

Uplink Block Diagonalization for Massive MIMO-OFDM Systems with Distributed Antennas

Leonel Arévalo[†], Rodrigo C. de Lamare[†], Martin Haardt[‡] and Raimundo Sampaio-Neto[†]

[†] Pontifical Catholic University of Rio de Janeiro (PUC-Rio)

Center for Studies in Telecommunications (CETUC) — Rio de Janeiro, Brazil

E-mails: {leonel_arevalo, delamare and raimundo}@cetuc.puc-rio.br

[‡] Ilmenau University of Technology

Communications Research Laboratory — Ilmenau, Germany

E-mail: martin.haardt@tu-ilmenau.de

Abstract—In this paper, we propose a block diagonalization (BD) procedure for the *uplink* of massive MIMO-OFDM systems to decompose the received signal vector, at the base station (BS), into independent single-user MIMO-OFDM signals. After *uplink* BD, the detection procedure can be performed separately for each active user, thus reducing the computational complexity when compared with standard detection techniques, where all the active users are coupled. In the proposed scheme, unlike prior work in which *downlink* BD algorithms are used, it is not necessary to use any precoding at the transmit side. The performance of the proposed scheme is evaluated in a realistic mobile radio channel for an indoor (local area) scenario with distributed antennas at the BS.

Keywords—*Uplink Block Diagonalization, Massive MIMO-OFDM systems, Distributed Antennas.*

I. INTRODUCTION

The fifth generation (5G) of wireless communications systems has stringent requirements in terms of data rates, latency and energy efficiency, and will be structured in the form of heterogeneous networks with an increasing number of smart terminals and their emerging applications [1]. In order to develop the foundations of 5G networks, which are expected to be operational around 2020, researchers have recently started to consider 5G technical challenges and investigate technical solutions.

In recent years, massive multiple-input multiple-output (MIMO) has been considered one of the most promising technologies for 5G wireless networks [2]-[4]. This technique involves the use of a large number of antennas, allowing the accommodation of more users, higher data rates and effective mitigation of fading. Specifically, massive MIMO consists of a large number of transmit antennas which serve a large number of users simultaneously and exploit a high degree of multiuser diversity. If the number of antennas at the base station (BS) is much larger than the number of users simultaneously served, the system performance can be greatly improved [5] due to the fact that the channels become quasi-orthogonal. Furthermore, it has been shown that massive MIMO can increase the spectral efficiency by more than tenfold and simultaneously improve the energy efficiency by a factor of 100 [6].

Distributed antenna systems (DAS) are a promising alternative for 5G cellular architectures with massive MIMO [7]. In this context, the BS will be equipped with a large number of antennas and some remote antennas arrays will be distributed

around the cell and connected to the BS via optical fiber. The remote antennas arrays are used to transmit and receive signals. The signals associated with different remote antennas arrays are processed at the BS. Recently, DAS have received more attention for their lower path loss effects, to improve the coverage and the spectral efficiency [8]. The energy consumption of users is reduced and the transmission quality is improved due to the shorter distances between users and some remote antennas arrays. For this vision of 5G wireless networks, which includes a combination of massive MIMO and distributed antenna systems, efficient detection techniques are necessary to increase the spectral efficiency. Despite many advantages, massive MIMO has as main challenge the signal processing complexity which basically depends on the number of active users, the number of antennas at the BS and the detection scheme used. Detection schemes proposed recently for MU-MIMO systems include those based on the successive interference cancellation (SIC) strategy such as multiple-feedback SIC (MF-SIC), multiple-branches SIC (MB-SIC) and multiple-branches lattice reduction SIC (MB-LR-SIC) that have shown significant performance improvements [9]-[11], but could be costly in massive MIMO due to the use large matrices that must be processed.

In this paper, we propose a new method for the *uplink* of massive MIMO-OFDM systems that allows the separation of the combined received signal vector of all active users, at the BS, into independent single user MIMO-OFDM signals. The proposed *uplink* block diagonalization (BD) technique computes a set of matrices that allows the decoupling of the streams of all users using the singular value decomposition (SVD). Previously, BD has been used for *downlink* MIMO precoding [12]-[15]. For the *uplink* BD proposed here, it is only necessary to know the channel matrix corresponding to the active users. We assume that channel state information (CSI) has been estimated at the BS. After *uplink* BD decouples the stream of all users, it is possible to use detection schemes that, due to their high computational cost, were not possible to be implemented when all users were coupled. The performance of the proposed scheme is evaluated in a realistic scenario with distributed antennas.

This paper is organized as follows. Section II examines the massive MIMO-OFDM system model. The proposed *uplink* BD is detailed in Section III. Section IV presents simulations results. Section V summarizes the conclusions.

