Robust Multi-branch Tomlinson-Harashima Source and Relay Precoding Scheme in Nonregenerative MIMO Relay Systems

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***Abstract*—This paper investigates a robust Tomlinson-Harashima precoding (THP) design for multiple-input multiple-output (MIMO) relay systems based on a multi-branch (MB) strategy. The proposed scheme employs a parallel MB structure at the source according to different pre-stored ordering patterns. For each parallel branch, the robust nonlinear transceiver design consists of a TH precoder at the source along with a linear precoder at the relay and a linear minimum-mean-squared-error (MMSE) receiver at the destination. By taking the channel uncertainties into account, the source and relay precoders are jointly optimised to minimise the MSE. We can finally use an iterative method to obtain the solution for the relay and the source precoders via Karush-Kuhn-Tucker (KKT) conditions. An appropriate selection rule is developed to choose the nonlinear transceiver corresponding to the best branch for data transmission. Simulation results demonstrate that the proposed MB-THP scheme outperforms existing transceiver designs with perfect and imperfect channel state information (CSI).**[1]

***Index Terms*—multiple-input multiple-output (MIMO), multiple branch (MB), channel state information (CSI), Tomlinson-Harashima precoding (THP)**

I. Introduction

Recently, the nonregenerative multiple-input multiple-output (MIMO) relay system has attracted intensive interest, since it has a great potential to increase the coverage of wireless communications under power and spectral constraints and can provide a significant improvement in terms of both spectral efficiency and link reliability [1]–[5]. Many works have been proposed for linear precoding techniques in MIMO relay systems [1], [2]. As an alternative to linear transceiver design, using nonlinear pre-filtering for MIMO relay channels has recently aroused a great attention. Compared with the linear transceiver design, nonlinear transceivers yield a better performance [3], [4]. The authors in [3] focused on the joint design of linear processors for a two-hop network with THP employed at the source. In [4], the direct link between the source and the destination node was also considered. Note that all the mentioned algorithms above require the perfectly known channel state information (CSI). However, in a practical system, CSI is usually imperfect, since channel estimation errors are inevitable. Thus, the errors should be taken into account in transceiver design. To overcome the problem, robust linear transceiver designs have been developed for MIMO relay systems [5].

To the best of our knowledge, there is a very small number of works investigating robust Tomlinson- Harashima precoding (THP) in MIMO relay systems. Conventional THP algorithms are only based on one particular cancellation order and are more sensitive to channel estimation errors than their linear counterpart [3], [4]. Therefore, in this paper, we propose a robust nonlinear multi-branch (MB) TH transceiver algorithm for MIMO relay systems in the presence of imperfect CSI. The original idea of an MB strategy was first proposed in [6] to utilize the potential extra diversity gains for DS-CDMA systems and then extended to precoding in [7]. The proposed scheme employs a parallel MB structure at the source according to the pre-stored ordering patterns. For each branch, the nonlinear transceiver design consists of a TH precoder at the source along with a linear precoder used at the relay and an minimum-mean-squared-error (MMSE) receiver at the destination. We employ the diagonalization method along with majorization theory to obtain the optimal relay and source precoders. The solution can finally be computed by using an iterative method via the Karush-Kuhn-Tucker (KKT) conditions. An appropriate selection rule is developed to choose the nonlinear transceiver corresponding to the best branch for data transmission. Simulations show that the robust precoder outperforms the conventional precoding algorithms.

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II. Proposed System Model

We consider a three-node nonregenerative MIMO relay network comprising of one source, one relay and one destination equipped with N_s, N_r and N_d antennas, respectively. Here, we assume that $N_s \leq \min\{N_r, N_d\}$ provides sufficient degrees of freedom for signal detection. For simplicity, we ignore the direct link between the source and the destination node.

This system consists of a TH source precoder, a linear relay precoder and a linear MMSE receiver, as shown in Fig. 1. The quantity $\mathbf{s}$ is the input signal vector with zero mean and $\mathrm{E}\left[\mathbf{s}\mathbf{s}^H\right] = \sigma_s^2 \mathbf{I}_{N_d}$, where $\mathrm{E}\left[\cdot\right]$ stands for the statistical expectation, $\mathbf{I}_{N_d}$ is an $N_d \times N_d$ identity matrix, and σ_s^2 is the average transmitted power per antenna at the source. The source signal $\mathbf{s} = \left[s_1, ..., s_{N_d}\right]^T$ is modulated by M-ary square quadrature amplitude modulation (QAM), where the real and imaginary parts of s_k belong to the set $\left\{\pm 1, \pm 3, ..., \pm\left(\sqrt{M}-1\right)\right\}$. Then the input signal is sorted to generate multiple branch signals by the pre-stored cancellation ordering patterns. We introduce the ordering transformation matrix $\mathbf{T}^{(l)}$, $l \in \{1, ..., L\}$, which corresponds to the ordering pattern employed in the l-th branch. The optimal ordering scheme conducts an exhaustive search $L = N_s!$, where ! is the factorial operator.

The reordered vector $\mathbf{s}^{(l)}=\mathbf{T}^{(l)}\mathbf{s}$ which is based on the l-th cancellation order is then recursively computed by a backward squared matrix $\mathbf{C}^{(l)}$ for the l-th branch and a nonlinear modulo operation in order to conduct a successive interference cancellation (SIC) operation.

