Set-Membership Adaptive Algorithms Based on Time-Varying Error Bounds for CDMA Interference Suppression

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***Abstract*—This paper presents set-membership (SM) adaptive algorithms based on time-varying error bounds for code-division multiple-access (CDMA) interference suppression. We introduce a modified family of SM adaptive algorithms for parameter estimation with time-varying error bounds. The considered algorithms include modified versions of the SM normalized least mean square (SM-NLMS), the affine projection (SM-AP), and the bounding ellipsoidal adaptive constrained (BEACON) recursive least-square technique. The important issue of error-bound specification is addressed in a new framework that takes into account parameter estimation dependency, multiaccess, and intersymbol interference (ISI) for direct-sequence CDMA (DS-CDMA) communications. An algorithm for tracking and estimating the interference power is proposed and analyzed. This algorithm is then incorporated into the proposed time-varying error bound mechanisms. Computer simulations show that the proposed algorithms are capable of outperforming previously reported techniques with a significantly lower number of parameter updates and a reduced risk of overbounding or underbounding.**

***Index Terms*—Adaptive filters, direct-sequence code-division multiple access (DS-CDMA), interference suppression, set-membership filtering (SMF).**

I. Introduction

Set-membership filtering (SMF) [1]–[4] represents a class of recursive estimation algorithms that, on the basis of a predetermined error bound, seeks a set of parameters that yield bounded filter output errors. These algorithms have been used in a variety of applications such as adaptive equalization [5] and multiaccess interference suppression [6], [7]. The SMF algorithms are able to combat conflicting requirements such as fast convergence and low misadjustment by introducing a modification on the objective function. In addition, these algorithms exhibit reduced complexity due to data-selective updates, which involve two steps: 1) information evaluation and 2) update of parameter estimates. If the filter update does not frequently occur, and information evaluation does not involve much computational complexity, the overall complexity can significantly be reduced.

The adaptive SMF algorithms usually achieve good convergence and tracking performance due to an adaptive step size for each update and reduced complexity resulting from data-selective updating. However, the performance of SMF techniques depends on the error bound specification, which is very difficult to obtain in practice due to the lack of knowledge of the environment and its dynamics. In wireless networks characterized by nonstationary environments, where users often enter and exit the system, it is very difficult to choose an error bound, and the risk of overbounding (when the error bound is larger than the actual one) and underbounding (when the error bound is smaller than the actual one) is significantly increased, leading to performance degradation. In addition, when the measured noise in the system is time varying, and the multiple-access interference (MAI) and the intersymbol interference (ISI) encountered by a receiver in a communication system are highly dynamic, the selection of an error bound is further complicated. This is particularly relevant for low-complexity estimation problems encountered in applications such as mobile terminals and wireless sensor networks [8], where the sensors have limited signal processing capabilities, and power consumption is of central importance. These problems suggest the deployment of mechanisms to automatically adjust the error bound to guarantee good performance and a small update rate (UR). It should also be remarked that most of the prior work on adaptive algorithms for interference suppression [18] is restricted to systems with short codes. However, the proposed adaptive techniques are also applicable to systems with long codes provided some modifications are carried out. For downlink scenarios, the designer can resort to chip equalization [23], followed by a despreader. For an uplink solution, channel estimation algorithms for aperiodic sequences [24], [25] are required, and the sample average approach for estimating the covariance matrix $\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^H(i)]$ of the observed data $\mathbf{r}(i)$ has to be replaced by $\hat{\mathbf{R}} = \mathbf{P}\mathbf{P}^H + \sigma^2\mathbf{I}$, which is constructed with a matrix $\mathbf{P}$ containing the effective signature sequence of users and the variance σ^2 of the receiver's noise [26].

Previous works on time-varying error bounds include the tuning of noise bounds in [9] and [10], the approach in [13], which assumes that the "true" error bound is constant, and the parameter-dependent error bound recently proposed in

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[11] and [12] with frequency-domain estimation algorithms. The techniques reported so far do not introduce any mechanism for tracking the MAI and the ISI and incorporating their power estimates in the error bound. In addition, the existing approaches with time-varying bounds have not been considered for more sophisticated adaptive filtering algorithms such as affine projection (AP)- and least-square (LS)-based techniques.

In this paper, we propose and analyze a low-complexity framework for tracking parameter evolution and MAI and ISI power levels that relies on simple channel and interference estimation techniques and encompasses a family of set-membership (SM) algorithms [2], [3], [6], [14] with time-varying error bounds. Specifically, we present modified versions of the SM normalized least mean square (SM-NLMS) [2], the affine projection [3] (SM-AP), and the bounding ellipsoidal adaptive constrained (BEACON) [6], [14] recursive LS (RLS) algorithm for parameter estimation. Then, we incorporate the proposed mechanisms of interference estimation and tracking into the time-varying error bounds. To evaluate the proposed algorithms, we consider a direct-sequence code-division multiple-access (DS-CDMA) interference suppression application and adaptive linear multiuser receivers in situations of practical interest.

This paper is organized as follows. Section II briefly describes the DS-CDMA system model and linear receivers. Section III reviews the SMF concept with time-varying error bounds and is devoted to the derivation of adaptive algorithms. Section IV presents the framework for time-varying error bounds and the proposed algorithms for channel, interference estimation, and tracking. Section V is dedicated to the analysis of the algorithms for channel, amplitude, interference estimation, and their tracking. Section VI shows and discusses the simulation results, whereas Section VII gives the conclusions.

II. DS-CDMA System Model and Linear Receivers

Let us consider the downlink of a symbol synchronous DS-CDMA system with K users, N chips per symbol, and L_p propagation paths [18]. We assume that the delay is a multiple of the chip rate, the channel is constant during each symbol interval, and the spreading codes are repeated from symbol to symbol. The received signal $r(t)$ after filtering by a chip-pulse matched filter and sampled at the chip rate yields the M-dimensional received vector

$$\mathbf{r}[i] = \sum_{k=1}^{K} A_k[i] b_k[i] \mathbf{C}_k \mathbf{h}[i] + \boldsymbol{\eta}[i] + \mathbf{n}[i] \qquad (1)$$

where $M = N + L_p - 1$, and $\mathbf{n}[i] = [n_1[i] \quad \cdots \quad n_M[i]]^T$ is the complex Gaussian noise vector with zero mean and covariance matrix $E[\mathbf{n}[i]\mathbf{n}^H[i]] = \sigma^2 \mathbf{I}$, where $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian transpose, respectively. The quantity $E[\cdot]$ stands for expected value, and the user k symbol is $b_k[i]$ and is assumed to be drawn from a general constellation. The amplitude of user k is $A_k[i]$, and $\boldsymbol{\eta}[i]$ is the ISI for user k. The $M \times L_p$ convolution matrix $\mathbf{C}_k$ that contains one-chip shifted versions of the signature sequence for user k expressed by $\mathbf{s}_k = [a_k(1) \quad \cdots \quad a_k(N)]^T$ and the $L_p \times 1$ vector $\mathbf{h}[i]$ with multipath components are described by

$$\mathbf{C}_k = \begin{bmatrix} a_k(1) & & \mathbf{0} \\ \vdots & \ddots & a_k(1) \\ a_k(N) & & \vdots \\ \mathbf{0} & \ddots & a_k(N) \end{bmatrix} \quad \mathbf{h}[i] = \begin{bmatrix} h_0[i] \\ \vdots \\ h_{L_p-1}[i] \end{bmatrix}. \qquad (2)$$

In this model, the ISI span and contribution $\boldsymbol{\eta}_k[i]$ are functions of the processing gain N and L_p. If $1 < L_p \leq N$, then three symbols would interfere in total, i.e., the current, previous, and successive symbols. In the case of $N < L_p \leq 2N$, then five symbols would interfere, i.e., the current, the two previous, and the two successive symbols. In most practical CDMA systems, we have that $1 < L_p \leq N$, and then, only three symbols are usually affected. See Universal Mobile Telecommunications System (UMTS) channel models [21], which reveal that the channel usually affects at most three symbols (it typically spans a few chips).

The multiuser linear receiver design corresponds to determining a finite-impulse response (FIR) filter $\mathbf{w}_k[i] = [w_0[i] \quad w_1[i] \quad \cdots \quad w_{M-1}]^T$ with M coefficients that provide an estimate of the desired symbol as given by

$$\hat{b}_k[i] = \mathrm{sgn}\left(\Re\left[\mathbf{w}_k^H[i]\mathbf{r}[i]\right]\right) = \mathrm{sgn}\left(\Re\left[z_k[i]\right]\right) \qquad (3)$$

where the quantity $\Re(\cdot)$ selects the real part, and $\mathrm{sgn}(\cdot)$ is the signum function. The quantity $z_k[i] = \mathbf{w}_k^H[i]\mathbf{r}[i]$ is the output of the receiver parameter vector $\mathbf{w}_k$ for user k, which is optimized according to a chosen criterion.