II. MIMO-OFDM SYSTEM MODEL

In this section, the basic concepts for massive MIMO-OFDM systems are presented, starting with the mathematical system model. It is also presented the indoor mobile radio access channel scenario with distributed antennas considered in this work.

A. Massive MIMO-OFDM Systems

We consider the *uplink* scenario of a massive MIMO-OFDM system with K active users, each equipped with N_{t_k} antennas. The users send signals to one BS with N_r antennas, where $N_r \geq N_t = \sum_{k=1}^K N_{t_k}$. In the transmission, an OFDM system with cyclic prefix (CP) is employed with subcarrier bandwidth Δ_f . After we remove the CP and apply a fast Fourier transformation (FFT), the received signal samples in the frequency domain are given by

$$\begin{aligned} \mathbf{y}^n &= \sum_{k=1}^K \mathbf{H}_k^n \mathbf{s}_k^n + \mathbf{n}^n \\ &= \mathbf{H}^n \mathbf{s}^n + \mathbf{n}^n, \end{aligned} \quad (1)$$

where $\mathbf{H}^n = [\mathbf{H}_1^n \ \mathbf{H}_2^n \ \dots \ \mathbf{H}_K^n]$ is the channel matrix in the frequency domain for the n -th FFT subcarrier and $\mathbf{s}^n = [(\mathbf{s}_1^n)^T \ (\mathbf{s}_2^n)^T \ \dots \ (\mathbf{s}_K^n)^T]^T$ is the transmitted signal vector of all active users, where $(\cdot)^T$ denotes the transpose operator and the symbols are taken from a modulation constellation $\mathcal{A} = \{a_1, a_2, \dots, a_M\}$. Each symbol carries N bits and $M = 2^N$. The symbol vector \mathbf{s}^n has zero mean and a covariance matrix $\mathbf{K}_{\mathbf{s}^n} = \sigma_s^2 \mathbf{I}$. The vector \mathbf{n}^n is the FFT of an $N_r \times 1$ zero mean complex circular symmetric Gaussian noise vector sample in the time domain, with covariance matrix $\mathbf{K}_{\mathbf{n}} = \mathbb{E}[\mathbf{n}(\mathbf{n})^H] = \sigma_n^2 \mathbf{I}$, where $\mathbb{E}[\cdot]$ and $(\cdot)^H$ represents the expected value and the Hermitian operator, respectively. It is worth noting that the index n will be removed from the subsequent equations to maintain simplicity.

B. Distributed Antennas for Indoor Scenario

In this part, we describe the DAS configuration for the indoor scenario [16]. The DAS configuration consists of N_r receive antennas at the BS, which are distributed in R remote antenna arrays over the coverage area. We consider that each array of antennas has N_r/R antennas. The DAS configuration, when compared with the centralized antenna configuration, can improve the spectral efficiency and power efficiency due to the low path loss effects and the short distances between users and some remote antenna arrays. On the other hand, the indoor scenarios are characterized by some features like the slow mobility of users with speeds between 3 and 5 km/h, that facilitate the channel knowledge with a good quality for longer periods of time. Also, the coverage area is relatively small and can be covered with DAS easily. The environment considered here combines the advantages of DAS and indoor scenarios. The main benefits of the DAS for an indoor scenario deployment are more even signal power distribution across the coverage area and significantly reduced spatial correlation between antennas at the BS. The possible applications include different public places, such as offices, airports and shopping malls. The DAS configuration for indoor scenario is illustrated in Figure 1. This indoor scenario consists of 40 rooms of size 10 m \times 10 m and two corridors that are 5 m wide.

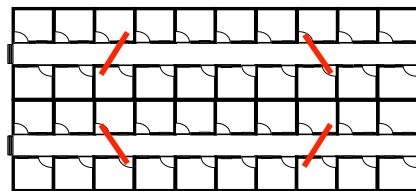


Fig. 1. Indoor scenario with distributed antennas [16]. View from top.

The red lines in Figure 1 represent a remote linear array of antennas which are distributed over the office and linked to one single BS via optical fiber that may use all of them jointly for transmission and reception. The users are assumed to be evenly distributed on the entire office floor [16]. The channels are modeled following the recommendations in the *WINNER II Channel Models* [17]. The *WINNER* channel model is a system level model, based on the OFDM access scheme, which can describe an arbitrary number of propagation environment realizations including the indoor mobile radio access with DAS considered here.