As shown in Fig. 1, $\mathrm{MOD}_m(\cdot)$ stands for the modulo operator which is used to constrain a value to be within the region $(-\sqrt{M},\sqrt{M}]$. The modulo operator that acts independently over the real and imaginary parts of its input according to the following rule

$$\mathrm{MOD}_m(x) = x - 2\sqrt{m}\left\lfloor \frac{x+\sqrt{m}}{2\sqrt{m}} \right\rfloor. \quad (1)$$

With $\mathbf{C}^{(l)}$ and the modulo operation in (1), the l-th branch channel symbols $\mathbf{x}_k^{(l)}$ are successively generated

$$\mathbf{x}_k^{(l)} = \mathbf{s}_k^{(l)} - \sum_{m=1}^{k-1}\mathbf{C}^{(l)}(k,m)\mathbf{x}_m^{(l)} + \mathbf{e}_k^{(l)}, \quad (2)$$

where $\mathbf{C}^{(l)}$ is a strictly lower triangular matrix and $\mathbf{e}^{(l)} = \left[e_1^{(l)},...,e_{N_d}^{(l)}\right]^T$ is the error of modulo operation for the l-th branch. The equation can be rewritten in matrix form as

$$\mathbf{x}^{(l)} = \mathbf{U}^{(l)^{-1}}\mathbf{v}^{(l)}, \quad (3)$$

where $\mathbf{U}^{(l)} = \mathbf{C}^{(l)}+\mathbf{I}$ is a unite triangular matrix (a triangular matrix with ones on the main diagonal) and $\mathbf{v}^{(l)} = \mathbf{s}^{(l)} + \mathbf{e}^{(l)}$. This leads to the l-th branch channel symbols $\mathbf{x}^{(l)}$ having slightly higher energy than $\mathbf{s}^{(l)}$. For moderate to high M this energy increase can be neglected [3], thus we still have $\mathrm{E}\left[\mathbf{x}^{(l)}\mathbf{x}^{(l)^H}\right] = \sigma_s^2\mathbf{I}_{N_d}$. And it is easy to get $\mathrm{E}\left[\mathbf{v}^{(l)}\mathbf{v}^{(l)^H}\right] = \sigma_s^2\mathbf{U}^{(l)}\mathbf{U}^{(l)^H}$. Based on a selection criterion, the optimum source precoder, relay precoder and receiver corresponding to the l_{opt}-th branch are chosen for data transmission. The signal transmission is carried out in two stages. In the first phase, the signal is processed by the selected precoding matrix $\mathbf{F}_s^{(l_{opt})} \in \mathbb{C}^{N_s\times N_d}$ for the l_{opt}-th branch. The received signal $\mathbf{y}_r^{(l_{opt})}$ corresponding to the l_{opt}-th cancellation order at the relay is given by

$$\mathbf{y}_r^{(l_{opt})} = \mathbf{H}_{sr}\mathbf{F}_s^{(l_{opt})}\mathbf{x}^{(l_{opt})} + \mathbf{n}_{sr}, \quad (4)$$

where $\mathbf{H}_{sr} \in \mathbb{C}^{N_r\times N_s}$ denotes the MIMO channel matrix between the source and the relay. The vector $\mathbf{n}_{sr}$ is the additive noise component at the relay which is modeled as a circularly symmetric complex Gaussian random vectors with zero-mean and correlation matrix $\mathrm{E}\left[\mathbf{n}_{sr}\mathbf{n}_{sr}^H\right] = \sigma_{n_{sr}}^2\mathbf{I}_{N_r}$, where $\sigma_{n_{sr}}^2$ is the average noise power at the relay.

In the second phase, the relay forwards the received signals to the destination after performing linear precoding, while the source keeps silent. Thus, the equivalent signal $\mathbf{y}_d^{(l_{opt})}$ received at the destination corresponding to the l_{opt}-th branch is given by

$$\begin{aligned}\mathbf{y}_d^{(l_{opt})} &= \mathbf{T}^{(l_{opt})}\mathbf{H}_{rd}\mathbf{F}_r^{(l_{opt})}\mathbf{H}_{sr}\mathbf{F}_s^{(l_{opt})}\mathbf{x}^{(l_{opt})} \\ &\quad + \mathbf{T}^{(l_{opt})}\mathbf{H}_{rd}\mathbf{F}_r^{(l_{opt})}\mathbf{n}_{sr}+\mathbf{T}^{(l_{opt})}\mathbf{n}_{rd} \end{aligned} \quad (5)$$

where $\mathbf{F}_r^{(l_{opt})} \in \mathbb{C}^{N_r\times N_r}$ is the selected relay precoder for the l_{opt}-th branch. $\mathbf{H}_{rd} \in \mathbb{C}^{N_d\times N_r}$ stands for the MIMO channel matrix between the relay and the destination. Mathematically, the equivalent channel matrix after a specific transmit pattern can be denoted as $\mathbf{H}_{rd}^{(l_{opt})} = \mathbf{T}^{(l_{opt})}\mathbf{H}_{rd}$. By transforming the channel matrix, the columns of the channel matrix $\mathbf{H}_{rd}$ are permutated [8]. The vector $\mathbf{n}_{rd}$ is the zero-mean complex Gaussian noise vector at the destination with $\mathrm{E}\left[\mathbf{n}_{rd}\mathbf{n}_{rd}^H\right] = \sigma_{n_{rd}}^2\mathbf{I}_{N_d}$, where $\sigma_{n_{rd}}^2$ denotes the destination received average noise power.

At the destination, the selected linear receiver $\mathbf{W}^{(l_{opt})}$ is then employed to detect the received signal. The detected signal is given as follows:

$$\hat{\mathbf{v}}^{(l_{opt})} = \mathbf{W}^{(l_{opt})}\mathbf{y}_d^{(l_{opt})}. \quad (6)$$