III. SM Adaptive Algorithms With Time-Varying Error Bounds and Problem Statement

In this section, we describe a framework that encompasses modified SM adaptive algorithms with time-varying error bounds for communication applications. In an SM filtering [2] framework, the parameter vector $\mathbf{w}_k[i]$ for user k in a multi-access system is designed to achieve a specified bound on the magnitude of the estimation error $e_k[i] = b_k[i] - \mathbf{w}_k^H[i]\mathbf{r}[i]$. As a result of this constraint, the SM adaptive algorithm will only perform filter updates for certain data. Let $\Theta_k[i]$ represent the set containing all $\mathbf{w}_k[i]$ that yields an estimation error upper bounded in magnitude by a time-varying error bound $\gamma_k[i]$. Thus, we can write

$$\Theta_k[i] = \bigcap_{(b_k[i],\mathbf{r}[i]) \in \boldsymbol{S}_k} \left\{\mathbf{w}_k \in \mathcal{C}^M : \mid e_k[i] \mid \leq \gamma_k[i]\right\} \qquad (4)$$

where $\mathbf{r}[i]$ is the observation vector, $\boldsymbol{S}_k$ is the set of all possible data pairs $(b_k[i], \mathbf{r}[i])$, and the set $\Theta_k[i]$ is referred to as the feasibility set for user k, and any point in it is a valid estimate $z_k[i] = \mathbf{w}_k^H[i]\mathbf{r}[i]$. Since it is not practical to predict all data pairs, adaptive methods work with the membership sets $\psi_{k,i} = \bigcap_{m=1}^{i} \mathcal{H}_{k,m}$ provided by the observations, where $\mathcal{H}_{k,m} = \{\mathbf{w}_k \in \mathcal{C}^M : |b_k[m] - z_k[m]| \leq \gamma_k[m]\}$ is the constraint set. It can be seen that the feasibility set $\Theta_k[i]$ is a subset of the exact membership set at any given time instant. The feasibility set $\Theta_k[i]$ is also the limiting set of the exact membership set, i.e., the two sets will be equal if the training signal traverses all signal pairs belonging to $\boldsymbol{S}_k$. The idea of the SM algorithms is to adaptively find an estimate that belongs to the feasibility set $\Theta_k[i]$. One alternative is to apply one of the many optimal bounding ellipsoidal (OBE) algorithms such as the BEACON [6], [14] RLS algorithm, which tries to approximate the exact membership set with ellipsoids. Another way is to compute a point estimate through projections using, for example, the information provided by the constraint set $\mathcal{H}_{k,i}$, as done by the SM-NLMS and SM-AP [3] algorithms. To devise an effective SM algorithm, the error bound $\gamma_k[i]$ must appropriately be chosen. Due to the time-varying nature of many practical environments, this error bound should also be adaptive and adjustable to certain characteristics of the environment for the SM estimation technique. The natural question that arises is how to design an efficient and effective mechanism to adjust $\gamma_k[i]$. In what follows, we will present a modified family of SM adaptive algorithms that rely on general time-varying error bounds. Specifically, we will consider the SM-NLMS [2], SM-AP [3], and BEACON [6], [14] algorithms, and we will modify them such that they will operate with general time-varying error bounds.

A. SM-NLMS Algorithm With Time-Varying Bounds

To derive an SM-NLMS adaptive algorithm with time-varying bounds using point estimates, we consider the following optimization problem:

$$\begin{aligned}&\text{minimize } \|\mathbf{w}_k[i+1]-\mathbf{w}_k[i]\|^2\\ &\text{subject to } \left(b_k[i]-\mathbf{w}_k^H[i+1]\mathbf{r}[i]\right)=g_k[i].\end{aligned} \tag{5}$$

To solve the foregoing constrained optimization problem, we resort to the method of Lagrange multipliers [1], [22], which yields the unconstrained cost function

$$\begin{aligned}\mathcal{L}=&\|\mathbf{w}_k[i+1]-\mathbf{w}_k[i]\|^2\\ &+2\Re\left[\lambda^*\left(b_k[i]-\mathbf{w}_k^H[i+1]\mathbf{r}[i]-g_k[i]\right)\right]\end{aligned} \tag{6}$$

where $*$ denotes complex conjugate, λ is a Lagrange multiplier, and $g_k[i]$ is the time-varying SM constraint for user k. Taking the gradient terms of (6) with respect to $\mathbf{w}_k[i+1]$ and $\lambda*$, and setting them to zero, leads us to a system of equations. Solving these equations yields

$$e_k[i]=b_k[i]-\mathbf{w}_k^H[i]\mathbf{r}[i] \tag{7}$$

$$\mathbf{w}_k[i+1]=\mathbf{w}_k[i]+\left(\mathbf{r}^H[i]\mathbf{r}[i]\right)^{-1}\left(e_k[i]-g_k[i]\right)^*\mathbf{r}[i] \tag{8}$$

where $e_k[i]$ is the error for user k. By choosing $g_k[i]$ such that $e_k[i]$ lies on the closest boundary of $\Theta_k[i]$ and considering a time-varying error bound $\gamma_k[i]$, i.e., $g_k[i]=\gamma_k[i]\mathrm{sgn}(e_k[i])$ [2], we obtain the following data-dependent update rule and step size:

$$\mathbf{w}_k[i+1]=\mathbf{w}_k[i]+\mu_w[i]e_k^*[i]\mathbf{r}[i] \tag{9}$$

$$\mu_w[i]=\begin{cases}\frac{1}{\mathbf{r}^H[i]\mathbf{r}[i]}\left(1-\gamma_k[i]/|e_k^*[i]|\right), & \text{if } |e_k^*[i]|>\gamma_k[i]\\ 0, & \text{otherwise.}\end{cases} \tag{10}$$

B. SM-AP Algorithm With Time-Varying Bounds

To describe a modified SM-AP algorithm with time-varying bounds, let us first define the observation matrix $\mathbf{Y}[i]=[\mathbf{r}[i]\ \cdots\ \mathbf{r}[i-P+1]]$, the desired output vector $\mathbf{b}_k[i]=[b_k[i]\ \cdots\ b_k[i-P+1]]^T$ that comprises P outputs, and the error vector

$$\begin{aligned}\mathbf{e}_k[i]&=\begin{bmatrix}b_k^*[i]-\mathbf{r}^H[i]\mathbf{w}_k[i]\\ \vdots\\ b_k^*[i]-\mathbf{r}^H[i-P+1]\mathbf{w}_k[i]\end{bmatrix}\\ &=\mathbf{b}_k^*[i]-\mathbf{Y}^H[i]\mathbf{w}_k[i].\end{aligned} \tag{11}$$

The SM-AP adaptive algorithm with time-varying bounds can be derived from the optimization problem

$$\begin{aligned}&\text{minimize } \|\mathbf{w}_k[i+1]-\mathbf{w}_k[i]\|^2\\ &\text{subject to } \left(\mathbf{b}_k[i]-\mathbf{Y}[i]\mathbf{w}_k^H[i+1]\mathbf{r}[i]\right)=\mathbf{g}_k[i].\end{aligned} \tag{12}$$

To solve the foregoing problem, we employ the method of Lagrange multipliers and consider the unconstrained cost function

$$\begin{aligned}\mathcal{L}=&\|\mathbf{w}_k[i+1]-\mathbf{w}_k[i]\|^2\\ &+2\Re\left[\left(\mathbf{b}_k[i]-\mathbf{Y}^H[i]\mathbf{w}_k[i+1]-\mathbf{g}_k[i]\right)^H\boldsymbol{\lambda}\right]\end{aligned} \tag{13}$$

where $\boldsymbol{\lambda}$ is the vector with Lagrange multipliers, and $\mathbf{g}_k[i]$ is a constraint vector. By calculating the gradient terms of (13) with respect to $\mathbf{w}_k[i+1]$ and $\boldsymbol{\lambda}$, setting them to zero, and solving the resulting equations, we arrive at the following algorithm:

$$\mathbf{t}_k[i]=\left(\mathbf{Y}^H[i]\mathbf{Y}[i]+\delta\mathbf{I}\right)^{-1}\left(\mathbf{e}_k[i]-\mathbf{g}[i]\right) \tag{14}$$

$$\mathbf{w}_k[i+1]=\mathbf{w}_k[i]+\mathbf{Y}[i]\mathbf{t}_k[i] \tag{15}$$

where δ is a small constant inserted in addition to the term $\mathbf{Y}^H[i]\mathbf{Y}[i]$ for improving robustness. If we select $\mathbf{e}_k[i]-\mathbf{g}_k[i]=(e_k[i]-\gamma\mathrm{sgn}(e_k[i]))\mathbf{u}=(1-\gamma_k[i]/|e_k[i]|)e_k[i]\mathbf{u}$, where the *a posteriori* errors $e_k[i-j]$ are kept constant for $j=1,\ldots,P-1$ and $\mathbf{u}=[1\ \ 0\ \ \cdots\ \ 0]^T$, we obtain the following recursion for the update of $\mathbf{t}_k[i]$:

$$\mathbf{t}_k[i]=\left(\mathbf{Y}^H[i]\mathbf{Y}[i]+\delta\mathbf{I}\right)^{-1}\left(1-\gamma/|e_k[i]|\right)e_k[i]\mathbf{u}. \tag{16}$$

Substituting (16) into (15) and using the bound constraint, we obtain the following SM-AP algorithm:

$$\mathbf{w}_k[i+1]=\mathbf{w}_k[i]+\mu_w[i]\mathbf{Y}[i]\left(\mathbf{Y}^H[i]\mathbf{Y}[i]+\delta\mathbf{I}\right)^{-1}e_k[i]\mathbf{u} \tag{17}$$

$$\mu_w[i]=\begin{cases}\left(1-\gamma_k[i]/|e_k[i]|\right), & \text{if } |e_k[i]|>\gamma_k[i]\\ 0, & \text{otherwise.}\end{cases} \tag{18}$$

The SM-AP algorithm described here has a computational complexity of UR $\times\ \mathcal{O}(PM+2K_{\mathrm{inv}}P^2)$, where K_{inv} is a factor required to invert a $P\times P$ matrix [1]. Note that SM-AP is a generalized case of the SM-NLMS, where P data vectors are used to increase the convergence speed.