III. UPLINK BLOCK DIAGONALIZATION

In this section, we describe the proposed *uplink* BD algorithm which allows the separation of the multiuser received signal vector in (1) into independent single-user signals. We assume that the channel matrix \mathbf{H} was previously estimated without errors at the BS. In the *uplink* BD, a matrix \mathbf{W}_k is calculated for each user in order to separate the k -th user received signal from the other users' signals. To compute the matrix \mathbf{W}_k , we define the matrix $\tilde{\mathbf{H}}_k$ excluding the k -th users' channel matrix as the following form:

$$\tilde{\mathbf{H}}_k = [\mathbf{H}_1 \ \dots \ \mathbf{H}_{k-1} \ \mathbf{H}_{k+1} \ \dots \ \mathbf{H}_K], \quad (2)$$

where $\tilde{\mathbf{H}}_k \in \mathbb{C}^{N_r \times (N_t - N_{t_k})}$. After that, the objective is to obtain a matrix \mathbf{W}_k that satisfies the following condition:

$$\mathbf{W}_k \tilde{\mathbf{H}}_k = \mathbf{0}, \quad \forall k \in (1 \dots K). \quad (3)$$

Performing the SVD of $\tilde{\mathbf{H}}_k$ we have

$$\tilde{\mathbf{H}}_k = \tilde{\mathbf{U}}_k \tilde{\Sigma}_k \tilde{\mathbf{V}}_k^H, \quad (4)$$

where $\tilde{\Sigma}_k \in \mathbb{C}^{N_r \times (N_t - N_{t_k})}$ is a rectangular diagonal matrix with the singular values of $\tilde{\mathbf{H}}_k$ on the diagonal, $\tilde{\mathbf{U}}_k \in \mathbb{C}^{N_r \times N_r}$ and $\tilde{\mathbf{V}}_k^H \in \mathbb{C}^{(N_t - N_{t_k}) \times (N_t - N_{t_k})}$ are unitary matrices. If r_k is the rank of $\tilde{\mathbf{H}}_k$, that corresponds to the number of non-zero singular values, i.e., $r_k = \text{rank}(\tilde{\mathbf{H}}_k) \leq N_t - N_{t_k}$, we can rewrite (4) as

$$\tilde{\mathbf{H}}_k = [\tilde{\mathbf{U}}_{1,k} \ \tilde{\mathbf{U}}_{0,k}] \tilde{\Sigma}_k [\tilde{\mathbf{V}}_{1,k} \ \tilde{\mathbf{V}}_{0,k}]^H, \quad (5)$$

where $\tilde{\mathbf{U}}_{0,k} \in \mathbb{C}^{N_r \times (N_r - r_k)}$ and $\tilde{\mathbf{V}}_{0,k}^H \in \mathbb{C}^{(N_t - N_{t_k} - r_k) \times (N_t - N_{t_k})}$ are composed with zero singular vectors and form an orthogonal basis for the left null space and the null space of $\tilde{\mathbf{H}}_k$, respectively. One solution for (3) is given by

$$\mathbf{W}_k = \tilde{\mathbf{U}}_{0,k}^H. \quad (6)$$

Computing (6) for the K active users we can construct the matrix \mathbf{W} as

$$\mathbf{W} = [\tilde{\mathbf{U}}_{0,1} \ \dots \ \tilde{\mathbf{U}}_{0,K}]^H \in \mathbb{C}^{K(N_r - r_k) \times N_r}. \quad (7)$$

Using (7) we transform the MU-MIMO received signal vector into parallel SU-MIMO signals as described by

$$\mathbf{y}_s = \mathbf{W}\mathbf{y} = \mathbf{W}\mathbf{H}\mathbf{s} + \mathbf{W}\mathbf{n} = \mathbf{H}_s\mathbf{s} + \mathbf{n}_s, \quad (8)$$

where $\mathbf{y}_s \in \mathbb{C}^{K(N_r-r_k) \times 1}$ is the equivalent received signal vector, $\mathbf{H}_s \in \mathbb{C}^{K(N_r-r_k) \times N_t}$ is the equivalent block diagonal channel matrix and $\mathbf{n}_s \in \mathbb{C}^{K(N_r-r_k) \times 1}$ is a Gaussian noise vector. For a more simple visualization, equation (8) can be rewritten as

$$\begin{bmatrix} \mathbf{y}_{s_1} \\ \vdots \\ \mathbf{y}_{s_K} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{s_1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{H}_{s_K} \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 \\ \vdots \\ \mathbf{s}_K \end{bmatrix} + \begin{bmatrix} \mathbf{n}_{s_1} \\ \vdots \\ \mathbf{n}_{s_K} \end{bmatrix}, \quad (9)$$

where $\mathbf{H}_{s_k} = \tilde{\mathbf{U}}_{0,k}^H \mathbf{H}_k \in \mathbb{C}^{(N_r-r_k) \times N_{t_k}}$ is the equivalent channel matrix of the k -th user after eliminating the multiuser interference and the noise vector $\mathbf{n}_{s_k} = \tilde{\mathbf{U}}_{0,k}^H \mathbf{n}$ is white and Gaussian. From (9) we can see that passing the received signal vector \mathbf{y} through the linear filter \mathbf{W} , the users can be decoupled at the BS, which allows the subsequent use of more complex detection schemes.