The well-known kronecker model is adopted for the covariance of the CSI mismatch [5]. We have the following expression

$$\mathbf{H}_{sr} = \bar{\mathbf{H}}_{sr} + \Delta\mathbf{H}_{sr}, \quad (7)$$

where $\bar{\mathbf{H}}_{sr}$ is the estimated channel matrices, while $\Delta\mathbf{H}_{sr}$ is the corresponding channel estimation error matrices, and $\Delta\mathbf{H}_{sr}$ can be written as $\Delta\mathbf{H}_{sr} = \mathbf{\Sigma}_{sr}^{1/2}\mathbf{H}_{i.i.d}\mathbf{\Psi}_{sr}^{1/2}$, where the elements of $\mathbf{H}_{i.i.d}$ are independent and identically distributed Gaussian random variables with zero mean and unit variance. Both the relay and destination have the estimated CSI. Thus, $\Delta\mathbf{H}_{sr}$ has the matrix-variate complex Gaussian distribution, which can be expressed as [5]

$$\Delta\mathbf{H}_{sr} \sim \mathcal{CN}_{N_r,N_s}\left(\mathbf{0}_{N_r,N_s}, \mathbf{\Sigma}_{sr}\otimes\mathbf{\Psi}_{sr}^T\right), \quad (8)$$

where $\mathbf{\Psi}_{sr}$ denotes the $N_s\times N_s$ covariance matrix of channel estimation error at the transmitter, while $\mathbf{\Sigma}_{sr}$ is the $N_r\times N_r$ covariance matrix of channel estimation error at the receiver. The factor $\otimes$ represents the operation of the Kronecker product. Similar definition can be applied on $\mathbf{H}_{rd}$ and $\Delta\mathbf{H}_{rd}$, respectively. The equivalent estimated channel matrix after the l_{opt}-th transmit ordering pattern can be denoted as $\bar{\mathbf{H}}_{rd}^{(l_{opt})} = \mathbf{T}^{(l_{opt})}\bar{\mathbf{H}}_{rd}$.

The final output is obtained by

$$\hat{\mathbf{s}}= \mathrm{Q}\left(\mathrm{MOD}\left(\mathbf{W}^{(l_{\mathrm{opt}})}\mathbf{y}_d^{(l_{\mathrm{opt}})}\right)\right) \quad (9)$$

where $\mathrm{Q}(\cdot)$ denotes the quantization operation.

III. Proposed Robust Transceiver Design and Selection Criterion

In this section, we propose the robust transceiver design and the selection criterion to choose the best branch for data transmission.

A. *Proposed Transceiver Design*

For each branch, we then focus on the problem that jointly design $\mathbf{F}_s^{(l)}$, $\mathbf{F}_r^{(l)}$, $\mathbf{W}^{(l)}$, $\mathbf{U}^{(l)}$ to minimize the total MSE under the sum power constraint at the source and relay. The system MSE matrix is defined as $\mathrm{E}[(\mathbf{W}^{(l)}\mathbf{y}_d^{(l)} - \mathbf{v}^{(l)})(\mathbf{W}^{(l)}\mathbf{y}_d^{(l)} - \mathbf{v}^{(l)})^H]$. Note that the expectation is taken with respect to the channel estimation errors and noise. By taking the statistical property, the MSE can be calculated as

$$\begin{aligned}&\mathrm{MSE}\left(\mathbf{U}^{(l)},\mathbf{F}_s^{(l)},\mathbf{F}_r^{(l)},\mathbf{W}^{(l)}\right) \\ &=\mathrm{tr}\left(\mathbf{W}^{(l)}\mathbf{A}^{(l)}\mathbf{W}^{(l)^H}\right) + \sigma_s^2\mathrm{tr}\left(\mathbf{U}^{(l)}\mathbf{U}^{(l)^H}\right) \\ &\quad - \sigma_s^2\mathrm{tr}\left(\mathbf{U}^{(l)}\mathbf{F}_s^{(l)^H}\bar{\mathbf{H}}_{sr}^H\mathbf{F}_r^{(l)^H}\bar{\mathbf{H}}_{rd}^{(l)^H}\mathbf{W}^{(l)^H}\right) \\ &\quad - \sigma_s^2\mathrm{tr}\left(\mathbf{W}^{(l)}\bar{\mathbf{H}}_{rd}^{(l)}\mathbf{F}_r^{(l)}\bar{\mathbf{H}}_{sr}\mathbf{F}_s^{(l)}\mathbf{U}^{(l)^H}\right) \end{aligned} \quad (10)$$

where

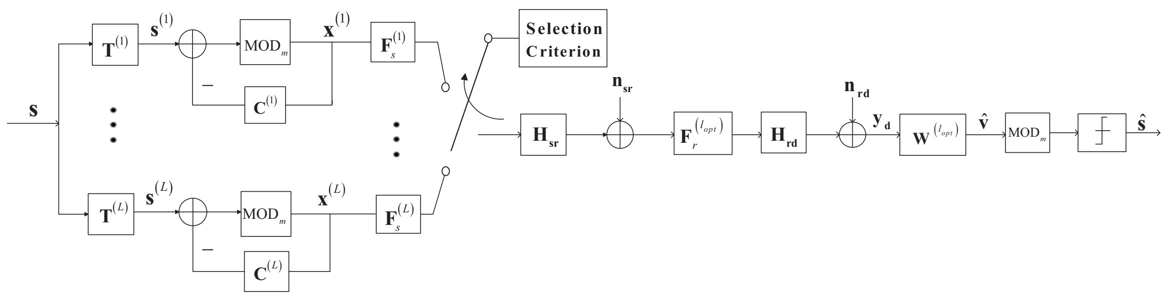

Fig. 1: MB-TH source and linear relay precoded AF MIMO relay system with MMSE receiver

$$\mathbf{A}^{(l)} \triangleq \bar{\mathbf{H}}_{rd}^{(l)}\mathbf{F}_r^{(l)}(\sigma_s^2\bar{\mathbf{H}}_{sr}\mathbf{F}_s^{(l)}\mathbf{F}_s^{(l)^H}\bar{\mathbf{H}}_{sr}^H + \sigma_s^2\alpha_1^{(l)}\mathbf{\Sigma}_{sr} + \sigma_{n_{sr}}^2\mathbf{I}_{N_r})\mathbf{F}_r^{(l)^H}\bar{\mathbf{H}}_{rd}^{(l)^H} + \alpha_2^{(l)}\hat{\mathbf{\Sigma}}_{rd} + \sigma_{n_{rd}}^2\mathbf{I}_{N_d} \quad (11)$$