C. BEACON Adaptive Algorithm With Time-Varying Bounds

Here, we propose a modification for a computationally efficient version of an OBE algorithm called the BEACON LS algorithm [6], which is closely related to a constrained LS optimization problem. The proposed technique amounts to deriving the BEACON algorithm equipped with time-varying bounds. Unlike other previously reported low-complexity algorithms [2], [3] and the modified SM-NLMS and SM-AP techniques described in the previous sections, the modified BEACON recursion has the potential to achieve a very fast convergence performance, which is relatively independent from the eigenvalue spread of the data covariance matrix, as compared with the stochastic gradient (SG) algorithms [1].

The proposed BEACON algorithm with a time-varying bound can be derived from the following optimization problem [6]:

$$\text{minimize} \sum_{l=1}^{i-1} \lambda^{i-l}[l]\, |b_k[l] - \mathbf{w}_k[i]^{\mathbf{r}}[l]|^2$$
$$\text{subject to } \left|\mathbf{b}_k[i] - \mathbf{w}_k[i]^H \mathbf{r}[i]\right|^2 = \gamma_k^2[i]. \tag{19}$$

The foregoing constrained problem can be recast as an unconstrained problem and be solved via an unconstrained LS cost function using the method of Lagrange multipliers given by

$$\mathcal{L} = \sum_{l=1}^{i-1} \lambda_k[l]^{i-l} \left|b_k[l] - \mathbf{w}_k^H[i]\mathbf{r}[l]\right|^2 + \lambda_k[i]\left(\left|b_k[i] - \mathbf{w}_k^H[i]\mathbf{r}[i]\right|^2 - \gamma_k^2[i]\right) \tag{20}$$

where $\lambda_k[l]$ plays the role of the Lagrange multiplier and forgetting factor at the same time for user k. The solution to the above optimization problem is obtained by taking the gradient terms with respect to $\mathbf{w}_k[i]$ and making them equal to zero. After some mathematical manipulations, we have

$$\mathbf{w}_k[i] = \mathbf{w}_k[i-1] + \lambda_k[i]\mathbf{P}_k[i]\mathbf{r}[i]\left(b_k[i] - \mathbf{w}_k^H[i]\mathbf{r}[i]\right)^*. \tag{21}$$

By using the constraint $b_k[i] - \mathbf{w}_k^H[i]\mathbf{r}[i] = (\xi_k[i]/|\xi_k[i]|)\gamma_k[i] = \xi_k[i] - (\lambda_k[i]\mathbf{G}_k[i](\xi_k[i]/|\xi_k[i]|)\gamma_k[i])$ and the matrix inversion lemma [1], we can arrive at the BEACON algorithm with time-varying bounds described by

$$\mathbf{P}_k[i] = \mathbf{P}_k[i-1] - \frac{\lambda_k[i]\mathbf{P}_k[i-1]\mathbf{r}[i]\mathbf{r}^H[i]\mathbf{P}_k[i-1]}{1 + \lambda_k[i]t_k[i]} \tag{22}$$
$$\mathbf{w}_k[i] = \mathbf{w}_k[i-1] + \lambda_k[i]\mathbf{P}_k[i]\mathbf{r}[i]\xi_k^*[i] \tag{23}$$

where the prediction error is $\xi_k[i] = b_k[i] - \mathbf{w}_k^H[i-1]\mathbf{r}[i]$, $t_k[i] = \mathbf{r}^H[i]\mathbf{P}_k[i-1]\mathbf{r}[i]$, and $\lambda_k[i] \geq 0$. To compute the optimal value for $\lambda_k[i]$, the algorithm considers the following cost function [6]:

$$\mathcal{C}_{\lambda_k[i]} = \lambda_k[i]\left[\frac{\xi_k[i]}{\gamma_k^2[i]}\left(\frac{1}{1 + \lambda_k[i]t_k[i]}\right) - 1\right]. \tag{24}$$

The minimization of the cost function in (24) leads to the innovation check of the proposed BEACON algorithm

$$\lambda_k[i] = \begin{cases} \frac{1}{t_k[i]}\left(\frac{|\xi_k[i]|}{\gamma_k[i]} - 1\right), & \text{if } |\xi_k^*[i]| > \gamma_k[i] \\ 0, & \text{otherwise.} \end{cases} \tag{25}$$

IV. Algorithms for Time-Varying Error Bounds, Interference Estimation, and Tracking

This section is devoted to time-varying error bounds that incorporate parameter and interference dependency. We propose a low-complexity framework for time-varying error bounds, interference estimation, and tracking. A simple and effective algorithm for taking into consideration parameter dependency is introduced and incorporated into the error bound. A procedure for estimating MAI and ISI power levels is also presented and employed in the adaptive error bound for SM algorithms. The proposed algorithms are based on the use of simple rules and parameters that behave as forgetting factors, regulate the pace of time averages, and selectively weigh some quantities.

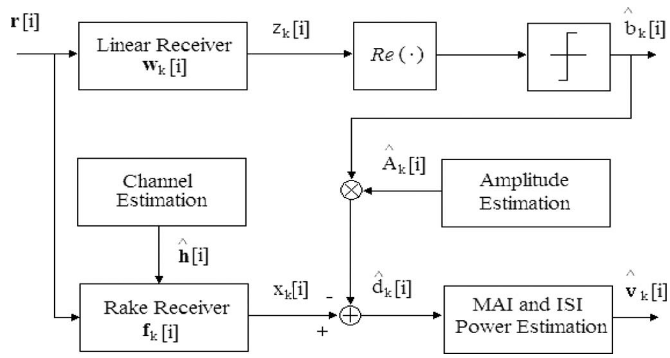

Fig. 1. Block diagram of the proposed interference estimation and tracking algorithm.

A. PDB

Here, we describe a parameter-dependent bound (PDB) that is similar to that proposed in [11] and considers the evolution of the parameter vector $\mathbf{w}_k[i]$ for the desired user (user k). The proposed PDB recursion computes a bound for SM adaptive algorithms and is described by

$$\gamma_k[i+1] = (1-\beta)\gamma_k[i] + \beta\sqrt{\alpha\, \|\mathbf{w}_k[i]\|^2\, \hat{\sigma}_v^2[i]} \tag{26}$$

where β is a forgetting factor that should be adjusted to ensure an appropriate time-averaged estimate of the evolutions of the parameter vector $\mathbf{w}_k[i]$, $\alpha\|\mathbf{w}_k[i]\|^2\hat{\sigma}_v^2[i]$ is the variance of the inner product of $\mathbf{w}_k[i]$ with $\mathbf{n}[i]$ that provides information on the evolution of $\mathbf{w}_k[i]$, α is a tuning parameter, and $\hat{\sigma}_v^2[i]$ is an estimate of the noise power. This kind of recursion helps avoiding too high or low values of the squared norm of $\mathbf{w}_k[i]$ and provides a smoother evolution of its trajectory for use in the time-varying bound. The noise power at the receiver should be estimated via a time-averaged recursion. In this paper, we will assume that it is known at the receiver.

B. PIDB

In this part, we develop an interference estimation and tracking procedure to be combined with a PDB and incorporated into a time-varying error bound for SM recursions. The MAI and ISI power estimation scheme, which is outlined in Fig. 1, employs both the RAKE receiver and the linear receiver described in (3) for subtracting the desired user signal from $\mathbf{r}[i]$ and estimating MAI and ISI power levels. With the aid of adaptive algorithms, we design the linear receiver, estimate the channel modeled as an FIR filter for the RAKE receiver, and obtain the detected symbol $\hat{b}_k[i]$, which is combined with an amplitude estimate

$\hat{A}_k[i]$ for subtracting the desired signal from the output $x_k[i]$ of the RAKE. Then, the difference $d_k[i]$ between the desired signal and $x_k[i]$ is used to estimate MAI and ISI power.

