn}\left(\Im\left[\mathbf{w}_k^H[i]\mathbf{r}[i]\right]\right) j\end{aligned} \tag{4}$$

where $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary parts, respectively, $\mathrm{sgn}(\cdot)$ is the signum function, and the system of interest uses unity-normalized QPSK modulation. The quantity $z_k[i] = \mathbf{w}_k^H[i]\mathbf{r}[i]$ is the output of the adaptive receiver $\mathbf{w}_k[i]$ for user k at the ith time instant, where $\mathbf{w}_k$ is optimized according to a chosen criterion. However, the $M \times 1$ dimensionality of

$\mathbf{w}[i]$ can become large in large systems, leading to computationally intensive implementations and slow convergence when full-rank adaptive algorithms are used. Reduced-rank and SM techniques offer solutions to these problems.

III. SM Reduced-Rank Framework

Reduced-rank techniques for communications achieve dimensionality reduction by projecting the $M \times 1$ received vector $\mathbf{r}[i]$ onto a reduced-rank signal subspace, for example, the Krylov subspace for the MSWF [13], [30] and the AVF [10], [11] with orthogonal auxiliary vectors. The tasks of interference suppression, symbol estimation, and detection can then be performed in the lower dimensionality signal subspace with a standard reduced-length adaptive filter. For user k in a multiuser system, this approach is mathematically expressed as

$$\hat{b}_k[i] = \mathrm{Q}\left(\left[\tilde{\mathbf{w}}_k^H[i]\mathbf{S}_{D_k}^H[i]\mathbf{r}[i]\right]\right) = \mathrm{Q}\left(\left[\tilde{\mathbf{w}}_k^H[i]\tilde{\mathbf{R}}_k[i]\right]\right) \quad (5)$$

where $\tilde{\mathbf{r}}_k[i] = \mathbf{S}_{D_k}^H[i]\mathbf{r}[i]$, and the $M \times D$ projection matrix $\mathbf{S}_{D_k}$ performs the dimensionality reduction. The $D \times 1$ vector $\tilde{\mathbf{w}}[i]$ performs the linear interference suppression, where D is the rank of signal subspace and, therefore, the dimensionality of the filter, where $D \ll M$. However, the majority of techniques prior to the proposition of JIO of adaptive filters relied, in some part, on SVD or a similarly complex task to generate the projection matrix $\mathbf{S}_{D_k}$ [7], [31].

Reduced-rank adaptive filtering based on JIO circumvents these complex tasks by considering the projection matrix and reduced-rank filter as adaptive structures and placing them in a joint optimization function. These two structures are then jointly and iteratively adapted to reach a solution. Expressing this condition as a conventional optimization problem, we arrive at

$$[\mathbf{S}_{D,opt}[i], \tilde{\mathbf{w}}_{opt}[i]] = \arg\min_{\mathbf{S}_D,\tilde{\mathbf{w}}} E\left[\left|b[i] - \tilde{\mathbf{w}}^H[i]\mathbf{S}_D^H[i]\mathbf{r}[i]\right|^2\right] \quad (6)$$

where the user index k has been omitted, and user 1 is assumed—a feature that will continue for the remainder of this paper. The MMSE expressions for these structures are then derived by fixing $\tilde{\mathbf{w}}[i]$ and $\mathbf{S}_D[i]$, in turn, and minimizing with respect to the other, resulting in the following expressions:

$$\tilde{\mathbf{w}}_{opt}[i] = \tilde{\mathbf{R}}^{-1}[i]\tilde{\mathbf{p}}[i] \quad (7)$$

$$\mathbf{S}_{D,opt}[i] = \mathbf{R}^{-1}[i]\mathbf{P}_D[i]\mathbf{R}_w^{-1}[i] \quad (8)$$

where $\tilde{\mathbf{R}}[i] = E[\tilde{\mathbf{r}}_{opt}[i]\tilde{\mathbf{r}}_{opt}^H[i]]$, $\tilde{\mathbf{p}}[i] = E[b^*[i]\tilde{\mathbf{r}}_{opt}[i]]$, $\mathbf{R}[i] = E[\mathbf{r}[i]\mathbf{r}^H[i]]$, and $\mathbf{R}_{\mathbf{w}}[i] = E[\tilde{\mathbf{w}}_{opt}[i]\tilde{\mathbf{w}}_{opt}^H[i]]$ are the reduced- and full-rank input signal autocorrelation matrices and reduced-rank filter autocorrelation matrix, respectively. In addition, $\mathbf{P}_D[i] = E[b^*[i]\mathbf{r}[i]\tilde{\mathbf{w}}_{opt}^H[i]]$ is the reduced-rank cross correlation matrix. The interdependency between $\tilde{\mathbf{w}}_{opt}[i]$ and $\mathbf{S}_{D,opt}[i]$ prohibits a closed-form solution; however, solutions can be reached by iterating (7) and (8) after suitable initialization, which does not annihilate the signal or destabilize the iterative process. The MMSE can then be obtained, as given by

$$\mathrm{MMSE} = \sigma_b^2 - \tilde{\mathbf{p}}^H[i]\tilde{\mathbf{R}}^{-1}[i]\tilde{\mathbf{p}}[i] \quad (9)$$

where $\sigma_b^2 = E[|b[i]|^2]$. The joint optimization structure of the MMSE function given by (6) opens up the possibility of a nonconvex error surface. However, this case is considered in [31], and although multiple solutions exist, there are no local minima when iteratively implemented, and therefore, the adaptive process is not sensitive to initialization.[2] The purely adaptive nature of JIO and its previous implementation with NLMS and RLS algorithms suits it well to integration with SM techniques—one step that is far less involved and problematic but also more complete than previous methods, which use alternative reduced-rank techniques [18]. This condition is due to the well-defined SM framework, which already exists for the algorithms used to implement the JIO schemes.

The generation of the JIO-SM framework resembles a standard SM scheme; however, two solution sets are required at each iteration. To create the JIO-SM framework, first, an expression for the soft symbol estimate and the two error bounds has to be defined as

$$z[i] = \tilde{\mathbf{w}}^H[i-1]\mathbf{S}_D^H[i-1]\mathbf{r}[i] \quad (10)$$

and

$$\begin{aligned} \left|b[i] - \tilde{\mathbf{w}}^H[i]\mathbf{S}_D^H[i-1]\mathbf{r}[i]\right|^2 &\le \gamma_{\tilde{w}}^2 \\ \left|b[i] - \tilde{\mathbf{w}}^H[i-1]\mathbf{S}_D^H[i]\mathbf{r}[i]\right|^2 &\le \gamma_S^2 \end{aligned} \quad (11)$$

where γ_S and $\gamma_{\tilde{w}}$ are the error bounds for the projection matrix and reduced-rank filter, respectively. The structures $\tilde{\mathbf{w}}[i-1]$ and $\mathbf{S}_D[i-1]$ refer to the previous estimate of the reduced-rank filter and projection matrix, respectively, in an iterative estimation procedure.

Then, we define a sample space χ that contains all possible data pairs b and $\mathbf{r}$. We can then define the feasibility sets $\Theta_{\tilde{\mathbf{w}}}$ and $\Theta_{\mathbf{S}_D}$ as subsets of χ that contain the values that fulfill each error bound in (11). These sets for the reduced-rank filter and the projection matrix are, respectively, expressed as

$$\begin{aligned} \Theta_{\tilde{w}} &\triangleq \bigcap_{(b,\mathbf{r})\in\chi} \tilde{\mathbf{w}} \in \mathbb{C}^D : \left|b - \tilde{\mathbf{w}}^H\mathbf{S}_D^H\mathbf{r}\right|^2 \le \gamma_{\tilde{w}}^2 \\ \Theta_{\mathbf{S}} &\triangleq \bigcap_{(b,\mathbf{r})\in\chi} \mathbf{S}_D \in \mathbb{C}^{M\times D} : \left|b - \tilde{\mathbf{w}}^H\mathbf{S}_D^H\mathbf{r}\right|^2 \le \gamma_S^2 \end{aligned} \quad (12)$$

where the alternative adaptive structure is assumed fixed in each.

The final step in the development requires us to apply the feasibility sets to a time-varying scenario; therefore, they contain all estimates that fulfill the respective error criteria at the ith time instant. These sets are termed the constraint sets and are given by

$$\begin{aligned} \mathcal{H}_{\tilde{w}}[i] &= \left\{\tilde{\mathbf{w}}[i] \in \mathbb{C}^D : \left|b[i] - \tilde{\mathbf{w}}^H[i]\mathbf{S}_D^H[i-1]\mathbf{r}[i]\right| \le \gamma_{\tilde{w}}\right\} \\ \mathcal{H}_S[i] &= \left\{\mathbf{S}_D[i] \in \mathbb{C}^{M\times D} : \left|b[i] - \tilde{\mathbf{w}}^H[i-1]\mathbf{S}_D^H[i]\mathbf{r}[i]\right| \le \gamma_S\right\}. \end{aligned} \quad (13)$$

[2]Provided that the signal is not annihilated.