$$\alpha_1^{(l)} \triangleq \mathrm{tr}\left(\mathbf{F}_s^{(l)}\mathbf{F}_s^{(l)^H}\mathbf{\Psi}_{sr}\right) \quad (12)$$

$$\alpha_2^{(l)} \triangleq \mathrm{tr}((\mathbf{F}_r^{(l)}(\sigma_s^2\bar{\mathbf{H}}_{sr}\mathbf{F}_s^{(l)}\mathbf{F}_s^{(l)^H}\bar{\mathbf{H}}_{sr}^H + \sigma_s^2\alpha_1^{(l)}\mathbf{\Sigma}_{sr} + \sigma_{n_{sr}}^2\mathbf{I}_{N_r})\mathbf{F}_r^{(l)^H}\mathbf{\Psi}_{rd})) \quad (13)$$

$$\hat{\mathbf{\Sigma}}_{rd} \triangleq \mathbf{T}^{(l)}\mathbf{\Sigma}_{rd}\mathbf{T}^{(l)^H}. \quad (14)$$

The optimal linear receiver $\mathbf{W}^{(l)}$ can be derived by solving $\frac{\partial}{\partial \mathbf{W}^{(l)*}}\mathrm{MSE}\left(\mathbf{U}^{(l)}, \mathbf{F}_s^{(l)}, \mathbf{F}_r^{(l)}, \mathbf{W}^{(l)}\right) = 0$, and it is given by

$$\mathbf{W}^{(l)} = \sigma_s^2\mathbf{U}^{(l)}\mathbf{F}_s^{(l)^H}\bar{\mathbf{H}}_{sr}^H\mathbf{F}_r^{(l)^H}\bar{\mathbf{H}}_{rd}^{(l)^H}\mathbf{A}^{(l)^{-1}}. \quad (15)$$

By substituting (15) into (10) and making use of the matrix inversion lemma [9], the MSE matrix can be represented as

$$\mathrm{MSE}\left(\mathbf{U}^{(l)}, \mathbf{F}_s^{(l)}, \mathbf{F}_r^{(l)}\right) = \mathrm{tr}\left(\mathbf{E}^{(l)}\right) \quad (16)$$

where

$$\mathbf{E}^{(l)} \triangleq \mathbf{U}^{(l)}(\sigma_s^{-2}\mathbf{I}_{N_s} + \mathbf{F}_s^{(l)^H}\bar{\mathbf{H}}_{sr}^H\mathbf{F}_r^{(l)^H}\bar{\mathbf{H}}_{rd}^{(l)^H}\mathbf{B}^{(l)^{-1}} \times \bar{\mathbf{H}}_{rd}^{(l)}\mathbf{F}_r^{(l)}\bar{\mathbf{H}}_{sr}\mathbf{F}_s^{(l)})^{-1}\mathbf{U}^{(l)^H} \quad (17)$$

$$\mathbf{B}^{(l)} \triangleq \bar{\mathbf{H}}_{rd}^{(l)}\mathbf{F}_r^{(l)}\left(\sigma_s^2\alpha_1^{(l)}\mathbf{\Sigma}_{sr} + \sigma_{n_{sr}}^2\mathbf{I}_{N_r}\right)\mathbf{F}_r^{(l)^H}\bar{\mathbf{H}}_{rd}^{(l)^H} + \alpha_2^{(l)}\hat{\mathbf{\Sigma}}_{rd} + \sigma_{n_{rd}}^2\mathbf{I}_{N_d}. \quad (18)$$

As is known to us, for a positive semi-definite matrix $\mathbf{M} \in \mathbb{C}^{N\times N}$, we have $|\mathbf{M}|^{1/N} \leq \mathrm{tr}(\mathbf{M})/N$. Only when $\mathbf{M}$ is a diagonal matrix with equal diagonal elements, the equality can be achieved. Here we use the fact that $\left|\mathbf{U}^H\mathbf{U}\right| = 1$, $|\mathbf{MN}| = |\mathbf{NM}|$ and $\left|\mathbf{M}^{-1}\right| = |\mathbf{M}|^{-1}$, the operator $|\cdot|$ here denotes the determinant of the matrix. By letting $\bar{\mathbf{H}}^{(l)} = \bar{\mathbf{H}}_{rd}^{(l)}\mathbf{F}_r^{(l)}\bar{\mathbf{H}}_{sr}$, we obtain the following bound on the $\mathrm{MSE}\left(\mathbf{U}^{(l)}, \mathbf{F}_s^{(l)}, \mathbf{F}_r^{(l)}\right)$:

$$\left|\left(\sigma_s^{-2}\mathbf{I}_{N_s} + \mathbf{F}_s^{(l)^H}\bar{\mathbf{H}}^{(l)^H}\mathbf{B}^{(l)^{-1}}\bar{\mathbf{H}}^{(l)}\mathbf{F}_s^{(l)}\right)\right|^{-1/N_s} \leq \mathrm{tr}\left\{\mathbf{U}^{(l)}\left(\sigma_s^{-2}\mathbf{I}_{N_s} + \mathbf{F}_s^{(l)^H}\bar{\mathbf{H}}^{(l)^H}\mathbf{B}^{(l)^{-1}}\bar{\mathbf{H}}^{(l)}\mathbf{F}_s^{(l)}\right)\mathbf{U}^{(l)^H}\right\}/N_s, \quad (19)$$

the expression of MSE in (16) can achieve the lower bound when $\mathbf{E}^{(l)} = \gamma\mathbf{I}_{N_s}$, where γ is a scaling parameter. By using the lower bound in (19) as our objective function and considering the power constraints at the source and relay based on the channel model, the problem can be simplified as

$$\max\left|\left(\sigma_s^{-2}\mathbf{I}_{N_s} + \mathbf{F}_s^{(l)^H}\bar{\mathbf{H}}^{(l)^H}\mathbf{B}^{(l)^{-1}}\bar{\mathbf{H}}^{(l)}\mathbf{F}_s^{(l)}\right)\right|$$
$$s.t.\quad \mathrm{tr}\left(\sigma_s^2\mathbf{F}_s^{(l)}\mathbf{F}_s^{(l)^H}\right) \leq P_s$$
$$\mathrm{tr}(\mathbf{F}_r^{(l)}(\sigma_s^2\bar{\mathbf{H}}_{sr}\mathbf{F}_s^{(l)}\mathbf{F}_s^{(l)^H}\bar{\mathbf{H}}_{sr}^H + \sigma_s^2\alpha_1^{(l)}\mathbf{\Sigma}_{sr} + \sigma_{n_{sr}}^2\mathbf{I}_{N_r})\mathbf{F}_r^{(l)^H}) \leq P_r. \quad (20)$$

