Reduced-Rank Techniques for Array Signal Processing and Communications : Design, Algorithms and Applications

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Outline

Part I :

– Introduction

– System model and rank reduction

– Reduced-Rank MMSE and LCMV Designs

– Eigen-decomposition techniques

– The multistage Wiener filter

Outline (continued)

Part II :

- Techniques based on joint and iterative optimisation of filters
- Joint interpolation, decimation and filtering
- Techniques based on joint and iterative optimisation of basis functions
- Model order selection
- Applications, perspectives and future work
- Concluding remarks

Introduction

– Reduced-rank detection and estimation techniques are a fundamental set of tools in signal processing and communications.

– Motivation of reduced-rank processing :

  – robustness against noise and model uncertainties,
  – computational efficiency,
  – decompositions of signals for design and analysis,
  – inverse problems,
  – feature extraction,
  – dimensionality reduction,
  – problems with short data record, faster training .

Introduction

- Main Goals of Reduced-Rank Methods :
  - simplicity, ease of deployment,
  - to provide minimal reconstruction error losses,
  - to allow simple mapping and inverse mapping functions,
  - to improve convergence and tracking performance for dynamic signals,
  - to reduce the need for storage of the coefficients of the estimator,
  - to provide amenable and stable adaptive implementation,

Introduction

- Communications :
  - Interference mitigation, synchronization, fading mitigation, channel estimation.
  - Parameter estimation with MMSE or LS criteria (Haykin [1]) :

$$\mathbf{w} = \mathbf{R}^{-1}\mathbf{p}$$

    where
    $\mathbf{w}$ is a parameter vector with M coefficients,
    $\mathbf{r}(i)$ is the $M \times 1$ input data vector,
    $\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^H(i)]$ is the $M \times M$ covariance matrix,
    $\mathbf{p} = E[d^*(i)\mathbf{r}(i)]$ and $d(i)$ is the desired signal.

  - Detection approaches using MMSE or LS estimates.
  - Problems : dimensionality of system, matrix inversion
  - How to improve performance ?

Introduction

- Array signal processing :
  - Beamforming, direction finding, information combining with sensors, radar and sonar (van Trees [2]).
  - Parameter estimation with LCMV criterion :

$$\mathbf{w} = \xi^{-1}\mathbf{R}^{-1}\mathbf{a}(\Theta_k)$$

    where
    $\mathbf{w}$ is a parameter vector with M coefficients,
    $\mathbf{r}(i)$ is the $M \times 1$ input data vector,
    $\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^H(i)]$ is the $M \times M$ covariance matrix,
    $\mathbf{a}(\Theta_k)$ is the $M \times 1$ array response vector and
    $\xi = \mathbf{a}(\Theta_k)^H\mathbf{R}^{-1}\mathbf{a}(\Theta_k)$.
  - Use of LCMV for beamforming and direction finding..
- Any idea ?
- Undermodelling ? $\rightarrow$ designer has to select the key features of $\mathbf{r}(i)$ $\rightarrow$ reduce-rank signal processing