1) Channel Estimation: Let us first present a simple channel estimation algorithm for designing the RAKE receiver. Consider the constraint matrix $\mathbf{C}_k$ that contains one-chip shifted versions of the signature sequence for user k defined in (2) and the assumption that the symbols $b_k[i]$ are independent and identically distributed (i.i.d.) and statistically independent from the symbols of the other users. The proposed channel estimation algorithm is based on the following cost function:

$$\begin{aligned}\mathcal{C}\left(\hat{\mathbf{h}}[i], \hat{A}_k[i]\right) &= E\left[\left\|\hat{A}_k[i]b_k[i]\mathbf{C}_k\hat{\mathbf{h}}[i] - \mathbf{r}[i]\right\|^2\right]\\ &= E\left[\left\|\hat{A}_k[i]b_k[i]\hat{\mathbf{f}}_k[i] - \mathbf{r}[i]\right\|^2\right] \qquad (27)\end{aligned}$$

where $\hat{\mathbf{h}}[i]$ is an estimate of the channel $\mathbf{h}[i]$, and $\hat{\mathbf{f}}_k[i] = \mathbf{C}_k\hat{\mathbf{h}}[i]$ is the RAKE receiver for user k with the estimated channel. By taking the gradient terms of (27), making them equal to zero, we can devise an SG channel estimation algorithm as follows:

$$\hat{\mathbf{h}}[i+1] = \hat{\mathbf{h}}[i] - \mu_h\hat{A}_k[i]\mathbf{C}_k^H b_k^*[i]\left(b_k[i]\mathbf{C}_k\hat{\mathbf{h}}[i] - \mathbf{r}[i]\right). \qquad (28)$$

2) Amplitude Estimation: The amplitude also has to be estimated at the receiver to provide this information for different tasks such as interference cancellation and power control. The proposed interference estimation and tracking algorithm needs some form of amplitude estimation to proceed with the estimation of the interference power. To estimate the amplitudes of the associated user signals, we describe the following optimization problem with the cost function in (27):

$$\begin{aligned}\hat{A}_k[i] &= \arg\min_{A_k[i]} \mathcal{C}\left(\hat{\mathbf{h}}[i], \hat{A}_k[i]\right)\\ &= \arg\min_{A_k[i]} E\left[\left\|A_k[i]\hat{b}_k[i]\mathbf{C}_k\hat{\mathbf{h}}[i] - \mathbf{r}[i]\right\|^2\right]. \qquad (29)\end{aligned}$$

To efficiently solve the foregoing problem, we describe a simple SG algorithm to estimate the amplitude of user k, as given by

$$\hat{A}_k[i+1] = \hat{A}_k[i] - \mu_A b_k^*[i]\hat{\mathbf{h}}^H[i]\mathbf{C}_k^H\left(\hat{A}_k[i]b_k[i]\mathbf{C}_k\hat{\mathbf{h}}[i] - \mathbf{r}[i]\right). \qquad (30)$$

3) Interference Estimation and Tracking: Let us consider the RAKE receiver with perfect channel knowledge whose parameter vector $\mathbf{f}_k[i] = \mathbf{C}_k\mathbf{h}[i]$ for user k (desired one) estimates the effective signature sequence at the receiver, i.e., $\mathbf{C}_k\mathbf{h}[i] = \tilde{\mathbf{s}}_k[i]$. The output of the RAKE receiver is given by

$$\begin{aligned}x_k[i] &= \mathbf{f}_k^H[i]\mathbf{r}[i]\\ &= \underbrace{A_k[i]b_k[i]\mathbf{f}_k^H[i]\tilde{\mathbf{s}}_k[i]}_{\text{desired signal}} + \underbrace{\sum_{\substack{j=2\\ j\neq k}}^{K} A_j[i]b_j[i]\mathbf{f}_j^H[i]\tilde{\mathbf{s}}_j[i]}_{\text{MAI}}\\ &\quad + \underbrace{\mathbf{f}_k^H[i]\boldsymbol{\eta}[i]}_{\text{ISI}} + \underbrace{\mathbf{f}_k^H[i]\mathbf{n}[i]}_{\text{noise}} \qquad (31)\end{aligned}$$

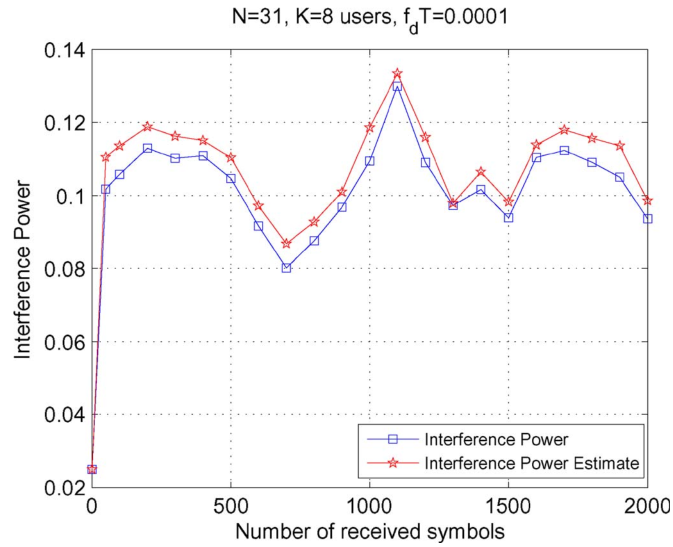

Fig. 2. Performance of the interference power estimation and tracking at $E_b/N_0 = 12$ dB and with $\beta = 0.05$.

where $\mathbf{f}_k^H[i]\tilde{\mathbf{s}}_k[i] = \rho_k[i]$, and $\mathbf{f}_k^H[i]\tilde{\mathbf{s}}_j[i] = \rho_{1,j}[i]$ for $j \neq 1$. The symbol ρ_k represents the cross correlation (or inner product) between the effective signature and the RAKE with perfect channel estimates. The symbol $\rho_{1,j}[i]$ represents the cross correlation between the RAKE receiver and the effective signature of user j. The second-order statistics of the output of the RAKE in (31) are described by

$$\begin{aligned}E\left[|x_k[i]|^2\right] &= A_k^2[i]\rho_p^2[i]\underbrace{E\left[|b_k[i]|^2\right]}_{\to 1}\\ &\quad + \underbrace{\sum_{\substack{j=1\\ j\neq k}}^{K}\sum_{\substack{l=1\\ j\neq k}}^{K} A_j^2[i]E\left[b_j[i]b_l^*[i]\right]\mathbf{f}_j^H\tilde{\mathbf{s}}_j\tilde{\mathbf{s}}_l^H\mathbf{f}_j}_{\to\sum_{j=1,j\neq k}^{K}\mathbf{f}_j^H\tilde{\mathbf{s}}_j\tilde{\mathbf{s}}_j^H[i]\mathbf{f}_j}\\ &\quad + \mathbf{f}_k^H E\left[\boldsymbol{\eta}[i]\boldsymbol{\eta}^H[i]\right]\mathbf{f}_k + \underbrace{\mathbf{f}_k^H E\left[\mathbf{n}[i]\mathbf{n}^H[i]\right]\mathbf{f}_k}_{\to\sigma^2\mathbf{f}_k^H\mathbf{f}_k}. \qquad (32)\end{aligned}$$

From the foregoing analysis, we conclude that through the second-order statistics one can identify the sum of the power levels of MAI, ISI, and noise terms. Therefore, our strategy is to obtain instantaneous estimates of MAI, ISI, and noise from the output of a RAKE receiver, subtract the detected symbol in (3) from this output [using the more reliable multiuser receiver $(\mathbf{w}_k[i])$], and track the interference (MAI + ISI + noise) power, as shown in Fig. 2. Let us define the difference between the output of the RAKE receiver and the detected symbol for user 1 as

$$\begin{aligned}d_k[i] &= x_k[i] - \hat{A}_k[i]\hat{b}_k[i]\\ &\approx \underbrace{\sum_{k=2}^{K} A_k[i]b_k[i]\mathbf{f}_k^H[i]\tilde{\mathbf{s}}_k[i]}_{\text{MAI}} + \underbrace{\mathbf{f}_k^H[i]\boldsymbol{\eta}[i]}_{\text{ISI}} + \underbrace{\mathbf{f}_k^H[i]\mathbf{n}[i]}_{\text{noise}}. \qquad (33)\end{aligned}$$