In order to find the explicit structure of the optimal $\mathbf{F}_s^{(l)}$ and $\mathbf{F}_r^{(l)}$, we discuss a scenario with either the covariance matrix of the channel estimation error at the transmitter or a scenario in which the receiver is an identity matrix, respectively. We focus on the former case i.e. $\mathbf{\Psi}_{sr} = \mathbf{I}_{N_s}$ and $\mathbf{\Psi}_{rd} = \mathbf{I}_{N_r}$ to describe the design of the precoders. The derivation for the latter case is straightforward. In this case, we have $\alpha_1^{(l)} = \mathrm{tr}(\mathbf{F}_s^{(l)}\mathbf{F}_s^{(l)^H})$ and $\alpha_2^{(l)} = \mathrm{tr}(\mathbf{F}_r^{(l)}(\sigma_s^2\bar{\mathbf{H}}_{sr}\mathbf{F}_s^{(l)}\mathbf{F}_s^{(l)^H}\bar{\mathbf{H}}_{sr}^H + \sigma_s^2\alpha_1^{(l)}\mathbf{\Sigma}_{sr} + \sigma_{n_{sr}}^2\mathbf{I}_{N_r})\mathbf{F}_r^{(l)^H})$. Based on the singular value decomposition (SVD) and the eigenvalue decomposition (EVD), we have the following expressions:

$$\tilde{\mathbf{H}}_{sr}^{(l)} \triangleq \tilde{\mathbf{\Lambda}}_{\Sigma_{sr}}^{(l)^{-\frac{1}{2}}}\mathbf{U}_{\Sigma_{sr}}^H\bar{\mathbf{H}}_{sr} = \tilde{\mathbf{U}}_{sr}^{(l)}\tilde{\mathbf{\Lambda}}_{sr}^{(l)}\tilde{\mathbf{V}}_{sr}^{(l)^H} \quad (21)$$

$$\tilde{\mathbf{H}}_{rd}^{(l)} \triangleq \tilde{\mathbf{\Lambda}}_{\Sigma_{rd}}^{(l)^{-\frac{1}{2}}}\mathbf{U}_{\Sigma_{rd}}^{(l)^H}\bar{\mathbf{H}}_{rd}^{(l)} = \tilde{\mathbf{U}}_{rd}^{(l)}\tilde{\mathbf{\Lambda}}_{rd}^{(l)}\tilde{\mathbf{V}}_{rd}^{(l)^H}, \quad (22)$$

where $\bar{\mathbf{H}}_{sr} = \mathbf{U}_{sr}\mathbf{\Lambda}_{sr}\mathbf{V}_{sr}^H$, $\bar{\mathbf{H}}_{rd}^{(l)} = \mathbf{U}_{rd}^{(l)}\mathbf{\Lambda}_{rd}^{(l)}\mathbf{V}_{rd}^{(l)^H}$, $\mathbf{\Sigma}_{sr} = \mathbf{U}_{\Sigma_{sr}}\mathbf{\Lambda}_{\Sigma_{sr}}\mathbf{U}_{\Sigma_{sr}}^H$, $\hat{\mathbf{\Sigma}}_{rd} = \mathbf{U}_{\Sigma_{rd}}^{(l)}\mathbf{\Lambda}_{\Sigma_{rd}}^{(l)}\mathbf{U}_{\Sigma_{rd}}^{(l)^H}$, $\tilde{\mathbf{\Lambda}}_{\Sigma_{sr}}^{(l)} = \alpha_1^{(l)}\mathbf{\Lambda}_{\Sigma_{sr}} + \sigma_{n_{sr}}^2\mathbf{I}_{N_r}$, $\tilde{\mathbf{\Lambda}}_{\Sigma_{rd}}^{(l)} = \alpha_2^{(l)}\mathbf{\Lambda}_{\Sigma_{rd}}^{(l)} + \sigma_{n_{rd}}^2\mathbf{I}_{N_d}$.

The optimal solutions of $\mathbf{F}_s^{(l)}$ and $\mathbf{F}_r^{(l)}$ are obtained when $\alpha_1^{(l)} = P_t/\sigma_s^2$ and $\alpha_2^{(l)} = P_r$. The precoding matrices have the structure as follows:

$$\mathbf{F}_s^{(l)} = \tilde{\mathbf{V}}_{sr}^{(l)}\mathbf{\Lambda}_s^{(l)}\mathbf{\Phi}_s^{(l)} \quad (23)$$

$$\tilde{\mathbf{F}}_r^{(l)} = \tilde{\mathbf{\Lambda}}_{rd}^{(l)}\mathbf{\Lambda}_r^{(l)}\tilde{\mathbf{U}}_{sr}^{(l)^H} \quad (24)$$

$$\mathbf{F}_r^{(l)} = \tilde{\mathbf{F}}_r^{(l)}\tilde{\mathbf{\Lambda}}_{\Sigma_{sr}}^{(l)^{-\frac{1}{2}}}\mathbf{U}_{\Sigma_{sr}}^H, \quad (25)$$