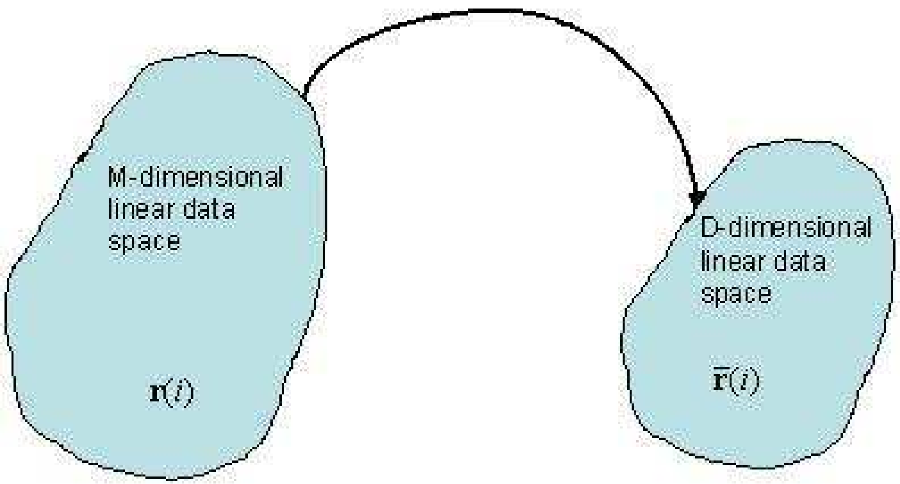
System Model and Rank Reduction

– Consider the following linear model

$$\mathbf{r}(i) = \mathbf{H}(i)\mathbf{s}(i) + \mathbf{n}(i)$$

where $\mathbf{s}(i)$ is a $M \times 1$ discrete-time signal organized in data vectors, $\mathbf{r}(i)$ is the $M \times 1$ input data, $\mathbf{H}(i)$ is a $M \times M$ matrix and $\mathbf{n}(i)$ is $M \times 1$ noise vector.

– Dimensionality reduction $\rightarrow$ an M-dimensional space is mapped into a D-dimensional subspace.



System Model and Rank Reduction

– A general reduced-rank version of $\mathbf{r}(i)$ can be obtained using a transformation matrix $\mathbf{S}_D$ (assumed fixed here) with dimensions $M \times D$, where D is the rank. Please see Haykin [1], Scharf-91 [3], Scharf and Tufts-87 [4], Scharf and van Veen-87[5].

– In other words, the mapping is carried out by the transformation matrix $\mathbf{S}_D$.

– The resulting reduced-rank observed data is given by

$$\bar{\mathbf{r}}(i) = \mathbf{S}_D^H \mathbf{r}(i)$$

where $\bar{\mathbf{r}}(i)$ is a $D \times 1$ vector.

– Challenge : How to efficiently (or optimally) design $\mathbf{S}_D$?

Historical Overview of Reduced-Rank Methods

– Origins of reduced-rank methods as a structured field :

- 1987 - Louis Scharf from University of Colorado defined the problem as "a transformation in which a data vector can be represented by a reduced number of effective features and yet retain most of the intrinsic information of the input data" Scharf and Tufts-87 [4], Scharf and van Veen-87[5].

- 1987- Scharf - Investigation and establishment of the bias versus noise variance trade-off.

Historical Overview of Reduced-Rank Methods

– Early Methods :

- Hotelling and Eckhart (see Scharf [3]) in the 1930's → first methods using eigen-decompositions or principal components.
- Early 1990's - applications of eigen-decomposition techniques for reduced-rank estimation in communications. See Haimovich and Bar-Ness [7], Wang and Poor [8], and Hua *et al.* [9].
- 1994 → Cai and Wang [6], Bell Labs : joint domain localised adaptive processing → radar-based scheme, medium complexity.

– Main problems of eigen-decomposition techniques :

- Require computationally expensive SVD or algorithms to obtain the eigenvalues and eigenvectors.
- Performance degradation with increase in the signal subspace.

Historical Overview of Reduced-Rank Methods

– 1997 - Goldstein and Reed [10], University of Southern California : cross-spectral approach.

- Appropriate selection of singular values which addresses the performance degradation.
- Remaining problem : eigen-decomposition.

– 1997 $\rightarrow$ Pados and Batallama [19]-[23], University of New York, Buffalo : auxiliary vector filtering (AVF) algorithm :

- does not require SVD.
- very fast convergence but complexity is still a problem.
- equivalence between the AVF (with orthogonal AVs) and the MSWF was established by Chen, Mitra and Schniter [17].

Historical Overview of Reduced-Rank Methods

– 1998/9 - Partial despreading (PD) of Singh and Milstein [18], University of California at San Diego :

- simple but suboptimal and restricted to CDMA multiuser detection.