Presuming that the error bounds are chosen to ensure that the constraint sets are nonempty ($\mathcal{H}_{\tilde{w}}[i], \mathcal{H}_S[i] \neq \emptyset$), every point within each set is a valid estimate of the structure. The objective of the SM algorithm is to then select a point that lies in the appropriate constraint set at each time instant.

With the set-theory foundation set out, it is now possible to construct the optimization functions that form the starting point of the algorithms' derivation. For both the NLMS- and LS-based schemes, their derivation begins with a constrained optimization problem formed on the principle of minimal disturbance [42]. This approach corresponds to minimizing the disturbance to the projection matrix and interference suppression filter at each update instant. Accordingly, the distance that is traversed across the sample space at each time instant to reach the current constraint set should be minimized. A natural progression from this condition is that, if the previous estimate lies in the current constraint set, it remains a valid estimate, and therefore, no update is required to satisfy the conditions of the cost function. The result is a sparsely updating algorithm that effectively discards data if they will not result in a sufficient level of innovation.

IV. Proposed Algorithms

In this section, the theory that is set out in Section III is interpreted as two optimization problems, leading to the formation of MSE and LS cost functions. Solving these optimization problems results in two algorithms called JIO-SM-NLMS and JIO-BEACON.

A. SM Reduced-Rank NLMS Algorithm

To derive JIO-SM-NLMS, we consider the following constrained optimization problem:

$$\begin{aligned}[\mathbf{S}_D[i], \tilde{\mathbf{w}}[i]] = \arg \min_{\mathbf{S}_D, \tilde{\mathbf{w}}} \|\tilde{\mathbf{w}}[i] - \tilde{\mathbf{w}}[i-1]\|^2 + \|\mathbf{S}_D[i] - \mathbf{S}_D[i-1]\|^2 \\ \text{subject to } b[i] - \tilde{\mathbf{w}}^H[i]\mathbf{S}_D^H[i-1]\mathbf{r}[i] = \gamma_{\tilde{w}} \\ b[i] - \tilde{\mathbf{w}}^H[i-1]\mathbf{S}_D^H[i]\mathbf{r}[i] = \gamma_S \end{aligned} \quad (14)$$

where the objective is to minimize the disturbance to the projection matrix and reduced-rank filter while satisfying the bounds imposed on the estimation error. To recast (14) as a more readily solvable unconstrained optimization problem, the method of Lagrange multipliers is used, yielding

$$\begin{aligned}\mathcal{L} = &\|\tilde{\mathbf{w}}[i] - \tilde{\mathbf{w}}[i-1]\|^2 + \|\mathbf{S}_D[i] - \mathbf{S}_D[i-1]\|^2 \\ &+ \lambda_1 \left(b[i] - \tilde{\mathbf{w}}^H[i]\mathbf{S}_D^H[i-1]\mathbf{r}[i] - \gamma_{\tilde{w}}\right) \\ &+ \lambda_2 \left(b[i] - \tilde{\mathbf{w}}^H[i-1]\mathbf{S}_D^H[i]\mathbf{r}[i] - \gamma_S\right). \end{aligned} \quad (15)$$

Taking the gradient with respect to the two adaptive structures and equating to zero, the following system of equations is reached:

$$\nabla_{\tilde{w}[i]} = 2\left(\tilde{\mathbf{w}}[i] - \tilde{\mathbf{w}}[i-1]\right) - \mathbf{S}_D^H[i-1]\mathbf{r}[i]\lambda_1 = 0 \quad (16)$$

$$\nabla_{S_D[i]} = 2\left(\mathbf{S}_D[i] - \mathbf{S}_D[i-1]\right) - \mathbf{r}[i]\tilde{\mathbf{w}}^H[i-1]\lambda_2 = 0. \quad (17)$$

Further manipulations then allow us to arrive at expressions for the reduced-rank filter, projection matrix, and Lagrange multipliers, given by

$$\lambda_1 = \frac{2\left(b[i] - \tilde{\mathbf{w}}^H[i-1]\mathbf{S}_D^H[i-1]\mathbf{r}[i] - \gamma_{\tilde{w}}\right)^*}{\mathbf{r}^H[i]\mathbf{S}_D[i-1]\mathbf{S}_D^H[i-1]\mathbf{r}[i]} \quad (18)$$

$$\lambda_2 = \frac{2\left(b[i] - \mathbf{w}^H[i-1]\mathbf{S}_D^H[i-1]\mathbf{r}[i] - \gamma_S\right)^*}{\mathbf{r}^H[i]\mathbf{r}[i]\tilde{\mathbf{w}}^H[i-1]\tilde{\mathbf{w}}[i-1]} \quad (19)$$

$$\tilde{\mathbf{w}}[i] = \tilde{\mathbf{w}}[i-1] + \underbrace{\frac{(e[i] - \gamma_{\tilde{w}})^*}{\mathbf{r}^H[i]\mathbf{S}_D[i-1]\mathbf{S}_D^H[i-1]\mathbf{r}[i]}}_{\bar{\mu}[i]}\mathbf{S}_D^H[i-1]\mathbf{r}[i] \quad (20)$$

$$\mathbf{S}_D[i] = \mathbf{S}_D[i-1] + \underbrace{\frac{(e[i] - \gamma_S)^*}{\mathbf{r}^H[i]\mathbf{r}[i]\tilde{\mathbf{w}}^H[i-1]\tilde{\mathbf{w}}[i-1]}}_{\bar{\eta}[i]}\mathbf{r}[i]\tilde{\mathbf{w}}^H[i-1] \quad (21)$$

where the *a priori* error $e[i]$ is given by

$$e[i] = b[i] - \tilde{\mathbf{w}}^H[i-1]\mathbf{S}_D^H[i-1]\mathbf{r}[i]. \quad (22)$$

For the reduced-rank interference suppression filter, but equally applicable to the projection matrix, the update term $\bar{\mu}$ will attempt to find the shortest path from $\tilde{\mathbf{w}}[i-1]$ to the bounding hyperplane of $\mathcal{H}_{\tilde{w}}[i]$ in accordance with the principle of minimal disturbance, as shown in Fig. 2. However, if $\tilde{\mathbf{w}}[i-1] \in \mathcal{H}_{\tilde{w}}[i]$, it is clear that the error bound constraint is satisfied; therefore, no update is necessary, and $\tilde{\mathbf{w}}[i] = \tilde{\mathbf{w}}[i-1]$. To assess the need for an update a simple innovation check (IC) that consists of an "*if*" statement that compares the *a priori* error to the bound is used. The update terms are then set to zero if the result of the conditional statement is found to be negative, effectively removing the update procedure. When placed into the familiar NLMS structure, we reach the following expressions for the adaptation of the reduced-rank interference suppression filter:

$$\tilde{\mathbf{w}}[i] = \tilde{\mathbf{w}}[i-1] + \mu[i]e^*[i]\mathbf{S}_D^H[i-1]\mathbf{r}[i] \quad (23)$$

where

$$\mu[i] = \begin{cases} \frac{1}{\mathbf{r}^H[i]\mathbf{S}_D[i-1]\mathbf{S}_D^H[i-1]\mathbf{r}[i]}\left(1 - \frac{\gamma_{\tilde{w}}}{|e^*[i]|}\right), & \text{if } |e^*[i]| \leq \gamma_{\tilde{w}} \\ 0, & \text{otherwise.} \end{cases} \quad (24)$$

Similarly, for the projection matrix, we have

$$\mathbf{S}_D[i] = \mathbf{S}_D[i-1] + \eta[i]e^*[i]\mathbf{r}[i]\tilde{\mathbf{w}}^H[i-1] \quad (25)$$

where

$$\eta[i] = \begin{cases} \frac{1}{\mathbf{r}^H[i]\mathbf{r}[i]\tilde{\mathbf{w}}^H[i-1]\tilde{\mathbf{w}}[i-1]}\left(1 - \frac{\gamma_S}{|e^*[i]|}\right), & \text{if } |e^*[i]| \leq \gamma_S \\ 0, & \text{otherwise.} \end{cases} \quad (26)$$

The full algorithm is then formed from the update equations of (22), (23), and (25), where the variable step sizes utilized are given by (24) and (26). The adaptation of the two structures is then mutually exclusive with the projection matrix taking

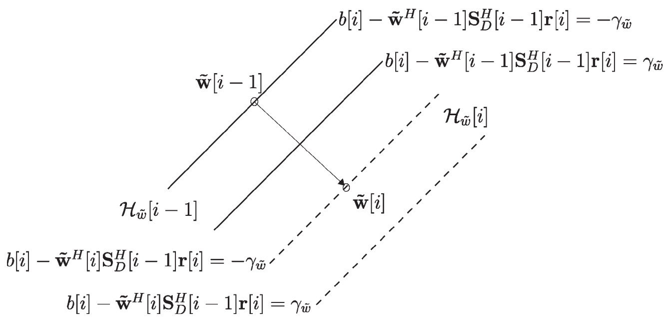

Fig. 2. Geometric interpretation of the JIO-SM-NLMS reduced-rank filter update.

priority; therefore, the reduced-rank filter has the opportunity to adapt when the projection matrix IC is negative.