where $\mathbf{\Lambda}_s^{(l)}$ and $\mathbf{\Lambda}_r^{(l)}$ are both diagonal matrices with the i-th diagonal elements $\lambda_{F_s,i}$ and $\lambda_{\tilde{F}_r,i}$, respectively, and $\mathbf{\Phi}_s^{(l)}$ is an unitary matrix yet to be determined. Then, we have $\mathbf{B}^{(l)} = \bar{\mathbf{H}}_{rd}^{(l)}\tilde{\mathbf{F}}_r^{(l)}\tilde{\mathbf{F}}_r^{(l)^H}\bar{\mathbf{H}}_{rd}^{(l)^H} + \mathbf{U}_{\Sigma_{rd}}^{(l)}\tilde{\mathbf{\Lambda}}_{\Sigma_{rd}}^{(l)}\mathbf{U}_{\Sigma_{rd}}^{(l)^H}$ and $\tilde{\mathbf{B}}^{(l)} = \tilde{\mathbf{H}}_{rd}^{(l)}\tilde{\mathbf{F}}_r^{(l)}\tilde{\mathbf{F}}_r^{(l)^H}\tilde{\mathbf{H}}_{rd}^{(l)^H} + \mathbf{I}_{N_d}$.

By substituting (23), (24) into (20), the problem can be simplified as follows:

$$\max\left|\left(\sigma_s^{-2}\mathbf{I}_{N_s} + \tilde{\mathbf{\Lambda}}_{sr}^{(l)^2}\mathbf{\Lambda}_s^{(l)^2}\mathbf{\Lambda}_r^{(l)^2}\tilde{\mathbf{\Lambda}}_{rd}^{(l)^2}\left(\tilde{\mathbf{\Lambda}}_{rd}^{(l)^2}\mathbf{\Lambda}_r^{(l)^2} + \mathbf{I}_{N_d}\right)^{-1}\right)\right|$$

$$s.t. \quad \mathrm{tr}\left(\sigma_s^2 \mathbf{\Lambda}_s^{(l)^2}\right) \le P_s$$

$$\mathrm{tr}\left(\mathbf{\Lambda}_r^{(l)^2}\left(\sigma_s^2 \mathbf{\Lambda}_s^{(l)^2} \tilde{\mathbf{\Lambda}}_{sr}^{(l)^2} + \mathbf{I}_{N_r}\right)\right) \le P_r. \quad (26)$$

Note that for a positive semi-definite matrix $\mathbf{M} \in \mathbb{C}^{N\times N}$, we have [9]

$$\det(\mathbf{M}) \le \prod_{i=1}^{N} \mathbf{M}(i,i), \quad (27)$$

the equality holds when $\mathbf{M}$ is a diagonal matrix. Thus, in order to maximize the determinant, we can try to design the joint source and relay precoders to let the matrix of the determinant be diagonal.

Let $\tilde{\lambda}_{1,i}$ and $\tilde{\lambda}_{2,i}$ be the ith diagonal element of $\tilde{\mathbf{\Lambda}}_{sr}^{(l)}$ and $\tilde{\mathbf{\Lambda}}_{rd}^{(l)}$, respectively, i=$1,\cdots,N_s$, from (26) we have the following results

$$\max \prod_{i=1}^{N_s}\left(\sigma_s^{-2} + \frac{\tilde{\lambda}_{1,i}^2 \tilde{\lambda}_{2,i}^2 \lambda_{F_s,i}^2 \lambda_{\tilde{F}_r,i}^2}{\tilde{\lambda}_{2,i}^2 \lambda_{\tilde{F}_r,i}^2 + 1}\right) \quad (28)$$

$$s.t. \quad \sum_{i=1}^{N_s} \sigma_s^2 \lambda_{F_s,i}^2 \le P_s \quad (29)$$

$$\sum_{i=1}^{N_s} \lambda_{\tilde{F}_r,i}^2 \left(\sigma_s^2 \lambda_{F_s,i}^2 \tilde{\lambda}_{1,i}^2 + 1\right) \le P_r. \quad (30)$$

We introduce

$$x_i \triangleq \sigma_s^2 \lambda_{F_s,i}^2 \quad (31)$$

$$y_i \triangleq \lambda_{\tilde{F}_r,i}^2 \left(\sigma_s^2 \lambda_{F_s,i}^2 \tilde{\lambda}_{1,i}^2 + 1\right). \quad (32)$$

In order to simplify the problem, we take the logarithm operation to the cost function. The solution to the objective function can be obtained by using an iterative waterfilling method via Karash-Kuhn-Tucker (KKT) conditions. For a given x_i, by solving (28) and (30), the optimum y_i can be obtained as follows:

$$y_i = \frac{1}{2\tilde{\lambda}_{2,i}^2}\left[\sqrt{\tilde{\lambda}_{1,i}^4 x_i^2 + 4\tilde{\lambda}_{1,i}^2 x_i \tilde{\lambda}_{2,i}^2 \mu_r} - \tilde{\lambda}_{1,i}^2 x_i - 2\right]^+ \quad (33)$$

where $[y]^+ = \max[0,y]$, and μ_r is the water level which satisfies the power constraint with equality at the relay in (30). By solving (28) and (29), the optimum x_i can be calculated as

$$x_i = \frac{1}{2\tilde{\lambda}_{1,i}^2}\left[\sqrt{\tilde{\lambda}_{2,i}^4 y_i^2 + 4\tilde{\lambda}_{1,i}^2 y_i \tilde{\lambda}_{2,i}^2 \mu_s} - \tilde{\lambda}_{2,i}^2 y_i - 2\right]^+ \quad (34)$$

where μ_s is the water level which satisfies the power constraint with equality at the source in (29). The algorithm can be implemented iteratively with initial values. Note that $\lambda_{F_s,i}$ and $\lambda_{\tilde{F}_r,i}$ can be calculated based on (31) and (32). We then focus on the derivation of the unitary matrix $\mathbf{\Phi}_s^{(l)}$ and the feedback matrix $\mathbf{U}^{(l)}$.