By taking expectations on $|d_k[i]|^2$ and taking into account the assumption that MAI, ISI, and noise are uncorrelated, we have

$$E\left[|d_k[i]|^2\right] \approx \sum_{k=2}^{K} \mathbf{f}_k^H[i]\tilde{\mathbf{s}}_k[i]\tilde{\mathbf{s}}_k^H[i]\mathbf{f}_k[i] + \mathbf{f}_k^H[i]E\left[\boldsymbol{\eta}[i]\boldsymbol{\eta}^H[i]\right]\mathbf{f}_k[i] + \sigma^2\mathbf{f}_k^H[i]\mathbf{f}_k[i] \quad (34)$$

where the above equation represents the interference power. Based on time averages of the instantaneous values of the interference power, we introduce the following algorithm to estimate and track $E[|d_k[i]|^2]$:

$$\hat{v}[i+1] = (1-\beta)\hat{v}[i] + \beta\,|d_k[i]|^2 \quad (35)$$

where β is a forgetting factor. To incorporate parameter dependency and interference power for computing a more effective bound, we propose the parameter and interference-dependent bound (PIDB)

$$\gamma_k[i+1] = (1-\beta)\gamma_k[i] + \beta\left(\sqrt{\tau\hat{v}^2[i]} + \sqrt{\alpha\|\mathbf{w}_k\|^2\hat{\sigma}_v^2[i]}\right) \quad (36)$$

where $\hat{v}[i]$ is the estimated interference power in the multiuser system, and τ is a weighting parameter that must be set. Similar to (26), the equations in (35) and (36) are time-averaged recursions that are aimed at tracking the quantities $|d_k[i]|^2$ and $(\sqrt{\tau\hat{v}^2[i]} + \sqrt{\alpha\|\mathbf{w}_k\|^2\hat{\sigma_v}^2[i]})$, respectively. The equations in (35) and (36) also avoid instantaneous values that are undesirably too high or too low and that may lead to inappropriate time-varying bound $\gamma_k[i]$.

V. Analysis of the Algorithms

In this section, we analyze the channel estimation and interference estimation algorithms described in the previous section. We focus on the convergence properties of the algorithms in terms of the step size parameters μ_h, μ_A, and β used for the channel, amplitude, and interference power estimators, respectively.

A. Channel Estimator

To analyze the SG channel estimator given in (28), let us first define an error vector $\boldsymbol{\epsilon}_h[i] = \hat{\mathbf{h}}[i] - \mathbf{h}_{\text{opt}}$. By subtracting $\mathbf{h}_{\text{opt}}$ from the SG recursion in (28), we get

$$\begin{aligned}\boldsymbol{\epsilon}_h&[i+1]\\ &= \boldsymbol{\epsilon}_h[i] - \mu_h\hat{A}_k[i]\mathbf{C}_k^H b_k^*[i]\left(\hat{A}_k[i]b_k[i]\mathbf{C}_k\hat{\mathbf{h}}[i] - \mathbf{r}[i]\right)\\ &= \boldsymbol{\epsilon}_h[i] - \mu_h\hat{A}_k[i]b_k^*[i]\mathbf{C}_k^H\left(\hat{A}_k[i]b_k[i]\mathbf{C}_k\left(\boldsymbol{\epsilon}_h[i] + \mathbf{h}_{\text{opt}}\right) - \mathbf{r}[i]\right)\\ &= \left[\mathbf{I} - \mu_h\hat{A}_k[i]\,|b_k[i]|^2\,\mathbf{C}_k^H\mathbf{C}_k\right]\boldsymbol{\epsilon}_h[i]\\ &\quad - \mu_h\hat{A}_k[i]b_k^*[i]\mathbf{C}_k^H\left(\hat{A}_k[i]b_k[i]\mathbf{C}_k\mathbf{h}_{\text{opt}} - \mathbf{r}[i]\right)\\ &= \left[\mathbf{I} - \mu_h\hat{A}_k[i]\,|b_k[i]|^2\,\mathbf{C}_k^H\mathbf{C}_k\right]\boldsymbol{\epsilon}_h[i]\\ &\quad - \mu_h\hat{A}_k[i]b_k^*[i]\mathbf{C}_k^H\mathbf{e}_{k,\text{opt}}[i]. \end{aligned} \quad (37)$$

By considering that the error vector $\boldsymbol{\epsilon}_h[i]$, $\mathbf{e}_{k,\text{opt}}[i]$, the signal components $\sum_{k=1}^K \mathbf{x}_k[i]$ from the data vector $\mathbf{r}[i] = \sum_{k=1}^K \mathbf{x}_k[i] + \mathbf{n}[i]$ given by (1) and $\mathbf{n}[i]$ are statistically independent and computing the covariance matrix of the error vector, i.e., $\mathbf{K}[i] = E[\boldsymbol{\epsilon}_h[i]\boldsymbol{\epsilon}_h^H[i]]$, we obtain

$$\mathbf{K}[i+1] = \left[\mathbf{I} - \mu_h\sigma_{A_k}^2\sigma_b^2\mathbf{C}_k^H\mathbf{C}_k\right]\mathbf{K}[i]\left[\mathbf{I} - \mu_h\sigma_{A_k}^2\sigma_b^2\mathbf{C}_k^H\mathbf{C}_k\right] + \mu_h^2\sigma_{A_k}^2\sigma_b^2\left\|\mathbf{C}_k^H\mathbf{C}_k\right\|^2\text{MSE}_{\min} \quad (38)$$

where $\sigma_b = E[|b_k[i]|^2]$, $\sigma_{A_k}^2 = E[|A_k[i]|^2]$, and $\text{MSE}_{\min} = E[\|\mathbf{e}_{k,\text{opt}}[i]\|^2]$ stands for the minimum mean-square error (MSE) achieved by the estimator. The recursive rule/algorithm in (28) is asymptotically unbiased and will converge to the optimum channel estimator $\mathbf{h}_{\text{opt}}$ if the step size μ_h satisfies the following condition:

$$0 < \mu_h < \frac{2}{\sigma_b^2\sigma_{A_k}^2\lambda_{\max}} \quad (39)$$

where $\lambda_{\max}$ is the largest eigenvalue of the matrix $\mathbf{C}_k^H\mathbf{C}_k$. The preceding condition concerning the step size μ_h arises from difference equations. The quantities generated in (38) represent a geometric series with a geometric ratio equal to $(1 - \mu_h\sigma_{A_k}^2\sigma_b^2\mathbf{C}_k^H\mathbf{C}_k)$. For stability or convergence of this algorithm, the magnitude of this geometric ratio must be less than 1 for all k $(|1 - \mu_h\sigma_{A_k}^2\sigma_b^2\mathbf{C}_k^H\mathbf{C}_k| < 1)$. This means that $0 < \mu_h < (2/\sigma_b^2\sigma_{A_k}^2\lambda_{\max})$. See [1] and [22] for further details.

B. Amplitude Estimator

To analyze the SG amplitude estimator described in (30), let us first define an error signal $\epsilon_{A,k}[i] = \hat{A}_k[i] - A_{k,\text{opt}}$. By subtracting $A_{k,\text{opt}}$ from the equation in (30), we obtain

$$\begin{aligned}\epsilon_{A_k}[i+1] &= \epsilon_{A_k}[i] - \mu_A b_k^*[i]\hat{\mathbf{h}}^H[i]\mathbf{C}_k^H\\ &\qquad\times\left(\hat{A}_k[i]b_k[i]\mathbf{C}_k\hat{\mathbf{h}}[i] - \mathbf{r}[i]\right)\\ &= \epsilon_{A_k}[i] - \mu_A b_k^*[i]\hat{\mathbf{h}}^H[i]\mathbf{C}_k^H\\ &\qquad\times\left(\left(\epsilon_{A,k}[i] + A_{k,\text{opt}}\right)b_k[i]\mathbf{C}_k\hat{\mathbf{h}}[i] - \mathbf{r}[i]\right)\\ &= \epsilon_{A_k}[i] - \mu_A\,|b_k[i]|^2\,\hat{\mathbf{h}}[i]\mathbf{C}_k^H\mathbf{C}_k\hat{\mathbf{h}}[i]\epsilon_{A_k}[i]\\ &\qquad - \mu_A b_k^*[i]\hat{\mathbf{h}}^H[i]\mathbf{C}_k^H\left(A_{k,\text{opt}}b_k[i]\mathbf{C}_k\hat{\mathbf{h}}[i] - \mathbf{r}[i]\right)\\ &= \left(1 - \mu_A\,|b_k[i]|^2\,\hat{\mathbf{h}}[i]\mathbf{C}_k^H\mathbf{C}_k\hat{\mathbf{h}}[i]\right)\epsilon_{A_k}[i]\\ &\qquad - \mu_A b_k^*[i]\hat{\mathbf{h}}^H[i]\mathbf{C}_k^H e_{A,\text{opt}}[i]. \end{aligned} \quad (40)$$

By considering that the error $\epsilon_A[i]$, $\mathbf{e}_{A,\text{opt}}[i] = A_{k,\text{opt}}b_k[i]\mathbf{C}_k\hat{\mathbf{h}}[i] - \mathbf{r}[i]$, the signal components $\sum_{k=1}^K \mathbf{x}_k[i]$ from the data vector $\mathbf{r}[i] = \sum_{k=1}^K \mathbf{x}_k[i] + \mathbf{n}[i]$ given by (1) and $\mathbf{n}[i]$ are statistically independent, and by computing the MSE, i.e., $\text{MSE}(A_k[i])[i] = K_{A_k}[i] = E[|\epsilon[i]|^2]$, we get

$$\text{MSE}\left(A_k[i]\right)[i+1] = \left(1 - \mu_A\sigma_b^2 E\left[\left\|\mathbf{h}_k^H[i]\mathbf{C}_k^H\right\|^2\right]\right)^2 K_{A_k}[i] + \mu_A^2\sigma_b^2 E\left[\left\|\mathbf{h}_k^H[i]\mathbf{C}_k^H\right\|^2\right]\text{MSE}_{A,\min} \quad (41)$$

where $\text{MSE}_{A,\min} = E[\|e_{A,\text{opt}}[i]\|^2]$. The cross multiplication between the terms will vanish as a result of the statistical independence between them. The general algorithm in (30) will asymptotically converge, and in an unbiased way to the $A_{k,\text{opt}}$ provided, the step size μ_A is chosen such that

$$0 < \mu_A < \frac{2}{\sigma_b^2 E\left[\left\|\mathbf{h}_k^H[i]\mathbf{C}_k^H\right\|^2\right]}. \quad (42)$$

The above range of values has to be tuned to ensure good convergence and tracking of the amplitude.