– 1997 - 2004 - Multistage Wiener filter (MSWF) of Goldstein, Reed and Scharf and its variants [12]-[16] :

- State-of-the-art in the field and benchmark.
- Very fast convergence, rank not scaling with system size.
- Complexity is still a problem as well as the existence of numerical instability for implementation.

Historical Overview of Reduced-Rank Methods

- 2004 $\rightarrow$ de Lamare and Sampaio-Neto ([25])- interpolated FIR filters with time-varying interpolators : low complexity, good performance but rank limited.
- 2005 $\rightarrow$ de Lamare and Sampaio-Neto - Novel approach - Joint interpolation, decimation and filtering (JIDF) scheme [27]-[29] - Best known scheme, flexible, smallest complexity in the field, patented.
- 2007 $\rightarrow$ de Lamare, Haardt and Sampaio-Neto - Robust MSWF [17] - Development of a robust version of the MSWF using the constrained constant modulus (CCM) design criterion.
- 2007 $\rightarrow$ de Lamare and Sampaio-Neto - Joint iterative optimisation of filters - (JIO) - Development of a generic reduced-rank scheme that is very good for mapping and inverse mapping [26].
- 2008 $\rightarrow$ de Lamare, Sampaio-Neto and Haardt [30] - Robust JIDF-type approach called BARC - Development of a robust version of the JIDF using the CCM design criterion.

MMSE Reduced-Rank Parameter Vector Design

– The MMSE filter is the vector $\mathbf{w} = \left[w_1 \; w_2 \; \ldots \; w_M\right]^T$, which is designed to minimize the MSE cost function

$$J = E\left[|d(i) - \mathbf{w}^H \mathbf{r}(i)|^2\right]$$

where $d(i)$ is the desired signal.

– The solution is $\mathbf{w} = \mathbf{R}^{-1}\mathbf{p}$, where $E[d^*(i)\mathbf{r}(i)]$ and $\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^H(i)]$.

– The parameter vector $\mathbf{w}$ can be also be estimated via adaptive algorithms, however ...

– The convergence speed and tracking of these algorithms depends on M and the eigenvalue spread. Thus, large M implies slow convergence.

– Reduced-rank schemes circumvent these limitations via reduction of number of coefficients and extraction of key features of data.

MMSE Reduced-Rank Parameter Vector Design

– Consider a reduced-rank input vector $\bar{\mathbf{r}}(i) = \mathbf{S}_D^H \mathbf{r}(i)$ as the input to a filter represented by the D vector $\bar{\mathbf{w}} = \left[\bar{w}_1 \;\; \bar{w}_2 \;\; \ldots \bar{w}_D\right]^T$ for time interval i.

– The filter output is

$$x(i) = \bar{\mathbf{w}}^H \mathbf{S}_D^H \mathbf{r}(i)$$

– The MMSE design problem can be stated as

$$\begin{aligned} \text{minimize} \;\; \mathcal{J}(\bar{\mathbf{w}}) &= E\Big[|d(i) - x(i)|^2\Big] \\ &= E\Big[|d(i) - \bar{\mathbf{w}}^H \mathbf{S}_D^H \mathbf{r}(i)|^2\Big] \end{aligned}$$

where $d(i)$ is the desired signal.

MMSE Reduced-Rank Parameter Vector Design

– The MMSE design with the reduced-rank parameters yields

$$\bar{\mathbf{w}} = \bar{\mathbf{R}}^{-1}\bar{\mathbf{p}}$$

where
$\bar{\mathbf{R}} = E[\bar{\mathbf{r}}(i)\bar{\mathbf{r}}^H(i)] = \mathbf{S}_D^H\mathbf{R}\mathbf{S}_D$ is the reduced-rank covariance matrix,
$\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^H(i)]$ is the full-rank covariance matrix,
$\bar{\mathbf{p}} = E[d^*(i)\bar{\mathbf{r}}(i)] = \mathbf{S}_D^H\mathbf{p}$ and $\mathbf{p} = E[d^*(i)\mathbf{r}(i)]$.