The lower bound of MSE is achieved when the objective function in (17) is a diagonal matrix with equal diagonal elements. Thus, the following equation must be met:

$$\mathbf{U}^{(l)}\left(\sigma_s^{-2}\mathbf{I}_{N_s} + \mathbf{F}_s^{(l)^H} \bar{\mathbf{H}}^{(l)^H} \mathbf{B}^{(l)^{-1}} \bar{\mathbf{H}}^{(l)} \mathbf{F}_s^{(l)}\right)\mathbf{U}^{(l)^H} = \bar{\sigma}^2 \mathbf{I}_{N_s}. \quad (35)$$

By substituting (23) and (25) into (35), we obtain $\mathbf{U}^{(l)}\mathbf{\Phi}_s^{(l)^H}\mathbf{\Sigma}^{(l)^{-1/2}}\mathbf{\Sigma}^{(l)^{-1/2}}\mathbf{\Phi}_s^{(l)}\mathbf{U}^{(l)^H} = \bar{\sigma}^2\mathbf{I}_{N_s}$, where we have $\mathbf{\Sigma}^{(l)} \triangleq (\sigma_s^{-2}\mathbf{I}_{N_s} + \tilde{\mathbf{\Lambda}}_{sr}^{(l)^2}\mathbf{\Lambda}_s^{(l)^2}\mathbf{\Lambda}_r^{(l)^2}\tilde{\mathbf{\Lambda}}_{rd}^{(l)^2}(\tilde{\mathbf{\Lambda}}_{rd}^{(l)^2}\mathbf{\Lambda}_r^{(l)^2} + \mathbf{I}_{N_d})^{-1})$. Then we define $\tilde{\mathbf{U}}^{(l)}$=$\bar{\sigma}\mathbf{U}^{(l)^{-H}}$ and apply the geometric mean decomposition (GMD) [10] on $\mathbf{\Sigma}^{(l)^{-1/2}}$, then obtain $\mathbf{\Sigma}^{(l)^{-1/2}} = \mathbf{Q}^{(l)}\tilde{\mathbf{U}}^{(l)}\mathbf{\Phi}_s^{(l)^H}$, where $\mathbf{Q}^{(l)}$ and $\mathbf{\Phi}_s^{(l)}$ are unitary matrices, and $\tilde{\mathbf{U}}^{(l)}$ is an upper triangular matrix with equal diagonal elements $\bar{\sigma}$, where $\bar{\sigma}^2$ is given by

$$\bar{\sigma}^2 = \prod_{i=1}^{N_s}\left(\sigma_s^{-2} + \frac{\tilde{\lambda}_{1,i}^2 \tilde{\lambda}_{2,i}^2 \lambda_{F_s,i}^2 \lambda_{\tilde{F}_r,i}^2}{\tilde{\lambda}_{2,i}^2 \lambda_{\tilde{F}_r,i}^2 + 1}\right)^{-1/N_s}. \quad (36)$$

From the equation above, it can be clearly seen that the equality is achieved. We then calculate $\mathbf{U}^{(l)} = \bar{\sigma}\tilde{\mathbf{U}}^{(l)^{-H}}$. With $\mathbf{\Phi}_s^{(l)}$ and $\mathbf{U}^{(l)}$, the source and relay precoders corresponding to the l-th cancellation order are obtained by (23) and (25). Subsequently, the MMSE receiver $\mathbf{W}^{(l)}$ can be derived by substituting (23) and (25) into (15).

Then, we consider the case that the covariance matrix of channel estimation error at the receiver side is identity matrix, i.e. $\mathbf{\Sigma}_{sr} = \sigma_e^2\mathbf{I}_{N_s}$ and $\mathbf{\Sigma}_{rd} = \sigma_e^2\mathbf{I}_{N_r}$. In this case, the precoding matrices have the structure as: $\mathbf{F}_s^{(l)} = \mathbf{V}_{sr}\mathbf{\Lambda}_s^{(l)}\mathbf{\Phi}_s^{(l)}$, $\mathbf{F}_r^{(l)} = \mathbf{V}_{rd}^{(l)}\mathbf{\Lambda}_r^{(l)}\mathbf{U}_{sr}^H$. One can obtain the similar solutions to the problem by using the aforementioned iterative method.

B. Selection Criterion for MB-THP Scheme

A proper selection criterion is of great importance for the MB-THP algorithm to achieve the transmit diversity gains of MIMO relay systems. The selection criterion chooses the best branch corresponding to the minimum Euclidean distance:

$$l_{opt} = \arg\min_{1\le l\le L} \mathbf{J}(l), \quad (37)$$

where the Euclidean distance for the l-th cancellation order is given by

$$\mathbf{J}(l) = \left\|\mathbf{s} - \hat{\mathbf{s}}^{(l)}\right\|^2 \quad (38)$$

where $\hat{\mathbf{s}}^{(l)}$=$\mathbf{T}^{(l)^T}\tilde{\mathbf{s}}^{(l)}$, $\hat{\mathbf{s}}^{(l)}$ is the transformed version of $\tilde{\mathbf{s}}^{(l)}$ back to the original order, the vector $\tilde{\mathbf{s}}^{(l)}$ denotes the noise-free pre-estimated values of the data at the transmitter using estimated CSI, it is given by

$$\tilde{\mathbf{s}}^{(l)} = \mathrm{MOD}\left(\tilde{\mathbf{y}}^{(l)}\right) \quad (39)$$

where $\tilde{\mathbf{y}}^{(l)}$=$\mathbf{W}^{(l)}\bar{\mathbf{H}}_{rd}^{(l)}\mathbf{F}_r^{(l)}\bar{\mathbf{H}}_{sr}\mathbf{F}_s^{(l)}\mathbf{x}^{(l)}$. The procedure of the proposed robust transceiver algorithm is summarized in Table I.