B. Reduced-Rank BEACON Algorithm

The BEACON algorithm operates by defining the constraint set as a degenerate ellipsoid specified by (11) at each time instant. An additional set that is denoted as the membership set can then be defined as the intersection of every constraint set up to the current time instant ($\cap_{l=1}^{i}\mathcal{H}[i]$) [35]. The OBE algorithm is then used to outerbound the membership set with the centroid of this ellipsoid, which can be taken as a point estimate, i.e., the ith filter value. The definition of the BEACON algorithm in these terms does not initially lend itself to straightforward integration with JIO; however, in [35], BEACON is shown to also be the solution to a constrained LS optimization—an interpretation that enables easier integration with the JIO.

After some initial manipulation, the constrained optimization problem is given by

$$\begin{aligned}[\mathbf{S}_D[i], \tilde{\mathbf{w}}[i]] = \arg \min_{\mathbf{S}_D, \tilde{\mathbf{w}}} \cdots \\ (\tilde{\mathbf{w}}[i] - \tilde{\mathbf{w}}[i-1])^H \mathbf{P}^{-1}[i] (\tilde{\mathbf{w}}[i] - \tilde{\mathbf{w}}[i-1]) \\ + (\mathbf{S}_D[i] - \mathbf{S}_D[i-1])^H \mathbf{C}^{-1}[i] \\ \times (\mathbf{S}_D[i] - \mathbf{S}_D[i-1]) \\ \text{subject to } \left|b[i] - \tilde{\mathbf{w}}^H[i]\mathbf{S}_D^H[i-1]\mathbf{r}[i]\right|^2 \leq \gamma_{\tilde{w}}^2 \\ \left|b[i] - \tilde{\mathbf{w}}^H[i-1]\mathbf{S}_D^H[i]\mathbf{r}[i]\right|^2 \leq \gamma_S^2\end{aligned} \tag{27}$$

where

$$\mathbf{P}[i] = \mathbf{P}[i-1] - \frac{\lambda_{\tilde{w}}[i]\mathbf{P}[i-1]\mathbf{S}_D^H[i-1]\mathbf{r}[i]\mathbf{r}^H[i]\mathbf{S}_D[i-1]\mathbf{P}[i-1]}{1+\lambda_{\tilde{w}}[i]G[i]} \tag{28}$$

$$\mathbf{C}[i] = \mathbf{C}[i-1] - \frac{\lambda_S[i]\mathbf{C}[i-1]\mathbf{r}[i]\tilde{\mathbf{w}}^H[i-1]\tilde{\mathbf{w}}[i-1]\mathbf{r}^H[i]\mathbf{C}[i-1]}{1+\lambda_S[i]F[i]}. \tag{29}$$

To continue with the derivation, (27) is recast as an unconstrained optimization problem with the use of the method of Lagrange multipliers, yielding

$$\begin{aligned}\mathcal{L} = &(\tilde{\mathbf{w}}[i] - \tilde{\mathbf{w}}[i-1])^H \mathbf{P}^{-1}[i] (\tilde{\mathbf{w}}[i] - \tilde{\mathbf{w}}[i-1]) \\ &+ (\mathbf{S}_D[i] - \mathbf{S}_D[i-1])^H \mathbf{C}^{-1}[i] (\mathbf{S}_D[i] - \mathbf{S}_D[i-1]) \\ &+ \lambda_{\tilde{w}}[i]\left(\left|b[i] - \tilde{\mathbf{w}}^H[i]\mathbf{S}_D^H[i-1]\mathbf{r}[i]\right|^2 - \gamma_{\tilde{w}}^2\right) \\ &+ \lambda_S[i]\left(\left|b[i] - \tilde{\mathbf{w}}^H[i-1]\mathbf{S}_D^H[i]\mathbf{r}[i]\right|^2 - \gamma_S^2\right).\end{aligned} \tag{30}$$

Forming a system of equations by taking the gradient of (30) with respect to the adaptive structures, we get

$$\begin{aligned}\nabla_{\tilde{w}}[i] = &(\tilde{\mathbf{w}}[i] - \mathbf{w}[i-1])\mathbf{P}^{-1}[i] - \lambda_{\tilde{w}}[i]\mathbf{r}^H[i]\mathbf{S}_D[i-1]\ldots \\ &\left(b[i] - \tilde{\mathbf{w}}^H[i]\mathbf{S}_D^H[i-1]\mathbf{r}[i]\right).\end{aligned} \tag{31}$$

$$\begin{aligned}\nabla_S[i] = &(\mathbf{S}_D[i] - \mathbf{S}_D[i-1])\mathbf{C}^{-1}[i] - \lambda_S[i]\mathbf{r}[i]\tilde{\mathbf{w}}^H[i-1]\ldots \\ &\left(d[i] - \tilde{\mathbf{w}}^H[i-1]\mathbf{S}_D^H[i]\mathbf{r}[i]\right).\end{aligned} \tag{32}$$

Further manipulation then allows us to reach intermediate expressions for the reduced-rank filter and projection matrix, respectively, i.e.,

$$\tilde{\mathbf{w}}[i] = \tilde{\mathbf{w}}[i-1] + \frac{\lambda_{\tilde{w}}[i]\mathbf{P}[i]\mathbf{r}^H[i]\mathbf{S}_D[i-1]\delta[i]\gamma_{\tilde{w}}}{|\delta[i]|} \tag{33}$$

$$\mathbf{S}_D[i] = \mathbf{S}_D[i-1] + \frac{\lambda_S[i]\mathbf{C}[i]\mathbf{r}[i]\tilde{\mathbf{w}}^H[i-1]\delta[i]}{1+\lambda_S[i]F[i]} \tag{34}$$

where

$$\delta[i] = b[i] - \tilde{\mathbf{w}}^H[i-1]\mathbf{S}_D^H[i-1]\mathbf{r}[i]. \tag{35}$$

To arrive at a recursive estimation procedure for each structure, $\mathbf{P}[i-1]$ and $\mathbf{C}[i-1]$ are used to calculate the auxiliary scalar variables $G[i]$ and $F[i]$, respectively, where

$$G[i] = \mathbf{r}^H[i]\mathbf{S}_D[i-1]\mathbf{P}[i-1]\mathbf{S}_D^H[i-1]\mathbf{r}[i] \tag{36}$$

$$F[i] = \tilde{\mathbf{w}}^H[i-1]\tilde{\mathbf{w}}[i-1]\mathbf{r}^H[i]\mathbf{C}[i-1]\mathbf{r}[i]. \tag{37}$$

Using the relationship

$$1+\lambda_{\tilde{w}}[i]G[i] = 1 + \frac{1}{G[i]}\left(\frac{|\delta[i]|}{\gamma_{\tilde{w}}} - 1\right)G[i] = \frac{|\delta[i]|}{\gamma_{\tilde{w}}} \tag{38}$$

the final reduced-rank filter update equations are reached, i.e.,

$$\tilde{\mathbf{w}}[i] = \tilde{\mathbf{w}}[i-1] + \frac{\lambda_{\tilde{w}}[i]\mathbf{P}[i]\mathbf{S}_D^H[i-1]\mathbf{r}[i]\delta[i]}{1+\lambda_{\tilde{w}}[i]G[i]} \tag{39}$$

where

$$\lambda_{\tilde{w}}[i] = \begin{cases}\frac{1}{G[i]}\left(\frac{|\delta[i]|}{\gamma_{\tilde{w}}} - 1\right), & \text{if } |\delta[i]| \geq \gamma_{\tilde{w}} \\ 0, & \text{otherwise}\end{cases} \tag{40}$$

and again, the *if* statement forms the IC. In an analogous manner to step size in the NLMS variant, the forgetting factor is set to zero, thus skipping the update procedure when the *if* statement returns a negative result. Similarly, for the projection matrix, the relationship given by

$$1+\lambda_S[i]F[i] = 1 + \frac{1}{F[i]}\left(\frac{|\delta[i]|}{\gamma_{\tilde{w}}} - 1\right)F[i] = \frac{|\delta[i]|}{\gamma_S} \tag{41}$$

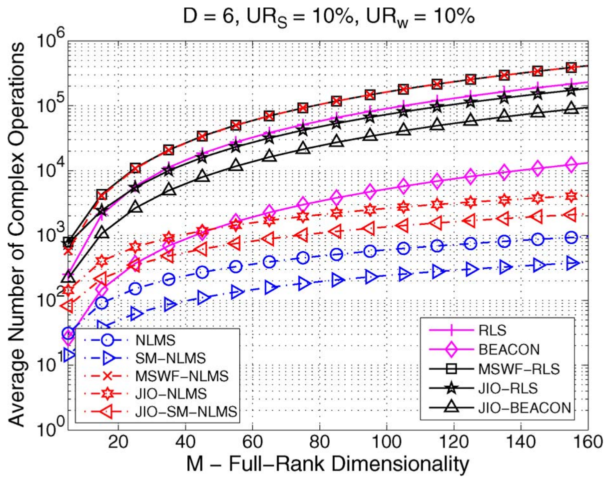

Fig. 3. Computational complexity of the proposed and existing algorithms.

is used to arrive at the recursive update procedure, i.e.,

$$\mathbf{S}_D[i] = \mathbf{S}_D[i-1] + \frac{\lambda_S[i]\mathbf{C}[i]\mathbf{r}[i]\tilde{\mathbf{w}}^H[i-1]\delta[i]}{1+\lambda_S[i]F[i]} \tag{42}$$

where

$$\lambda_S[i] = \begin{cases} \frac{1}{F[i]}\left(\frac{|\delta[i]|}{\gamma_S} - 1\right), & \text{if } |\delta[i]| \geq \gamma_S \\ 0, & \text{otherwise.} \end{cases} \tag{43}$$

The final algorithm then iteratively operates using (35), (39), and (42), where the variable forgetting factors are given by (40) and (43). The projection matrix adaptation again takes priority over the reduced-rank filter in the same manner as the JIO-SM-NLMS algorithm.