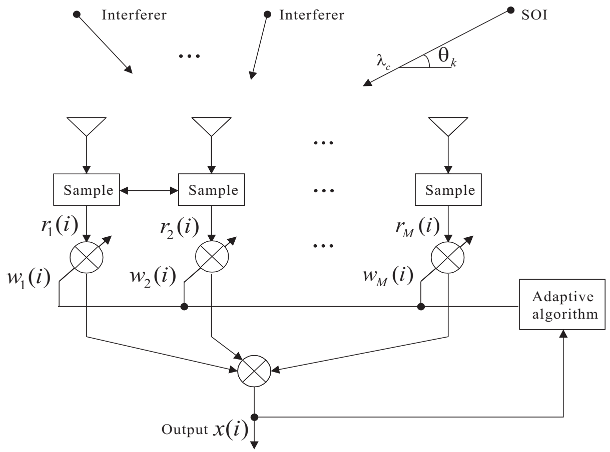
– The associated MMSE for a rank D estimator is expressed by

$$\text{MMSE} = \sigma_d^2 - \bar{\mathbf{p}}^H\bar{\mathbf{R}}^{-1}\bar{\mathbf{p}} = \sigma_d^2 - \mathbf{p}^H\mathbf{S}_D(\mathbf{S}_D^H\mathbf{R}\mathbf{S}_D)^{-1}\mathbf{S}_D^H\mathbf{p}$$

where σ_d^2 is the variance of $d(i)$.

LCMV Reduced-Rank Parameter Vector Design

– Consider a uniform linear array (ULA) of M elements. There are K narrowband sources impinging on the array ($K < M$) with directions of arrival (DOA) θ_l for $l = 1, 2, \ldots, K$.



– Reduced-rank array processing : The output of the array is

$$x(i) = \bar{\boldsymbol{w}}^H \bar{\boldsymbol{r}}(i) = \bar{\boldsymbol{w}}^H(i) \boldsymbol{S}_D^H \boldsymbol{r}(i)$$

LCMV Reduced-Rank Parameter Vector Design

– In order to design the reduced-rank filter $\bar{\boldsymbol{w}}(i)$ we consider the following optimization problem

$$\text{minimize } E\Big[|\bar{\boldsymbol{w}}^H \boldsymbol{S}_D^H \boldsymbol{r}(i)|^2\Big] = \bar{\boldsymbol{w}}^H \boldsymbol{S}_D^H \boldsymbol{R} \boldsymbol{S}_D \bar{\boldsymbol{w}}$$
$$\text{subject to } \bar{\boldsymbol{w}}^H \boldsymbol{S}_D^H \boldsymbol{a}(\theta_k) = 1$$

– Approach used to obtain a solution : Lagrange multiplier method

$$\mathcal{L}(\bar{\boldsymbol{w}}, \lambda) = E\Big[|\bar{\boldsymbol{w}}^H \boldsymbol{S}_D^H \boldsymbol{r}(i)|^2\Big] + 2\Re[\lambda(\bar{\boldsymbol{w}}^H \mathrm{S}_D^H \boldsymbol{a}(\theta_k) - 1)]$$

– The solution to this design problem is

$$\bar{\boldsymbol{w}} = \frac{(\boldsymbol{S}_D^H \boldsymbol{R} \boldsymbol{S}_D)^{-1} \boldsymbol{S}_D^H \boldsymbol{S}_D^H \boldsymbol{a}(\theta_k)}{\boldsymbol{a}^H(\theta_k) \boldsymbol{S}_D(i) (\boldsymbol{S}_D^H \boldsymbol{R} \boldsymbol{S}_D)^{-1} \boldsymbol{S}_D^H \boldsymbol{a}(\theta_k)} = \frac{\bar{\boldsymbol{R}}^{-1} \bar{\boldsymbol{a}}(\theta_k)}{\bar{\boldsymbol{a}}^H(\theta_k) \bar{\boldsymbol{R}}^{-1} \bar{\boldsymbol{a}}(\theta_k)}$$