IV. Simulation Results

In this section, we evaluate the performance of the proposed precoding scheme. In the following, we consider a nonregenerative MIMO relay system with $N_s = N_r = N_d = 4$. By using the exponential model [5], the channel estimation error covariance matrices can be expressed as: $\mathbf{\Psi}_{sr} = \mathbf{\Psi}_{rd} = \delta^{|i-j|}$ and $\mathbf{\Sigma}_{sr} = \mathbf{\Sigma}_{rd} = \sigma_e^2\gamma^{|i-j|}$, where δ and γ denote the correlation coefficients, and σ_e^2 is the estimation error variance. The estimated channel $\bar{\mathbf{H}}_{sr}$ is generated by the distribution as follows: $\bar{\mathbf{H}}_{sr} \sim \mathcal{CN}_{N_r,N_s}\left(\mathbf{0}_{N_r,N_s}, \frac{(1-\sigma_e^2)}{\sigma_e^2}\mathbf{\Sigma}_{sr}\otimes\mathbf{\Psi}_{sr}^T\right)$, similar definition can be applied on $\bar{\mathbf{H}}_{rd}$, such that channel realizations have unit variance. Let SNR_{sr} and SNR_{rd} denote,

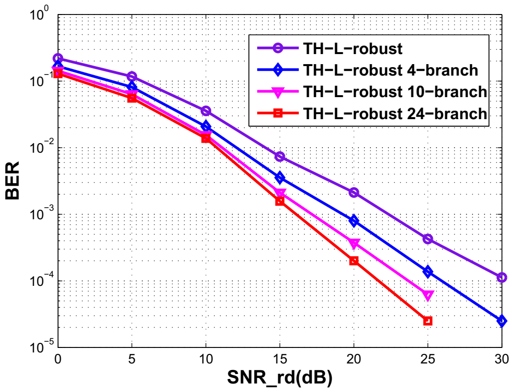

Fig. 2: BER performance comparison for different multi-branch ordering schemes

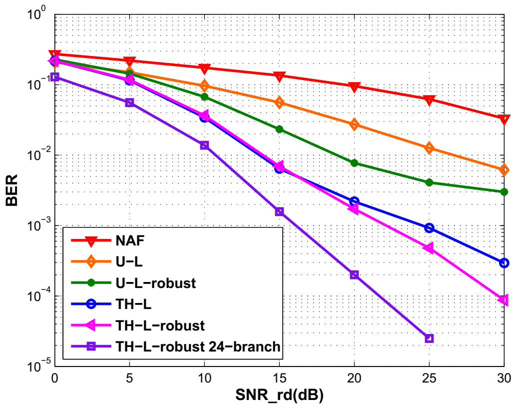

Fig. 3: BER performance comparison for conventional precoding techniques and the proposed MB-TH precoding algorithm ($\delta = \gamma = 0, \sigma_e^2 = 0.001$)

respectively, the signal-to-noise ratios (SNR) per receive antenna of the source-to-relay and the relay-to-destination links. Here, we let $\mathrm{SNR}_{sr} = 30$ dB and vary SNR_{rd}. Also, we use 16-QAM modulation as the modulation scheme.

TABLE I: Proposed Robust Transceiver Algorithm

1	**for** each parallel branch l, $l \in \{1, ..., L\}$.
2	Solve for the unknown diagonal matrices $\mathbf{\Lambda}_s^{(l)}$ and $\mathbf{\Lambda}_r^{(l)}$ in the optimal precoding structure using (33) and (34) via KKT conditions.
3	Compute $\mathbf{\Phi}_s^{(l)}$ and the feedback matrix $\mathbf{U}^{(l)}$ based on (23), (25) and (35).
4	Derive the optimal structure of $\mathbf{F}_s^{(l)}$ and $\mathbf{F}_r^{(l)}$ given by (23) and (25).
5	Compute the receiver $\mathbf{W}^{(l)}$ by using the obtained $\mathbf{F}_s^{(l)}$, $\mathbf{F}_r^{(l)}$ and $\mathbf{U}^{(l)}$.
5	Compute the squared Euclidean distance for the l-th cancellation order, $\mathbf{J}(l) = \left\|\mathbf{s} - \hat{\mathbf{s}}^{(l)}\right\|^2$.
6	**end**
7	Choose the optimum branch by using the selection criterion (37) for data transmission.

All the simulation results are averaged over 5000 independent realizations of the true channel matrices $\mathbf{H}_{sr}$ and $\mathbf{H}_{rd}$. Fig. 2 shows the BER performance versus the SNR for comparing the proposed MB-TH transceiver scheme, i.e. 4-, 10-, 24- cancellation ordering branches, respectively. The best performance is achieved with the proposed scheme with 24 ordering branches, i.e. the optimal scheme. The BER decreases as the number of branches increases.

We also compare the proposed robust MB-THP algorithm with the following five MIMO relay precoding algorithms: 1) an unprecoded system with a Wiener filter (NAF); 2) the linear relay precoded system without source precoding (U-L) [1]; 3) the robust linear relay precoded system without source precoding (U-L-robust) [5]; 4) the TH source and linear relay precoded system (TH-L) [3]; 5) the proposed robust TH source and linear relay precoded system (TH-L-robust). As shown in Fig. 3, the proposed robust MB-TH source and linear relay precoding algorithm using the optimum ordering scheme outperforms the existing transceiver designs in terms of BER. Meanwhile, the performance of the proposed robust algorithm considering the estimation error is better than that of the algorithm based on estimated channels only.

V. Conclusion

In this paper, the robust MB-TH transceiver design in MIMO relay networks with imperfect CSI has been addressed. The proposed MB structure is equipped with several parallel branches based on pre-stored ordering patterns. For each branch, the transceiver is composed of a TH precoder at the source, a linear precoder at the relay and an MMSE receiver at the destination. The solution for the precoders has been finally obtained by using an iterative method via the KKT conditions. An appropriate selection rule has been developed to choose the nonlinear transceiver corresponding to the best branch for data transmission. Simulations have shown that the proposed robust design outperforms the existing unprecoded/precoded systems without taking the channel uncertainties into account.

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