C. Interference Estimation and Tracking Algorithm

Let us describe in a general form the mechanisms for interference estimation and tracking given in (26) and (36). We can write without loss of generality that

$$\gamma_k[i+1] = (1-\beta)\gamma_k[i] + \beta\text{Po}[i] \quad (43)$$

where $\text{Po}[i]$ can account either for PDB, as described in Section IV-A, or for PIDB, as detailed in Section IV-B.

Our goal is to establish the convergence of a general stochastic recursion, as given in (43), in terms of the MSE at iteration $[i]$, as described by

$$\text{MSE}\left(\gamma[i]\right)[i] = E\left[\left|\gamma_k[i] - \gamma_{k,\text{opt}}\right|^2\right] \quad (44)$$

where $\gamma_{k,\text{opt}}$ is the optimal parameter estimate for $\gamma_k[i]$.

We will show that under certain conditions on β, the sequence converges to the optimal $\gamma_{k,\text{opt}}$ in the mean-square sense. Let us define the error

$$\epsilon_\gamma[i] = \gamma_k[i] - \gamma_{k,\text{opt}}$$

and substitute the preceding equation into (44), which yields

$$\begin{aligned}\epsilon_\gamma[i+1] &= (1-\beta)\epsilon_\gamma[i] + \beta\left(\text{Po}[i] - \gamma_{k,\text{opt}}\right)\\ &= (1-\beta)\epsilon_\gamma[i] + \beta e_{k,\text{opt}}[i]. \quad (45)\end{aligned}$$

The MSE at time instant $[i+1]$ is given by

$$\begin{aligned}&\text{MSE}\left(\gamma[i+1]\right)[i+1]\\ &= \epsilon_\gamma^*[i+1]\epsilon_\gamma[i+1]\\ &= |\epsilon_\gamma[i+1]|^2\\ &= (1-\beta)^2\epsilon_\gamma^*[i]\epsilon_\gamma[i] + \beta^2 e_{k,\text{opt}}[i]e_{k,\text{opt}}^*[i]\\ &\quad + (1-\beta)\beta\epsilon_\gamma[i]e_{k,\text{opt}}^*[i] + (1-\beta)\beta\epsilon_\gamma^*[i]e_{k,\text{opt}}[i]. \quad (46)\end{aligned}$$

By taking the expected value on both sides and assuming that $\epsilon_\gamma[i]$ and $e_{k,\text{opt}}[i]$ are statistically independent, we have

$$\begin{aligned}E\left[\text{MSE}[i+1]\right] &= E\left[|\epsilon_\gamma[i+1]|^2\right]\\ &= K_\gamma[i+1]\\ &= (1-\beta)K_\gamma[i](1-\beta) + \beta^2\left|e_{k,\text{opt}}[i]\right|^2 \quad (47)\end{aligned}$$

where $|e_{k,\text{opt}}[i]|^2$ is the minimum MSE achieved by the estimator.

We can notice that the cross-multiplication terms will vanish as a result of the statistical independence between the terms. The general recursive rule in (43) will asymptotically converge provided the step size β is chosen such that

$$0 < \beta < 2. \quad (48)$$

The above range of values has to be adjusted to ensure good convergence and tracking of the parameter dependency and/or the interference modeled here as the quantity $\text{Po}[i]$.

VI. Simulations

In this section, we assess the performance of the proposed and existing adaptive algorithms in several scenarios of practical interest:

1) the NLMS, the SM-NLMS [2], the SM-NLMS of Guo and Huang [11], and the proposed SM-NLMS with the parameter-dependent (PDB) and parameter- and interference-dependent (PIDB) time-varying bounds;
2) the AP [1], the SM-AP [3], and the proposed SM-AP with PDB and PIDB time-varying bounds;
3) the BEACON [6] and the proposed BEACON algorithms with PDB and PIDB time-varying bounds.

We consider, for the sake of simplicity, binary phase-shift-keying (BPSK) modulation, a DS-CDMA system with Gold sequences of length $N = 31$, and typical fading channels found in mobile communications systems that can be modeled according to Clarke's model [20]. The channels experienced by different users are identical since we focus on a downlink scenario, and the desired receiver processes the transmissions intended to other users (MAI) over the same channel as its own signal. The channel coefficients are $h_l[i] = p_l\alpha_l[i]$, where $\alpha_l[i]$ $(l = 0, 1, \ldots, L_p - 1)$ are obtained with Clarke's model [20]. In particular, we employ standard SG adaptive algorithms for channel estimation and RAKE design to concentrate on the comparison between the analyzed algorithms for receiver parameter estimation. We show the results in terms of the normalized Doppler frequency f_dT (cycles/symbol) and use three-path channels with relative powers given by 0, −3, and −6 dB, where in each run the spacing between paths is obtained from a discrete uniform random variable between one and two chips. The channel coefficients $h_l[i]$ $(l = 0, 1, \ldots, L_p - 1)$ are constant during each symbol interval and change according to Clarke's model over time. Since the channel is modeled as an FIR filter, we employ a channel estimation filter with six taps as an upper bound for the experiments. Note that the delays of the channel taps are multiples of the chip rate and are random. Their range coincides with the maximum length of the estimation filter, which is six taps.

The parameters of the algorithms are optimized and shown for each example, the system has a power distribution for the interferers that follows a log-normal distribution with associated standard deviation of 3 dB, and experiments are averaged over 100 independent runs. The receivers are trained with

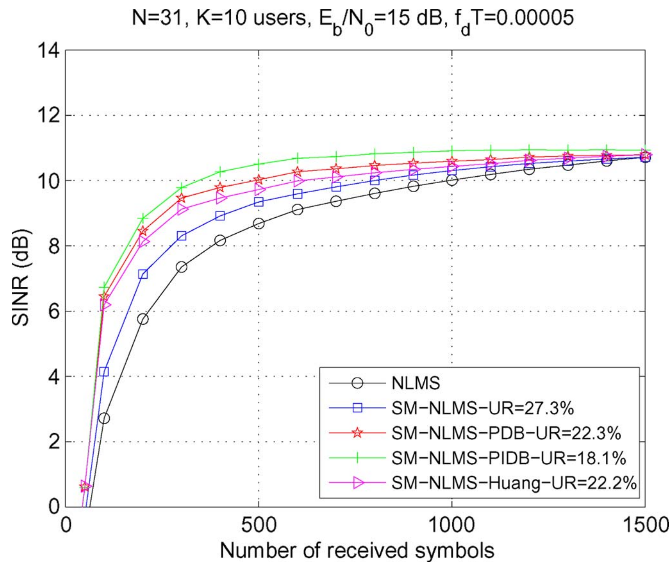

Fig. 3. SINR performance of NLMS algorithms at $E_b/N_0 = 15$ dB with $\beta = 0.05$, $\alpha = 8$, and $\tau = 2$.

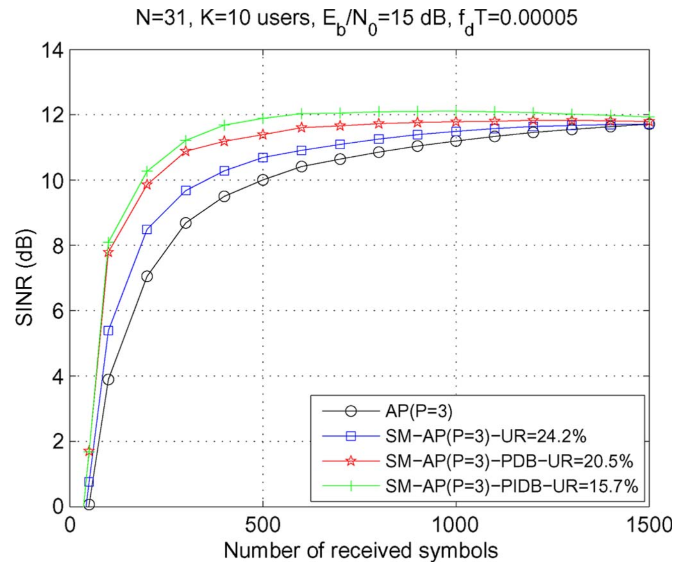

Fig. 4. SINR performance of AP algorithms at $E_b/N_0 = 15$ dB with $\beta = 0.05$, $\alpha = 8$, and $\tau = 2$.