C. Computational Complexity

The potential complexity reductions made by the proposed algorithms are closely related to the frequency of updates or update rates, denoted UR_S and UR_w for the projection matrix and reduced-rank interference suppression filter, respectively. These terms are defined as the fraction of received symbols that result in an update of their respective structure. Fig. 3 shows a comparison between the complexity of the conventional full- and reduced-rank algorithms and of the proposed schemes. The results shown in the figure are based on update rates of 10% for all SM schemes—a rate that is readily achievable during the training of the algorithms—and a rank of D = 6. The accompanying analytical expressions for the algorithm complexities are given by Table I.

In Fig. 3, we can see that the complexity savings of approximately an order of magnitude are possible for JIO-BEACON and approximately 63% for JIO-SM-NLMS, both of which are substantial savings with regard to power consumption in mobile and wireless sensor networks.

D. Rank Adaptation Algorithm

The dimensionality of the subspace of a reduced-rank algorithm has an impact on performance, in a manner analogous to conventional adaptive filtering. This condition therefore allows the rank of the proposed schemes to act as an additional optimization parameter. By adjusting the rank of the subspace depending on the stage of operation, it is possible to obtain performance improvements. In practical scenarios, this case corresponds to using lower ranks during the convergence of algorithms to aid the convergence and increased ranks for steady-state operations. Such methods have been proposed [11], [13], [20], [30], but the application of an automatic rank selection algorithm to a SM scheme has not been featured. The integration of a rank adaptation feature involves the formulation of a cost function that allows the optimum rank to be determined. In [15], an exponentially weighted LS *a posteriori* cost function was used, and a similar approach will be used here. However, the adaptation of the rank will only be permitted when the *a priori* error exceeds the appropriate bounds and an update is performed. The chosen rank D_{opt} will be constrained to lie between D_{min} and D_{max} and selected according to the expression

$$D_{opt}[i] = \begin{cases} \underset{D_{min} \leq D \geq D_{max}}{\arg\min} \mathcal{R}\left(\tilde{\mathbf{w}}_D[i], \mathbf{S}_D[i]\right), & \text{if } |e[i]| \geq \gamma \\ D[i-1], & \text{otherwise} \end{cases} \tag{44}$$

where

$$\mathcal{R}\left(\tilde{\mathbf{w}}_D[i], \mathbf{S}_D[i]\right) = \sum_{l=1}^{i} \beta^{i-l} \left|b[l] - \tilde{\mathbf{w}}_D^H[i]\mathbf{S}_D^H[i]\mathbf{r}[l]\right|^2. \tag{45}$$

The values of D_{min} and D_{max} are chosen to offer optimum performance throughout the data record over which the algorithm operates, and β is the exponential weighting factor for ensuring a smooth transition between subspace ranks. Fig. 4 displays the relationship between the rank and performance of the NLMS-based schemes with optimized parameters. We can see that the optimum range of ranks equates to $D_{min} = 2$ and $D_{max} = 10$, and therefore, these bounds will be used for relevant simulations that will later be featured in this paper.

E. Adaptive Variable Error Bound

The determination of the error bounds for a SM scheme is a complex task and requires knowledge of the parameters of the considered system. Inappropriate error bound selection results in the possibility of underbounding and overbounding, and associated performance degradation and complexity increases [34], [36]. By incorporating selected system parameters into the formulation of a variable bound for the proposed algorithms, it is possible to not only reduce the probability of encountering bounding problems but also remove the need for an accurate determination of error bounds prior to the operation of the schemes. In this paper, we concentrate on the implementation of parameter-dependent bounds that require knowledge of the projection matrix and reduced-rank filter, both of which are available at the receiver, and the noise variance of the system. For the implementation given here, we assume knowledge of the noise variance; however, it is readily obtainable through noise

TABLE I
COMPUTATIONAL COMPLEXITY OF THE PROPOSED AND EXISTING ALGORITHMS

Algorithm	Average number of complex operations per iteration: Additions	Multiplications
NLMS	$3M-1$	$3M+2$
RLS	$4M^2$	$5M^2+5M+2$
SM-NLMS	$UR_w(2M+4)+M+1$	$UR_w(2M)+M+1$
BEACON	$UR_w(3M^2+M+7)+M+1$	$UR_w(2M^2+M)+M+1$
MSWF-NLMS	$M^2(D+2)+M(D+1)-D-2$	$M^2(D+2)+M(2D+3)+3D+3$
MSWF-RLS	$M^2(D+2)+M(D+1)+4D^2-D-1$	$M^2(D+2)+M(2D+3)+3D+3$
JIO-NLMS	$M(2D+1)-3D-4$	$M(2D+1)+5D+5$
JIO-RLS	$3M^2+M(3D-2)+3D^2$	$4M^2+M(2D+1)+8D^2+4D+6$
JIO-SM-NLMS	$UR_S(M(D+1)+D-1)+UR_w(2D)+DM+4$	$UR_S(M(D+1)+2D+5)+UR_w(2D+4)+MD+D$
JIO-BEACON	$UR_S(M^2-M+D^2+4D+1)+UR_w(6D^2+2)+DM+1$	$UR_S(2M^2D+2MD+D^2+4D+10)+$ $UR_w(6D^2+2D+7)+MD+D+1$

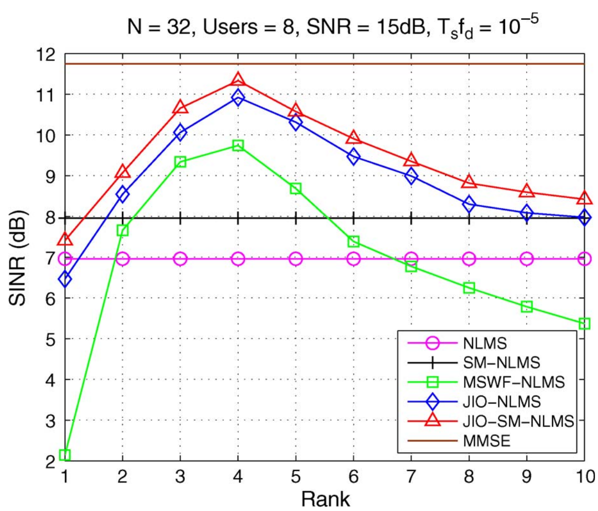

Fig. 4. NLMS scheme rank comparison.

estimation algorithms [43], [44]. The variable error bounds for the projection matrix and reduced-rank filter are given by

$$\gamma_S[i+1] = (1-\beta)\gamma_S[i] + \beta\sqrt{\alpha_S \|\mathbf{S}_D[i]\tilde{\mathbf{w}}[i]\| \hat{\sigma}_n^2[i]} \quad (46)$$

$$\gamma_{\tilde{w}}[i+1] = (1-\beta)\gamma_{\tilde{w}}[i] + \beta\sqrt{\alpha_{\tilde{w}} \|\mathbf{S}_D[i]\tilde{\mathbf{w}}[i]\| \hat{\sigma}_n^2[i]} \quad (47)$$

where β is a forgetting factor, and α_S and $\alpha_{\tilde{w}}$ are tuning parameters for the projection matrix and reduced-rank filter bounds, respectively. The motivation behind the formation of the variable bound expressions lies in providing the SM process with information on the noise at the output of the filtering process—an approximation given by the term $\sqrt{\alpha_S\|\mathbf{S}_D[i]\tilde{\mathbf{w}}[i]\|\hat{\sigma}_n^2[i]}$. Time averaging is then performed by β to ensure stable transitions between error bound values and overall stability. For added protection against the risk of overbounding and inaccurate symbol estimation, a ceiling value is implemented with regard to the bounds. For the power-normalized QPSK system considered in this paper, these ceiling values are set at $\gamma_{S_{max}} = 0.7$ and $\gamma_{\tilde{w}_{max}} = 0.65$ for both the JIO-SM-NLMS and JIO-BEACON schemes.