A. Detection Algorithms

In this subsection, we examine signal detection algorithms for massive MIMO-OFDM systems. To detect each user's data stream independently, we assume that the *uplink* BD algorithm described in (2)-(8) was previously employed.

1) *Linear Detectors*: In linear detectors, the equivalent received signal vector for the k -th user $\mathbf{y}_{s_k} \in \mathbb{C}^{(N_r-r) \times 1}$ is filtered by a linear filter to eliminate the channel effects [18]. The two linear detectors considered here are given by

$$\mathbf{G}_k^\chi = \begin{cases} (\mathbf{H}_{s_k}^H \mathbf{H}_{s_k})^{-1} \mathbf{H}_{s_k}^H, & \chi = \text{Zero Forcing} \\ (\mathbf{H}_{s_k}^H \mathbf{H}_{s_k} + \sigma_n^2 \mathbf{I})^{-1} \mathbf{H}_{s_k}^H, & \chi = \text{MMSE} \end{cases} \quad (10)$$

where $\mathbf{H}_{s_k} \in \mathbb{C}^{(N_r-r) \times N_{t_k}}$ is the equivalent channel matrix of the k -th user. The linear hard decision of \mathbf{s}_k is carried out as follows:

$$\hat{\mathbf{s}}_k = \mathbb{C}(\mathbf{G}_k^\chi \mathbf{y}_{s_k}), \quad (11)$$

where the function $\mathbb{C}(x)$ returns the point of the complex signal constellation closest to x . The linear detectors have a lower computational complexity when compared with the non-linear detectors, however, due to the impact of interference and noise, linear detectors offer a limited performance.

2) *Successive Interference Cancellation SIC*: The SIC detector for the k -th user in (9) consists of a bank of linear detectors, each detects a selected component $s_{k,i}$ of \mathbf{s}_k . The component obtained by the first detector is used to reconstruct the corresponding signal vector which is then subtracted from the equivalent received signal to further reduce the interference in the input to the next linear receive filter. The successively canceled received data vector that follows a chosen ordering in the i -th stage is given by

$$\mathbf{y}_{s_k,i} = \mathbf{y}_{s_k} - \sum_{j=1}^{i-1} \mathbf{h}_{s_k,j} \hat{s}_{k,j}, \quad (12)$$

where $\mathbf{h}_{s_k,j}$ correspond to the columns of the channel matrix \mathbf{H}_{s_k} and $\hat{s}_{k,j}$ is the estimated symbol obtained in the output of the j -th linear detector.

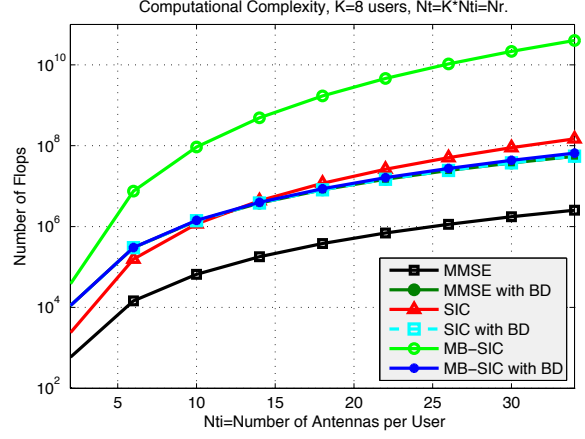


Fig. 2. Computational Complexity for *uplink* BD.