200 symbols and then switch to decision-directed mode for each data packet. We address the dynamic channel by adjusting the receiver weights with the training sequences (with length equal to 200 symbols), and then, we exploit the decision-directed mode to track the channel variations. If the channel does not vary too fast, then the adaptive receivers can track it with this scheme, as will be shown in what follows.

A. Interference Estimation and Tracking

To evaluate the effectiveness of the proposed interference estimation and tracking algorithms that are incorporated into the time-varying error bounds for tracking the MAI and ISI powers, we carried out an experiment, which is depicted in Fig. 2. In this scenario, the proposed algorithm estimates of the MAI and ISI powers are compared with the actual interference power levels that are generated by the simulations. The results obtained show that the proposed algorithms are very effective for estimating and tracking the interference power in dynamic environments. Specifically, the estimation error does not exceed in average 5% of the estimated power level, as depicted in Fig. 2. Indeed, the algorithms are capable of accurately estimating the interference power levels and tracking them, which can be verified by the proximity between the curves obtained with the estimation algorithms and the actual values.

B. SINR Performance

In this part, the performance of the proposed algorithms is assessed in terms of output signal-to-interference-plus-noise ratio (SINR), which is defined as

$$\text{SINR}[i] = \frac{\boldsymbol{w}^H[i] R_s[i] \boldsymbol{w}[i]}{\boldsymbol{w}^H[i] \boldsymbol{R}_I[i] \boldsymbol{w}[i]} \quad (49)$$

where $R_s[i]$ is the autocorrelation matrix of the desired signal, and $\boldsymbol{R}_I[i]$ is the cross-correlation matrix of the interference

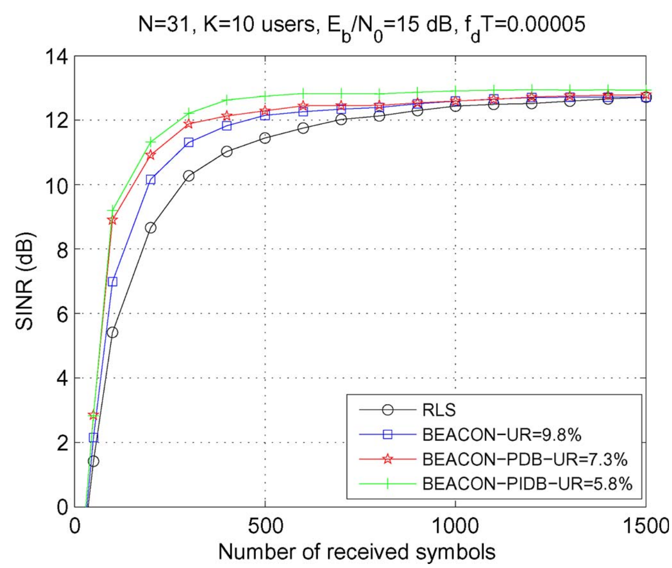

Fig. 5. SINR performance of RLS and BEACON algorithms at $E_b/N_0 = 15$ dB with $\beta = 0.05$, $\alpha = 5$, and $\tau = 1.5$.

and noise in the environment. The goal here is to evaluate the convergence performance of the proposed algorithms with time-varying error bounds for the modified SM-NLMS, SM-AP, and BEACON techniques. Specifically, we consider examples where the adaptive receivers converge to about the same level of SINR, which illustrates in a fair way the speed of convergence of the proposed algorithms and the existing ones. We also measure the UR of all the SM-based algorithms as an important complexity issue.

The SINR convergence performance of NLMS, AP, and BEACON algorithms is illustrated via computer experiments in Figs. 3–5, respectively. The curves in Fig. 3 show that the proposed SM-NLMS algorithms with PIDB time-varying error bounds achieve the fastest convergence, followed by the proposed SM-NLMS-PDB, SM-NLMS-Huang [11], conventional

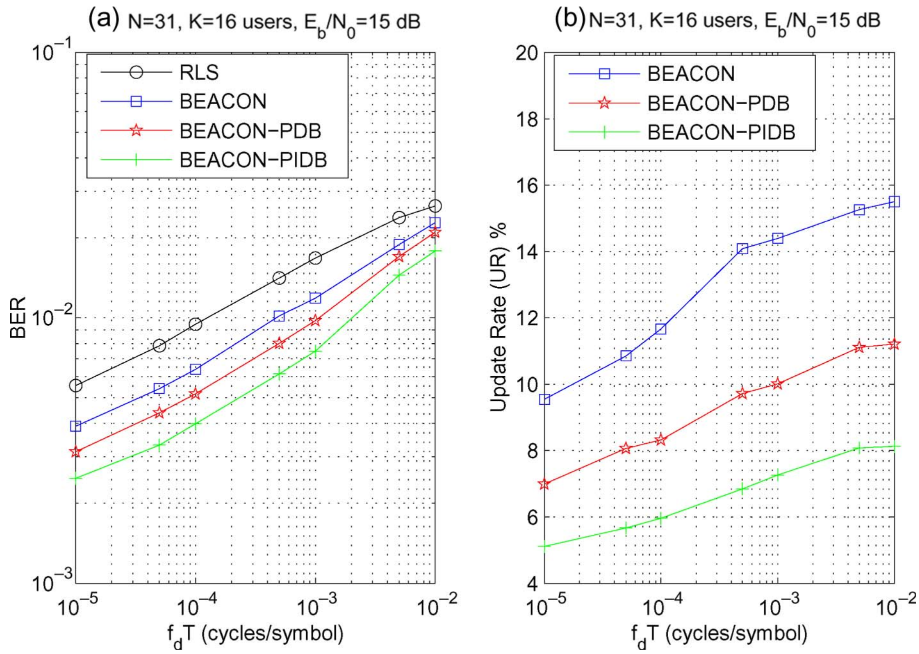

Fig. 6. (a) BER performance versus f_{dT} and (b) UR versus f_{dT} for the BEACON algorithms for a forgetting factor $\lambda = 0.997$ for the conventional RLS algorithm. The proposed BEACON-PDB and BEACON-PIDB algorithms use $\beta = 0.05$, $\alpha = 5$, and $\tau = 1.5$.

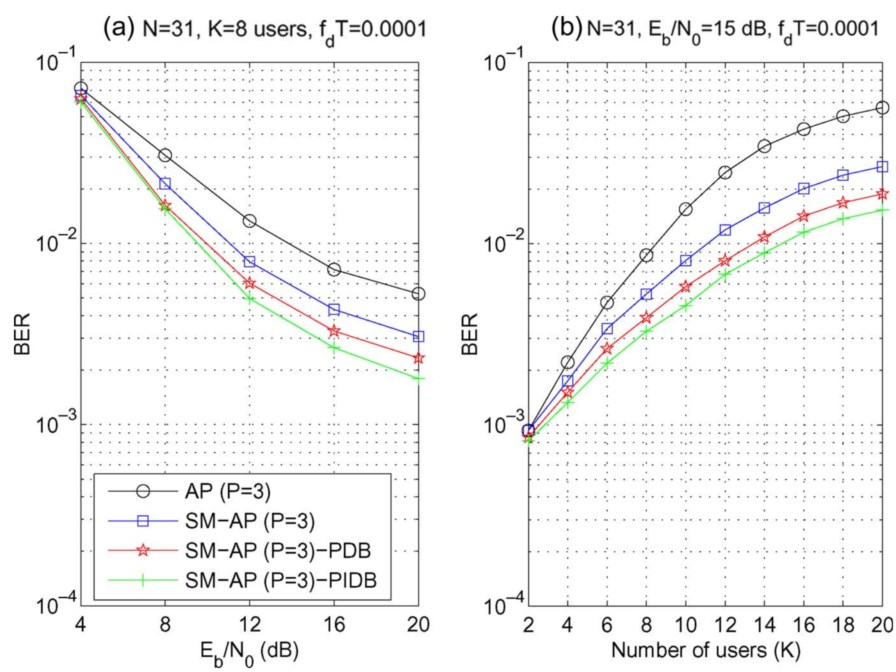

Fig. 7. BER performance versus (a) E_b/N_0 (in decibels) and (b) K for AP algorithms with $P = 3$.

SM-NLMS [2], and NLMS [1] recursions. Although the proposed SM-NLMS-PIDB and SM-NLMS-PDB algorithms enjoy the fastest convergence rates, they exhibit remarkably lower UR properties, saving significant computational resources and being substantially more economical than the conventional SM-NLMS algorithm.