V. Analysis

The MSE analysis of SM schemes presents a number of unique and challenging problems compared to conventional adaptive algorithms. The variable convergence parameters and the sparse updates significantly increase the complexity of the analysis. However, methods for partially taking account of these SM features have been presented in [45] and [46] for system identification, including a "probability of update" term that accounts for the sparse adaptation and simplifying assumptions about the variable convergence parameters. The analysis of JIO-SM is substantially more complex than conventional SM analysis; however, the aforementioned methods will still be used to aid the analysis. We begin by incorporating the probability of update terms into the recursive equations for the adaptation of the reduced-rank interference suppression filter and the projection matrix. This method yields the following expressions

$$\tilde{\mathbf{w}}[i] = \tilde{\mathbf{w}}[i-1] + \mu[i]P_{\tilde{w}_{up}}e^*[i]\mathbf{S}_D^H[i-1]\mathbf{r}[i] \quad (48)$$

$$\mathbf{S}_D[i] = \mathbf{S}_D[i-1] + \eta[i]P_{S_{up}}e^*[i]\mathbf{r}[i]\tilde{\mathbf{w}}^H[i-1] \quad (49)$$

for JIO-NLMS and

$$\tilde{\mathbf{w}}[i] = \tilde{\mathbf{w}}[i-1] + \frac{\lambda_{\tilde{w}}[i]P_{\tilde{w}_{up}}\mathbf{P}[i]\mathbf{S}_D^H[i-1]\mathbf{r}[i]\delta[i]}{1+\lambda_{\tilde{w}}[i]G[i]} \quad (50)$$

$$\mathbf{S}_D[i] = \mathbf{S}_D[i-1] + \frac{\lambda_S[i]P_{S_{up}}\mathbf{C}[i]\mathbf{r}[i]\tilde{\mathbf{w}}^H[i-1]\delta[i]}{1+\lambda_S[i]F[i]} \quad (51)$$

for JIO-BEACON. $P_{S_{up}}$ and $P_{\tilde{w}_{up}}$ are the probability of update terms for the projection matrix and reduced-rank interference suppression filter, respectively, and are the analytical embodiment of the update rates. To increase the accuracy and practicality of analyzing the SM schemes, we remove their dependency on the *a priori* error by confining the analysis to the excess error under steady-state conditions. This method allows more accurate models of the probability of update to be formed, because it can be assumed that $P_{S_{up}/\tilde{w}_{up}}$ are constant and reflect the probability of the steady-state error that exceeds the appropriate bound.

A. Stability

With regard to much of the JIO material covered in this paper, the analysis of its application to adaptive interference suppression is limited. However, with methods that are inspired by beamforming analysis in [47], we can begin to approach the stability analysis of the proposed JIO-SM-NLMS algorithm.

The spectral-radius technique can be used for JIO-SM-NLMS but depends on obtaining recursive relationships for the error weight vector and error weight matrix for the reduced-rank filter and projection matrix, respectively. To do this approach, we take expanded versions of (48) and (49), which include $P_{\tilde{w}_{up}}$, $P_{S_{up}}$, and the optimized step sizes, and substitute into the reduced-rank filter error weight vector $\boldsymbol{\varepsilon}_{\tilde{w}}[i+1] = \tilde{\mathbf{w}}[i+1] - \tilde{\mathbf{w}}_{opt}[i+1]$ and the projection matrix error weight matrix $\boldsymbol{\varepsilon}_{S_D}[i+1] = \mathbf{S}_D[i+1] - \mathbf{S}_{D,opt}[i+1]$ equations, yielding

$$\begin{aligned}\boldsymbol{\varepsilon}_{\tilde{w}}[i+1] = &\left(\mathbf{I} - \mu[i]P_{\tilde{w}_{up}}\mathbf{S}_D^H[i]\mathbf{r}[i]\mathbf{r}^H[i]\mathbf{S}_D^H[i]\right)\boldsymbol{\varepsilon}_{\tilde{w}}[i] \\ &+ \mu[i]P_{\tilde{w}_{up}}b^*[i]\mathbf{S}_D^H[i]\mathbf{r}[i] \\ &- \mu[i]P_{\tilde{w}_{up}}\mathbf{S}_D^H[i]\mathbf{r}[i]\mathbf{r}^H[i]\mathbf{S}_D^H[i]\tilde{\mathbf{w}}_{opt} \qquad (52)\end{aligned}$$

$$\begin{aligned}\boldsymbol{\varepsilon}_S[i+1] = &\boldsymbol{\varepsilon}_S[i]\left(\mathbf{I} - \eta[i]P_{S_{up}}\mathbf{r}[i]\mathbf{r}^H[i]\right) \\ &- \eta[i]P_{S_{up}}\mathbf{r}[i]\tilde{\mathbf{w}}^H[i]\mathbf{r}^H[i]\mathbf{S}_D[i]\boldsymbol{\varepsilon}_{\tilde{w}}[i] \\ &- \eta[i]P_{S_{up}}\mathbf{r}[i]\tilde{\mathbf{w}}^H[i]\mathbf{r}^H[i]\mathbf{S}_D[i]\tilde{\mathbf{w}}_{opt} \\ &+ \eta[i]P_{S_{up}}b^*[i]\mathbf{r}[i]\tilde{\mathbf{w}}^H[i]. \qquad (53)\end{aligned}$$

The expectations of (52) and (53) are then taken, and a recursive expression is reached, i.e.,

$$\begin{bmatrix}\mathrm{E}\left(\boldsymbol{\varepsilon}_{\tilde{w}}[i+1]\right) \\ \mathrm{E}\left(\boldsymbol{\varepsilon}_S[i+1]\right)\end{bmatrix} = \mathbf{B}\begin{bmatrix}\mathrm{E}\left(\boldsymbol{\varepsilon}_{\tilde{w}}[i]\right) \\ \mathrm{E}\left(\boldsymbol{\varepsilon}_S[i]\right)\end{bmatrix} + \mathbf{T} \qquad (54)$$

where

$$\mathbf{B} = \begin{bmatrix}\mathbf{I} - \mu[i]P_{\tilde{w}_{up}}\tilde{\mathbf{R}} & \mathbf{0} \\ -\eta[i]P_{S_{up}}\mathrm{E}\left(\mathbf{r}[i]\tilde{\mathbf{w}}^H[i]\mathbf{r}^H[i]\mathbf{S}_D[i]\right) & I - \mu[i]P_{S_{up}}\mathbf{R}\end{bmatrix} \qquad (55)$$

$$\mathbf{T} = \begin{bmatrix}\mu[i]P_{\tilde{w}_{up}}(\tilde{\mathbf{p}} - \tilde{\mathbf{R}}\tilde{\mathbf{w}}_{opt}) \\ \eta[i]P_{S_{up}}\mathbf{p}\tilde{\mathbf{w}}^H[i] - \eta[i]P_{S_{up}}\mathrm{E}\left(\mathbf{r}[i]\tilde{\mathbf{w}}^H[i]\mathbf{r}^H[i]\mathbf{S}_D[i]\right)\end{bmatrix} \qquad (56)$$

and $\tilde{\mathbf{p}} = \mathrm{E}(b^*[i]\tilde{\mathbf{r}}[i])$ and $\tilde{\mathbf{R}} = \mathrm{E}(\tilde{\mathbf{r}}[i]\tilde{\mathbf{r}}^H[i])$. The stability of the algorithm can then be determined from the spectral radius of (54). For convergence, the eigenvalues of $\mathbf{B}^H\mathbf{B}$ should not exceed 1 at each time instant—one factor that is partly ensured by the operation of the SM algorithms and its optimized step sizes. Numerical studies can then verify (54) and its ability to determine the stability.

B. Steady-State MSE

In this section, we study the steady-state MSE of the proposed schemes and develop expressions that allow the steady-state MSE to more accurately be predicted compared to using the *a priori* error bound.

1) JIO-SM-NLMS: The interdependency of the adaptive structures in JIO pose several problems when approaching the analysis; consequently, a semianalytical steady-state error solution is sought. The analysis begins by forming an M-dimensional expression for the MSE, where an equivalent M-dimensional low-rank filter is obtained by an inverse mapping of the reduced-rank interference suppression filter, given by

$$\mathbf{w}[i] = \mathbf{S}_D[i]\tilde{\mathbf{w}}[i]. \qquad (57)$$

The MSE can then be expressed as

$$J[i] = \mathrm{E}\left[\left|b[i] - \mathbf{w}^H[i]\mathbf{r}[i]\right|^2\right]. \qquad (58)$$

After straightforward manipulation, the MSE's dependency on the full-rank error weight matrix can be obtained as

$$J[i] = J_{min}[i] + \mathrm{tr}\left(\mathrm{E}\left[\boldsymbol{\varepsilon}_w^H[i]\mathbf{r}[i]\mathbf{r}^H[i]\boldsymbol{\varepsilon}_w[i]\right]\right) \qquad (59)$$

where $\boldsymbol{\varepsilon}_w[i] = \mathbf{w}[i] - \mathbf{w}_{opt}$, and $J_{min} = E[|b[i] - \mathbf{w}_{opt}^H\mathbf{r}[i]|]$. The second term in (59) is equal to the excess MSE and can be rearranged into a form that is appropriate for the analysis and pursuit of an expression for the steady-state error, i.e.,

$$J[i] = J_{min}[i] + \underbrace{\mathrm{tr}\left(\mathrm{E}\left[\mathbf{r}[i]\mathbf{r}^H[i]\boldsymbol{\varepsilon}_w[i]\boldsymbol{\varepsilon}_w^H[i]\right]\right)}_{\mathrm{J_{ex}}}. \qquad (60)$$