where the reduced-rank covariance matrix is $\bar{\boldsymbol{R}} = E[\bar{\boldsymbol{r}}(i)\bar{\boldsymbol{r}}^H(i)] = \boldsymbol{S}_D^H \boldsymbol{R} \boldsymbol{S}_D$ and the reduced-rank steering vector is $\bar{\boldsymbol{a}}(\theta_k) = \boldsymbol{S}_D^H \boldsymbol{a}(\theta_k)$.

LCMV Reduced-Rank Parameter Vector Design

– The associated minimum variance (MV) for a LCMV parameter vector/filter with rank D is

$$\begin{aligned}\text{MV} &= \frac{1}{\bar{\boldsymbol{a}}^H(\theta_k)\bar{\boldsymbol{R}}^{-1}\bar{\boldsymbol{a}}(\theta_k)} \\ &= \frac{1}{\boldsymbol{a}(\theta_k)^H\boldsymbol{S}_D(\boldsymbol{S}_D^H\boldsymbol{R}\boldsymbol{S}_D)^{-1}\boldsymbol{S}_D^H\boldsymbol{a}(\theta_k)}\end{aligned}$$

– The above expression can be used for direction finding by replacing the angles θ_k with a time-varying parameter (ω) in order to scan the possible angles.

– It can also be employed for general applications of spectral estimation including spectral sensing.

Eigen-Decomposition Techniques

– Why are eigen-decomposition techniques used ?

– For MMSE parameter estimation and a rank D estimator we have

$$\text{MMSE} = \sigma_d^2 - \mathbf{p}^H \mathbf{S}_D (\mathbf{S}_D^H \mathbf{R} \mathbf{S}_D)^{-1} \mathbf{S}_D^H \mathbf{p}$$

– Taking the gradient of MMSE with respect to $\boldsymbol{S}_D$, we get

$$\boldsymbol{S}_{D,\text{opt}} = [\boldsymbol{v}_1 \ldots \boldsymbol{v}_D]$$

– For MV parameter estimation and a rank D estimator we have

$$\text{MV} = \frac{1}{\boldsymbol{a}(\theta_k)^H \boldsymbol{S}_D (\boldsymbol{S}_D^H \boldsymbol{R} \boldsymbol{S}_D)^{-1} \boldsymbol{S}_D^H \boldsymbol{a}(\theta_k)}$$

– Taking the gradient of MV with respect to $\boldsymbol{S}_D$, we get

$$\boldsymbol{S}_{D,\text{opt}} = [\boldsymbol{v}_1 \ldots \boldsymbol{v}_D]$$

Eigen-Decomposition Techniques

– Rank reduction is accomplished by eigen- decomposition on the input data covariance matrix

$$\boldsymbol{R} = \boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{V}^H,$$

where
$\boldsymbol{V} = [\boldsymbol{v}_1 \ldots \boldsymbol{v}_M]$ and
$\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1, \ldots, \lambda_M)$.

– Early techniques : selection of eigenvectors $\boldsymbol{v}_j$ $(j = 1, \ \ldots, M)$ corresponding to the largest eigenvalues λ_j
$\rightarrow$ Transformation matrix is