By observing the results for the AP and the BEACON algorithms, which are shown in Figs. 4 and 5, one can notice that the results corroborate those found for the NLMS algorithms. It should be noted that despite their higher complexity than NLMS algorithms, the AP and BEACON techniques have faster convergence, better SINR steady-state performance, and lower URs.

C. BER Performance

In this section, we focus on the bit error rate (BER) performance of the proposed algorithms. We consider a simulation setup where the data packets transmitted have 1500 symbols, and the adaptive receivers and algorithms are trained with 200 symbols and then switch to decision-directed mode, in which they continue to adapt and track the channel variations.

First, we consider a study of the BER performance and the impact on the UR of the fading rate of the channel (f_{dT}) in the experiment shown in Fig. 6. We observe from the curves in Fig. 6(a) that the new BEACON algorithms obtain substantial gains in BER performance over the original BEACON [6] and the RLS algorithm [1] for a wide range of fading rates. In addition, as the channel becomes more hostile, the performance of the analyzed algorithms approaches one another, indicating that the adaptive techniques are encountering difficulties in dealing with the changing environment and interference. With regard to the UR, the curves in Fig. 6(b) illustrate the impact of the fading rate on the UR of the algorithms. Indeed, it is again verified that the proposed BEACON-PDB and BEACON-PIDB algorithms can obtain significant savings in terms of UR, allowing the mobile receiver to share its processing power with other important functions and to save battery life.

The BER performance versus the signal-to-noise ratio (E_b/N_0) and the number of users (K) is illustrated in Fig. 7 for the AP with $P = 3$. The results confirm the excellent performance of the proposed PIDB time-varying error bound for a variety of scenarios, algorithms, and loads. The PIDB technique allows significantly superior performance while reducing the UR of the algorithm and saving computations. We can also notice that significant performance and capacity gains can be obtained by exploiting data reuse. From the curves, it can be seen that the proposed PIDB mechanism with SM-AP with $P = 3$ can save up to 3 dB and up to 4 dB, as compared with PDB and SM-AP with fixed bounds, respectively, for the same BER performance. In terms of system capacity, we verify that the PIDB approach can accommodate up to four more users as compared with the PDB technique for the same BER performance.

VII. Conclusion

We have proposed SM adaptive algorithms based on time-varying error bounds. Adaptive algorithms for tracking MAI and ISI power and taking into account parameter dependency were incorporated into the new time-varying error bounds. The simulations show that the new algorithms outperform previously reported techniques and exhibit a reduced number of updates. The proposed algorithms can have a significant impact on the design of low-complexity receivers for spread-spectrum systems, as well as for future multiple-input–multiple-output systems employing either CDMA or orthogonal frequency-division multiplexing as the multiple-access technology. The proposed algorithms are particularly relevant to future wireless cellular, ad hoc, and sensor networks, where their potential to save computational resources may play a significant role, given the limited battery resources and processing capabilities of mobile units and sensors.

References

[1] P. S. R. Diniz, *Adaptive Filtering: Algorithms and Practical Implementations*, 2nd ed. Boston, MA: Kluwer, 2002.

[2] S. Gollamudi, S. Nagaraj, S. Kapoor, and Y. F. Huang, "Set-membership filtering and a set-membership normalized LMS algorithm with an adaptive step size," *IEEE Signal Process. Lett.*, vol. 5, no. 5, pp. 111–114, May 1998.

[3] S. Werner and P. S. R. Diniz, "Set-membership affine projection algorithm," *IEEE Signal Process. Lett.*, vol. 8, no. 8, pp. 231–235, Aug. 2001.

[4] P. S. R. Diniz and S. Werner, "Set-membership binormalized data-reusing LMS algorithms," *IEEE Trans. Signal Process.*, vol. 51, no. 1, pp. 124–134, Jan. 2003.

[5] S. Gollamudi, S. Kapoor, S. Nagaraj, and Y. F. Huang, "Set-membership adaptive equalization and an updator-shared implementation for multiple channel communications systems," *IEEE Trans. Signal Process.*, vol. 46, no. 9, pp. 2372–2385, Sep. 1998.

[6] S. Nagaraj, S. Gollamudi, S. Kapoor, and Y. F. Huang, "BEACON: An adaptive set-membership filtering technique with sparse updates," *IEEE Trans. Signal Process.*, vol. 47, no. 11, pp. 2926–2941, Nov. 1999.

[7] S. Nagaraj, S. Gollamudi, S. Kapoor, Y. F. Huang, and J. R. Deller, Jr., "Adaptive interference suppression for CDMA systems with a worst-case error criterion," *IEEE Trans. Signal Process.*, vol. 48, no. 1, pp. 284–289, Jan. 2000.

[8] I. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "A survey on sensor networks," *IEEE Commun. Mag.*, vol. 40, no. 8, pp. 102–114, Aug. 2002.

[9] D. Maksarov and J. P. Norton, "Tuning of noise bounds in parameter set estimation," in *Proc. Int. Conf. Identification Eng. Syst.*, Swansea, U.K., Mar. 27–29, 1996, pp. 584–593.

[10] S. Gazor and K. Shahtalebi, "A new NLMS algorithm for slow noise magnitude variation," *IEEE Signal Process. Lett.*, vol. 9, no. 11, pp. 348–351, Nov. 2002.

[11] L. Guo and Y. F. Huang, "Set-membership adaptive filtering with parameter-dependent error bound tuning," in *Proc. IEEE Int. Conf. Acoust. Speech Signal Process.*, 2005, pp. iv/369–iv/372.

[12] L. Guo and Y. F. Huang, "Frequency-domain set-membership filtering and its applications," *IEEE Trans. Signal Process.*, vol. 55, no. 4, pp. 1326–1338, Apr. 2007.

[13] T. M. Lin, M. Nayeri, and J. R. Deller, Jr., "Consistently convergent OBE algorithm with automatic selection of error bounds," *Int. J. Adapt. Control Signal Process.*, vol. 12, pp. 302–324, Jun. 1998.

[14] J. R. Deller, S. Gollamudi, S. Nagaraj, and Y. F. Huang, "Convergence analysis of the QUASI-OBE algorithm and the performance implications," in *Proc. IFAC Int. Symp. Syst. Identification*, Santa Barbara, CA, Jun. 2000, vol. 3, pp. 875–880.

[15] S. Dasgupta and Y. F. Huang, "Asymptotically convergent modified recursive least-squares with data dependent updating and forgetting factor for systems with bounded noise," *IEEE Trans. Inf. Theory*, vol. IT-33, no. 3, pp. 383–392, May 1987.

[16] J. R. Deller, M. Nayeri, and M. Liu, "Unifying the landmark developments in optimal bounding ellipsoid identification," *Int. J. Adapt. Control Signal Process.*, vol. 8, no. 1, pp. 43–60, Jan./Feb. 1994.

[17] D. Joachim, J. R. Deller, and M. Nayeri, "Multiweight optimization in OBE algorithms for improved tracking and adaptive identification," in *Proc. IEEE Int. Acoust., Speech Signal Process.*, Seattle, WA, May 1998, vol. 4, pp. 2201–2204.

[18] M. L. Honig and H. V. Poor, "Adaptive interference suppression," in *Wireless Communications: Signal Processing Perspectives*, H. V. Poor and G. W. Wornell, Eds. Englewood Cliffs, NJ: Prentice-Hall, 1998, ch. 2, pp. 64–128.

[19] S. Haykin, *Adaptive Filter Theory*, 4th ed. Englewood Cliffs, NJ: Prentice–Hall, 2002.

[20] T. S. Rappaport, *Wireless Communications*. Englewood Cliffs, NJ: Prentice–Hall, 1996.

[21] *Third Generation Partnership Project (3GPP)*. Specs. 25.101, 25.211-25.215, vers. 5.x.x.

[22] D. P. Bertsekas, *Nonlinear Programming*, 2nd ed. Belmont, MA: Athena Scientific, 1999.

[23] A. Klein, G. Kaleh, and P. Baier, "Zero forcing and minimum mean-square-error equalization for multiuser detection in code-division multiple-access channels," *IEEE Trans. Veh. Technol.*, vol. 45, no. 2, pp. 276–287, May 1996.

[24] S. Buzzi and H. V. Poor, "Channel estimation and multiuser detection in long-code DS/CDMA systems," *IEEE J. Sel. Areas Commun.*, vol. 19, no. 8, pp. 1476–1487, Aug. 2001.

[25] Z. Xu and M. Tsatsanis, "Blind channel estimation for long code multiuser CDMA systems," *IEEE Trans. Signal Process.*, vol. 48, no. 4, pp. 988–1001, Apr. 2000.

[26] P. Liu and Z. Xu, "Joint performance study of channel estimation and multiuser detection for uplink long-code CDMA systems," *EURASIP J. Wireless Commun. Netw.—Special Issue on Innovative Signal Transmission and Detection Techniques for Next Generation Cellular CDMA System*, vol. 2004, no. 1, pp. 98–112, Aug. 2004.

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