The first step is to reach a recursive expression for the full-rank equivalent filter error vector. To do this technique, we substitute (23) and (25) into (57) and then subtract the optimum full-rank equivalent filter, yielding

$$\begin{aligned}\boldsymbol{\varepsilon}_w[i+1] = &\boldsymbol{\varepsilon}_w[i] + \frac{\mu[i]P_{\tilde{w}_{up}}e^*[i]}{\tilde{\mathbf{r}}[i]\tilde{\mathbf{r}}[i]}\mathbf{S}_D[i]\mathbf{S}_D^H[i]\mathbf{r}[i] \\ &+ \frac{\eta[i]P_{S_{up}}e^*[i]}{\tilde{\mathbf{w}}^H[i]\tilde{\mathbf{w}}[i]\mathbf{r}^H\mathbf{r}[i]}\mathbf{r}[i]\tilde{\mathbf{w}}^H[i]\tilde{\mathbf{w}}[i] \\ &+ \frac{\mu[i]\eta[i]P_{\tilde{w}_{up}}P_{S_{up}}e^*[i]e^*[i]}{\tilde{\mathbf{r}}^H[i]\tilde{\mathbf{r}}^H[i]\tilde{\mathbf{w}}^H[i]\tilde{\mathbf{w}}[i]\mathbf{r}^H\mathbf{r}[i]} \\ &\times \mathbf{r}[i]\tilde{\mathbf{w}}^H[i]\mathbf{S}_D^H[i]\mathbf{r}[i]. \qquad (61)\end{aligned}$$

At this point in the derivation, certain simplifying assumptions can be made as a result of the proposed algorithm structure, particularly the mutually exclusive updates. This step can be expressed by

$$P\left((\mu[i] \neq 0) \cap (\eta[i] \neq 0)\right) = 0. \qquad (62)$$

Therefore, we can assume that $\mu[i]\eta[i] = 0$ and accordingly remove terms. The next step is to substitute (61) into the expression for J_{ex} in (60). The convenient manipulation available due to the trace of expectation operators allows a number of terms to be simplified and removed, resulting in a recursive expression for J_{ex}. Then, assuming that $P_{S_{up}}[i]$, $P_{\tilde{w}_{up}}[i]$, and $J_{ex}[i]$ are constant under the conditions

$$\begin{aligned}\lim_{i\to\infty} P_{S_{up}}[i] &= P_{S_{up}} \\ P_{\tilde{w}_{up}}[i] &= P_{\tilde{w}_{up}} \\ J_{ex}[i] &= J_{ex} \qquad (63)\end{aligned}$$

we can arrive at an expression for the steady-state excess error of the JIO-SM-NLMS algorithm, i.e.,

$$J_{ex} = J_{\min} \frac{\left(\mu^2 P_{\tilde{w}_{up}}^2 + \eta^2 P_{S_{up}}^2\right)}{\mu P_{\tilde{w}_{up}}(2 - \mu P_{\tilde{w}_{up}}) + \eta P_{S_{up}}(2 - \eta P_{S_{up}})} \quad (64)$$

where $\mu = E[\mu[i]]$, and $\eta = E[\eta[i]]$.

Unlike the majority of existing SM analysis [45], [46], which concentrate on system identification, the analysis here is in relation to an interference suppression scenario, and therefore, certain simplifying assumptions about the properties of the input signal cannot be made. This condition results in a steady-state error expression, which, although different, has a similar structure.

As we can see, (64) depends on $P_{S_{up}/\tilde{w}_{up}}$ and has to be obtained to arrive at an analytical expression. Assuming that the additive noise is white and Gaussian and that the estimation error has reached its steady-state value, the variations in J_{ex} can also be assumed to be Gaussian. Therefore, the probability that the steady-state error exceeds the bound can be modeled by a complementary Gaussian cumulative distribution or Q-function. Ideally, (64) would be used to obtain an accurate value for the steady-state error and, therefore, the probability that it exceeds the error bound. However, its dependency on $P_{S_{up}/\tilde{w}_{up}}$ prohibits such an approach. The alternative is to approximate the upper and lower bounds on the probability of update for the projection matrix based on J_{min}, γ_S, and σ_n^2. A lower bound can then be approximated by

$$P_{Sup_{min}} = 2Q\left(\frac{\gamma_S}{\sqrt{\sigma_n^2 + J_{min}}}\right) \quad (65)$$

and an upper bound can be approximated by

$$P_{Sup_{max}} = 2Q\left(\frac{\gamma_S}{\sqrt{\sigma_n^2 + \gamma_S^2}}\right) \quad (66)$$

where the factor of 2 ensures that $Q(0) = P_{up} = 1$ for $\gamma_S = 0$. The difference in application from the existing SM analysis also has an impact here, because it is unrealistic to assume that the minimum error is bounded by the noise variance. Consequently, the lower bound is assumed to be the error from the optimum equivalent full-rank filter with the addition of the noise variance. As before, the upper bound of the error is approximated by the sum of the SM error bound and the noise variance. However, as will become clear in Section V-B, using γ_S as an upper bound is not a satisfactory approximation of the error for the γ_S of interest, and in supporting simulations, the update rate is considerably better modeled by (65) than (66); therefore, (65) will be used for the remainder of this paper. Due to the higher update priority given to the projection matrix, the update characteristics of the reduced-rank filter $P_{\tilde{w}_{up}}$ differ from P_{Sup} and is therefore more suitably modeled using a semianalytical approach, where $P_{\tilde{w}_{up}}$ can be approximated from comparable simulations. This approach minimizes the divergence of the theory from the simulations.

The second pair of quantities required for the calculation of (64) are the expectation of the step sizes. In [46], the worst case scenario step size is used, and a similar approach will be taken in this paper, where the step size can take any value in the range (0, 1) but will be set to $\eta = 1$ and $\mu = 0.1$.

2) JIO-BEACON: The analysis presented in Section V-B1 extends the currently available analysis for SM schemes and will now be applied to the JIO-BEACON algorithm. The derivation presented here for JIO-BEACON follows a similar method and begins by forming an expression for the full-rank-equivalent filter error weight vector so that J_{ex} can be calculated from (60). By substituting (33) and (34) into (57) and again subtracting the optimum full-rank-equivalent filter, we reach

$$\begin{aligned}\boldsymbol{\varepsilon}_w[i+1] = {} & \boldsymbol{\varepsilon}_w[i] + \mathbf{S}_D[i] \frac{P_{\tilde{w}_{up}}}{G[i]} \lambda_{\tilde{w}}[i] \mathbf{P}[i] \tilde{\mathbf{r}}[i] e^*[i] \\ & + \frac{P_{S_{up}}}{1 + \lambda_S[i]} \left(\frac{\lambda_S[i]}{F[i]} \mathbf{C}[i] \mathbf{A}[i] \delta^*[i]\right) \tilde{\mathbf{w}}[i] \\ & + \frac{P_{S_{up}} P_{\tilde{w}_{up}} \lambda_S[i] \lambda_{\tilde{w}}[i]}{(1 + \lambda_S[i])\, G[i]} \\ & \left(\frac{\lambda_S[i]}{F[i]} \mathbf{C}[i] \mathbf{A}[i] \delta^*[i]\right) \ldots \mathbf{P}[i] \tilde{\mathbf{R}}[i] e^*[i] \end{aligned} \quad (67)$$

where $\mathbf{A}[i] = \mathbf{r}[i] \tilde{\mathbf{w}}^H[i]$. Using an equivalent simplification to the simplification found in the JIO-SM-NLMS analysis, i.e.,

$$P\left((\lambda_S[i] \neq 0) \cap (\lambda_{\tilde{w}}[i] \neq 0)\right) = 0 \quad (68)$$

we can then arrive at an expression for JIO-BEACON steady-state error as

$$J_{ex} = J_{\min} \frac{P_{\tilde{w}_{up}}^2 \lambda_{\tilde{w}}^2 + \frac{P_{S_{up}}^2 \lambda_S^2}{(1+\lambda_S)^2}}{2P_{\tilde{w}_{up}} \lambda_{\tilde{w}} + 2P_{S_{up}} \lambda_S - P_{\tilde{w}_{up}}^2 \lambda_{\tilde{w}}^2 - \frac{P_{S_{up}}^2 \lambda_S^2}{(1+\lambda_S)^2}}. \quad (69)$$

The expressions in (67)–(69) are included here for mathematical completeness and are similar to JIO-SM-NLMS. The main differences between (69) and (64) stem from the fact that the JIO-SM-NLMS uses a variable step size, whereas JIO-BEACON employs a variable forgetting factor.

VI. Simulations

In this section, the performance of the algorithms presented in this paper is compared to existing full- and reduced-rank schemes. Comparisons will be made in terms of convergence and tracking performance using the signal-to-interference-plus-noise ratio (SINR) and bit error rate (BER) as metrics. Throughout all simulations, the JIO- and MSWF-based schemes have a reduced dimensionality subspace of ranks 4 and 6, respectively. Each simulation is averaged over 2000 independent runs, and where the channel is nonstationary, the fading rate is given by the dimensionless normalized fading parameter $T_s f_d$, which is specified in each plot. All MMSE-based filters, full and reduced rank, are initialized as $w[0] = \delta \times (\mathbf{1})$, and all LS-based filters are initialized as $w[0] = (\mathbf{0})$ and $w(1) = 1$. Projection matrices will be initialized as $\mathbf{S}_D[0] = \zeta \times \left[\begin{smallmatrix}\mathbf{I}\\ \mathbf{0}\end{smallmatrix}\right]$, where δ and ζ are small positive constants.