3) *Multiple-Branch SIC Detection*: In the multi-branch scheme [10] for the k -th user, different orderings are explored for SIC, each ordering is referred to as a branch, so that a detector with L branches produces a set of L estimated vectors. Each branch uses a column permutation matrix \mathbf{P}_k . The estimate of the signal vector of branch l , $\hat{\mathbf{x}}_k^{(l)}$, is obtained using a SIC receiver based on a new channel matrix $\mathbf{H}_{s_k}^{(l)} = \mathbf{H}_{s_k} \mathbf{P}_k^{(l)}$. The order of the estimated symbols is rearranged to the original order by

$$\hat{\mathbf{s}}_k^{(l)} = \mathbf{P}_k^{(l)} \hat{\mathbf{x}}_k^{(l)}, \quad l = 1, \dots, L. \quad (13)$$

A higher detection diversity can be obtained by selecting the most likely symbol vector based on the ML selection rule, that is

$$\hat{\mathbf{s}}_k = \arg \min_l \|\mathbf{y}_{s_k} - \mathbf{H}_{s_k} \hat{\mathbf{s}}_k^{(l)}\|^2, \quad l = 1, \dots, L. \quad (14)$$

B. Computational complexity

In this subsection, the computational complexity of *uplink* BD using different detection schemes is evaluated. The SIC based receivers all use MMSE detection. Furthermore, the single-branch SIC and the first branch of the MB-SIC employ norm-based ordering. We compute the number of FLOPs per received vector \mathbf{y} using the Lightspeed Matlab toolbox [19]. In Fig. 2 we plot the required number of FLOPs versus the number of antennas per user N_{t_k} for a fixed number of users $K = 8$. We consider QPSK modulation. The proposed *uplink* BD with MB-SIC has a lower complexity than the MB-SIC with all users coupled. For the SIC detector we can see that *uplink* BD has a lower complexity when the number of antennas per user is increased. The computational cost is always dominated by BD with a percentage that depends on the detector used. It is worth noting that the curves displayed in Fig. 2 will have a substantial decrease if the channel does not change over a time period. It would increase the gap, in terms of computational cost, for the detection schemes using *uplink* BD.

IV. NUMERICAL RESULTS

In this section, the BER performance of the *uplink* BD is evaluated using different detection schemes which include

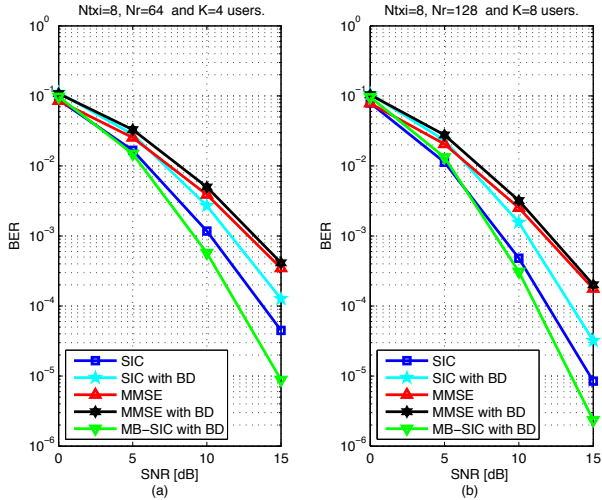


Fig. 3. BER vs SNR for massive MIMO-OFDM system with distributed antennas arrays. We use QPSK modulation, $K = 4$ users, $R = 4$ remote antennas arrays, $N_{t_i} = 8$ antennas per user.

MMSE, SIC and MB-SIC. The massive MIMO-OFDM channel with distributed antennas and 64 subcarriers channel has been generated by the WIM2 channel model [20]. The SNR per transmitted information bit is defined as $10 \log_{10} \frac{N_t \sigma_s^2}{N \sigma_n^2}$, where σ_s^2 is the common transmitted symbol energy and N is the number of information bits per transmitted symbol. The data transmitted vector \mathbf{s}_k of the k -th user is considered with 8 streams using QPSK modulation. The simulation curves correspond to an average of 3,000 simulation runs, with $640N_t$ symbols transmitted per run. In Fig. 3 (a) and Fig. 3 (b), we compare the performances of the SIC detector and the proposed *uplink* BD with MMSE, SIC and MB-SIC detectors, assuming perfectly known channel state information. The figures indicate that the performance of the SIC detector with *uplink* BD is worse than that of the SIC detector, but its computational complexity is lower for high number of antennas. Note that the results for the MB-SIC detector with *uplink* BD show a very good performance with a computational complexity lower than the MB-SIC and the SIC detectors without *uplink* BD.

V. CONCLUSIONS

In this paper we have proposed an orthogonalization procedure for *uplink* massive MIMO-OFDM systems that allows one to separate the users' signals into independent parallel single user signals at the BS, applying an SVD and assuming that the channel matrix was previously estimated. We evaluate the proposed system in a realistic channel scenario with distributed antennas, following the recommendation of the WINNER II channel model. With the proposed *uplink* BD scheme, it is possible to handle different classes of users in heterogeneous networks and to use different modulation and/or detection schemes for each class. The main advantage of *uplink* BD is the reduction in the computational cost of efficient detection schemes that, for its high computational complexity, are not viable to be implemented when the signals received from all active users are coupled.

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