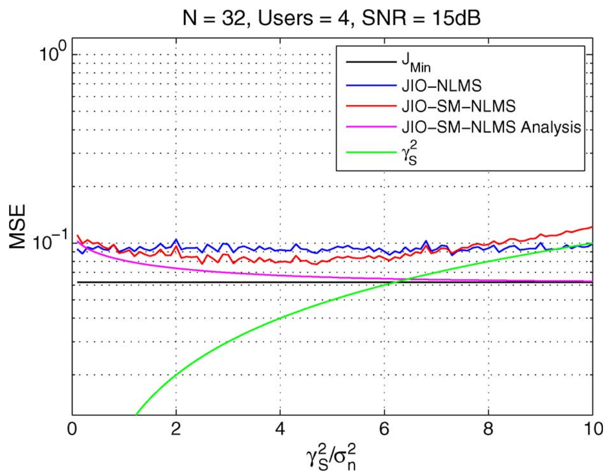

Fig. 5. Analytical MSE performance.

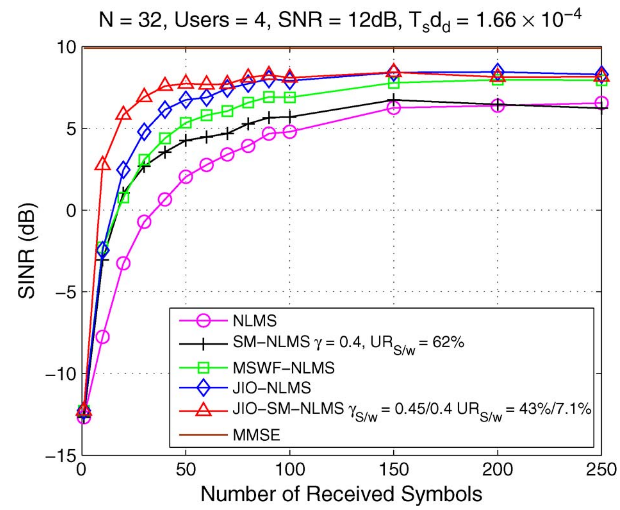

Fig. 6. SINR performance comparison of the MSE algorithms with 150 training symbols.

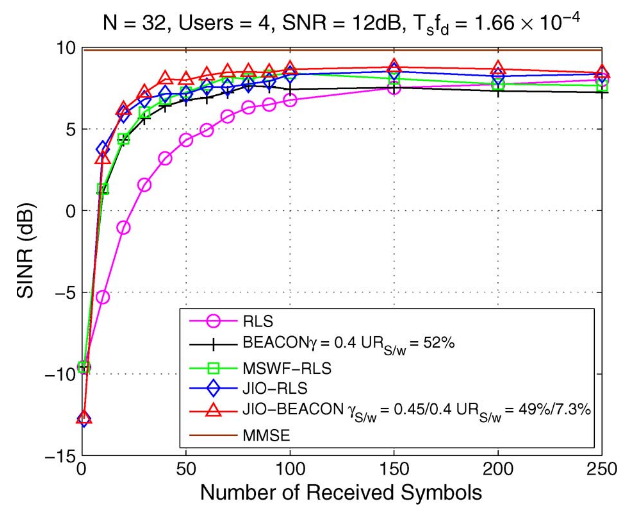

Fig. 7. SINR performance comparison of the LS algorithms with 100 training symbols.

A. Analytical MSE Performance

In this section, the analytical expressions and approximations derived in Section V-B for the JIO-SM-NLMS steady-state error are validated through comparison to the simulations of the proposed schemes. The JIO-SM-NLMS applied here makes use of a fixed estimation error bound and therefore exhibits a specific MSE performance characteristic where the optimum MSE performance is obtained when the bound is small but nonzero, a value which we define as $\gamma_{S,opt}$. Consequently, two methods of estimating the MSE are required; for small γ_S, the MSE expression $J[\infty] \approx J_{min} + J_{ex}$ can be used to provide an accurate lower bound, but for $\gamma_S > \gamma_{S,opt}$, $J[\infty] = \gamma^2$ acts as an increasingly accurate error approximation.

In the following simulations, a lightly loaded system with a spreading gain of 32 is used and operates in a stationary environment with a signal-to-noise ratio (SNR) of 15 dB. The simulated and analytical MSE are plotted against the projection matrix error bound, which has been normalized by the noise power σ_n^2, and the reduced-rank filter error bound is set to $\gamma_S - 0.05$.

As shown in Fig. 5, the analytical MSE provides a lower and significantly more accurate bound on the MSE of the JIO-SM schemes for $\gamma_S < \gamma_{S,opt}$ compared to γ_S^2, where $\gamma_{S,opt}^2/\sigma_n^2 \sim 4$, and therefore verifies the method of analysis presented in this paper. However, as aforementioned, γ_S^2 acts as a more accurate bound for a larger steady-state error, and therefore, the use of either technique depends on the relative level of the error bound.

B. SINR and BER Performance

In the simulations presented, each algorithm has an initial period of training and then switches to decision-directed operation. The step sizes throughout all the simulations were set to $\mu = 0.25$ for the full-rank NLMS and MSWF-NLMS, and $\eta = 0.25$ and $\mu = 0.1$ for the reduced-rank interference suppression filter and projection matrix adaptation for JIO-NLMS, respectively. The exponential forgetting factor for the LS-based schemes is $\lambda = 0.998$ for the convergence, multiuser, and SNR performance simulations.

The performance of the stochastic gradient-based schemes is shown in Fig. 6. The convergence of the proposed scheme is shown to exceed the conventional JIO and MSWF algorithms while having a considerably lower computational complexity. The proposed scheme is also shown to achieve an equal steady-state SINR compared to the conventional implementation.

Fig. 7 gives the performance of the LS-based schemes and shows that the proposed scheme exhibits improved convergence performance compared to the conventional JIO-RLS. It also reaches a maximum SINR close to the MSE while achieving a significant 50% reduction in complexity. In addition, the performance after convergence shows that JIO-BEACON outperforms JIO-RLS, indicating that the SM scheme has maintained the capability to mitigate the effects of a fading channel.

Fig. 8 gives the performance of the proposed and existing reduced-rank algorithms when the spreading sequence length is increased to 64 and the system is heavily loaded. Both of the proposed schemes show an improvement over the MSWF schemes, and the JIO-SM-NLMS exceeds the performance of the conventional JIO while making complexity savings. The

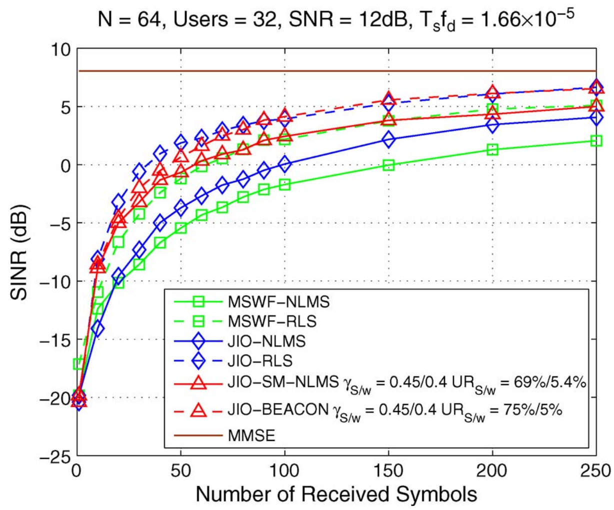

Fig. 8. SINR performance comparison of the LS and MSE algorithms with 250 training symbols and increased spreading sequence length.

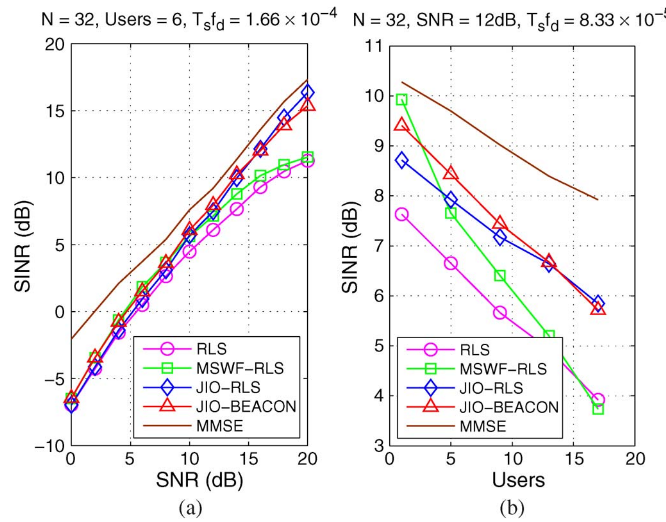

Fig. 10. LS schemes: SNR and multiuser performance after 100 training symbols. (a) SINR versus system SNR. (b) SINR versus system loading.

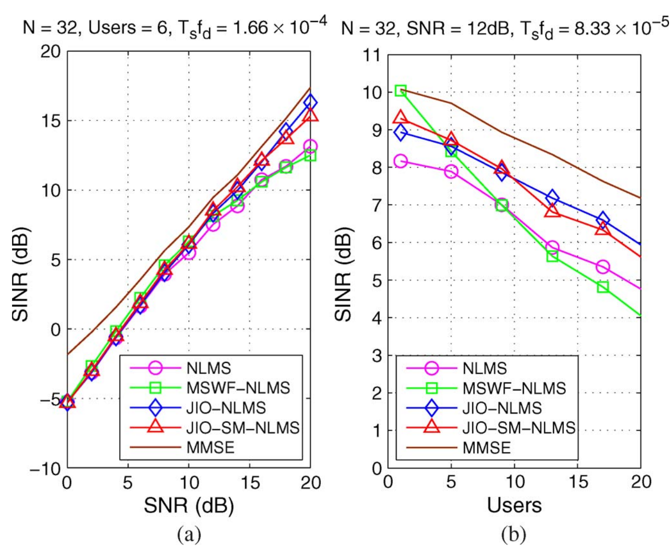

Fig. 9. MSE schemes: SNR and multiuser performance after 150 training symbols. (a) SINR versus SNR. (b) SINR versus system loading.

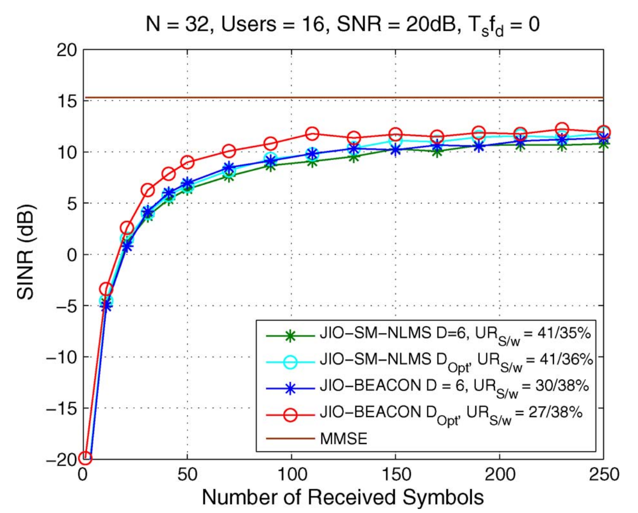

Fig. 11. Performance comparison of the automatic rank selection algorithms.

performance of BEACON-JIO has dropped compared with JIO-RLS when processing these more highly spread signals, but complexity savings are still made.

Fig. 9(a) and (b) shows the SINR performance of the proposed MSE-based scheme versus the system SNR and loading, respectively, for a fading channel after 150 training symbols. Again, the JIO-SM performs better than the conventional scheme at practical SNRs while achieving a reduction in complexity. However, at high SNR, its performance declines in line with the other schemes. The performance of the scheme under increasing system loads is good and exceeds the conventional scheme at moderate system loads. However, its performance degrades similar to MSWF when interference suppression becomes more challenging.

The SINR performance versus system SNR and loading for the LS-based schemes is shown in Fig. 10(a) and (b). Again, the simulations have been conducted with a fading channel and 100 training symbols. In Fig. 10(a), the proposed scheme performs well and closely matches the performance of the conventional JIO for SNRs of interest and makes substantial complexity savings. The performance of the scheme in Fig. 10(b) at low system loading is good but degrades at higher loading levels compared with the conventional JIO-RLS.

The SINR plot in Fig. 11 shows the performance gains that are possible when the automatic rank selection feature from Section IV-D is incorporated into the proposed algorithms. We can see that the automatic rank selection improves the steady-state SINR performance while also exceeding the convergence performance of the fixed-rank algorithms. The update rates associated with the schemes in Fig. 11 differ from the previous simulations because of the larger gap between the error bounds placed on the adaptive structures. In this simulation, the projection matrix and reduced-rank filter bounds are $\gamma_S = 0.6$ and $\gamma_w = 0.3$, respectively; therefore, an increase in the probability of the reduced-rank filter updating results.

The performance and complexity improvements brought about by an adaptive variable error bound are illustrated in

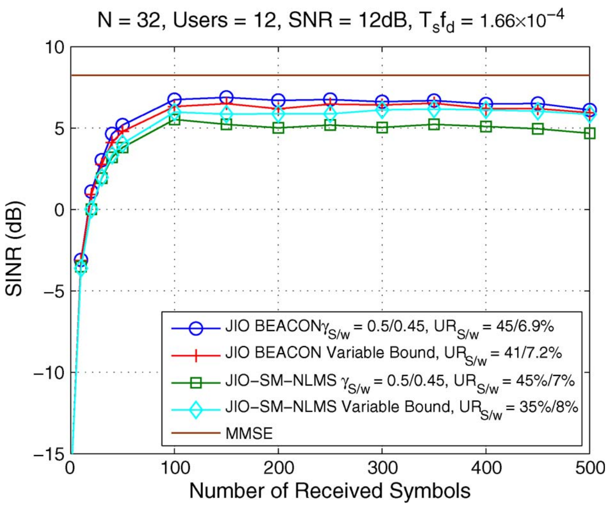

Fig. 12. SINR performance of the proposed JIO-SM-NLMS and JIO-BEACON algorithms with variable γ_S and $\gamma_{\tilde{w}}$, where $\alpha_S = 5$, and $\alpha_{\tilde{w}} = 4$, and a training sequence of 100 symbols.

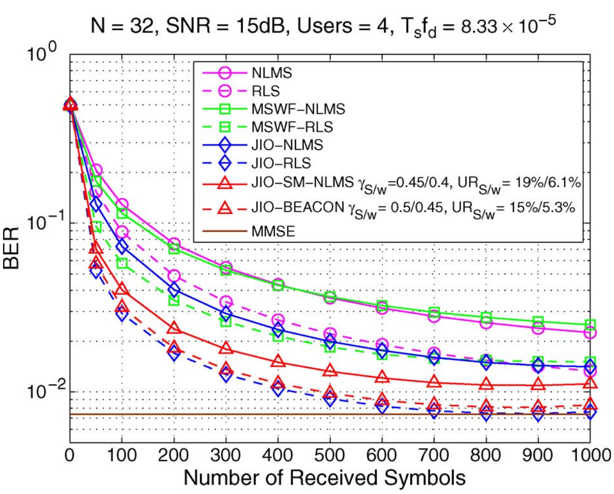

Fig. 13. BER performance comparison.

Fig. 12. Improvements in both convergence and steady-state SINR of the JIO-SM-NLMS result while also achieving a reduction in complexity. For the JIO-BEACON scheme, an increase in the steady-state performance is achieved, along with an increase in the maximum obtainable SINR, but at the cost of a marginally higher projection matrix updated ratio. However, this increase in complexity is lower than the complexity associated with simply tightening the fixed error bound.

Fig. 13 shows the uncoded BER performance of the proposed schemes and existing reduced-rank schemes along with the two most common full-rank schemes. The schemes are trained with 500 symbols and then switched to a decision-directed operation. As we can see, both of the SM reduced-rank algorithms exceed the convergence of the MSWF schemes and reach a lower steady-state error; however, as has been documented, MSWF-NLMS fails to tridiagonalize its covariance matrix and therefore struggles in this scenario. JIO-BEACON exhibits excellence performance and, along with the NLMS implementation, achieves significant complexity savings, which increase with the SNR.

VII. Conclusion

This paper has presented a SM reduced-rank framework based on the JIO of receive parameters. The sparse updates and optimized convergence parameters associated with SM schemes were brought to the adaptation of the rank-reduction matrix and the reduced-rank interference suppression filter. LS, NLMS, automatic rank adaptation, and adaptive error bound algorithms were presented, along with their application to interference suppression in the uplink of a time-varying multiuser DS-CDMA system. A novel analysis of the proposed schemes has been given, along with the limitations of applying existing SM analysis techniques to the proposed algorithms. Simulations have then been presented and illustrate that the performance of the proposed schemes closely matches the existing reduced-rank schemes while achieving a significant reduction in computational complexity.

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Patrick Clarke (S’09) received the M.Eng. degree in electronic engineering from the University of York, York, U.K., where he is currently working toward the Ph.D. degree with the Communications Research Group, Department of Electronics.

His research interests include adaptive signal processing and cooperative communications.

Rodrigo C. de Lamare (S’99–M’04–SM’10) received the Diploma degree in electronic engineering from the Federal University of Rio de Janeiro, Rio de Janeiro, Brazil, in 1998 and the M.Sc. and Ph.D. degrees in electrical engineering from the Pontifical Catholic University of Rio de Janeiro, Rio de Janeiro, in 2001 and 2004, respectively.

Since January 2006, he has been with the Communications Research Group, Department of Electronics, University of York, York, U.K., where he is currently a Senior Lecturer in communications engineering. He serves as the Associate Editor for the *EURASIP Journal on Wireless Communications and Networking*. His research interests include communications and signal processing, for which he has published more than 200 papers in refereed journals and conference proceedings.

Dr. de Lamare served as the General Chair of the Seventh IEEE International Symposium on Wireless Communications Systems, which was held in York in September 2010.