$$\boldsymbol{S}_D(i) = [\boldsymbol{v}_1 \ldots \boldsymbol{v}_D]$$

Eigen-Decomposition Techniques

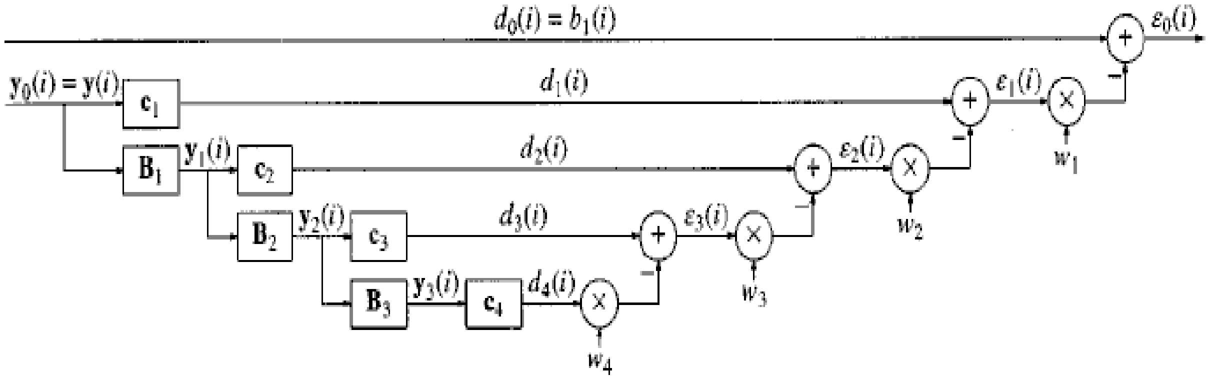
– Cross-spectral approach of Goldstein and Reed : choose eigenvectors that minimise the design criterion
$\longrightarrow$ Transformation matrix is

$$\boldsymbol{S}_D(i) = [\boldsymbol{v}_i \dots \boldsymbol{v}_t]$$

– Problems : Complexity $O(M^3)$, optimality implies knowledge of $\boldsymbol{R}$ but this has to be estimated.
– Complexity reduction : adaptive subspace tracking algorithms (popular in the end of the 90s) but still complex and susceptible to tracking problems.
– Can we skip or circumvent an eigen-decomposition ?

The Multi-stage Wiener Filter

– Rank reduction is accomplished by a successive refinement procedure that generates a set of basis vectors, i.e. the signal subspace, known



– Design : use of nested filters $\boldsymbol{c}_j$ $(j = 1, \ldots, M)$ and blocking matrices $\boldsymbol{B}_j$ for the decomposition $\rightarrow$ Projection matrix is

$$\boldsymbol{S}_D(i) = [\boldsymbol{p}, \boldsymbol{R}p, ..., \boldsymbol{R}^{D-1}\boldsymbol{p}]$$

– Advantages : rank D does not scale with system size, very fast convergence.

– Problems : complexity slightly inferior to RLS algorithms, not robust to signature mismatches in blind operation.

A Robust Multi-stage Wiener Filter

– Rank reduction is accomplished by a similar successive refinement procedure to original MSWF. However, the design is based on the CCM criterion (de Lamare, Haardt and Sampaio-Neto [ ]).

– Transformation matrix :

$$\mathbf{S}_D(i) = \Big[\mathbf{q}(i),\ \mathbf{R}(i)\mathbf{q}(i),\ \ldots,\ \mathbf{R}^{(D-1)}(i)\mathbf{q}(i)\Big]$$

– The reduced-rank CCM parameter vector with rank D is

$$\bar{\mathbf{w}}(i+1) = \Big(\mathbf{S}_D^H(i)\mathbf{R}(i)\mathbf{S}_D(i)\Big)^{-1}\mathbf{S}_D^H(i)\mathbf{q}(i),$$

where

$$\mathbf{q}(i) = \mathbf{d}(i) - (\mathbf{p}^H(i)\mathbf{R}^{-1}(i)\mathbf{p}(i))^{-1}(\mathbf{p}^H(i)\mathbf{R}^{-1}(i)\mathbf{d}(i) - \nu\ )\mathbf{p}(i),$$

$$\mathbf{d}(i) = E\Big[x^*(i)\boldsymbol{S}_D^H(i)\mathbf{r}(i)\Big]$$

Applications : Interference Suppression for CDMA

– We assess BER performance of the supervised LS, the CMV-LS and the CCM-LS and their full-rank and reduced-rank versions.

– The DS-CDMA system uses random sequences with $N = 64$.

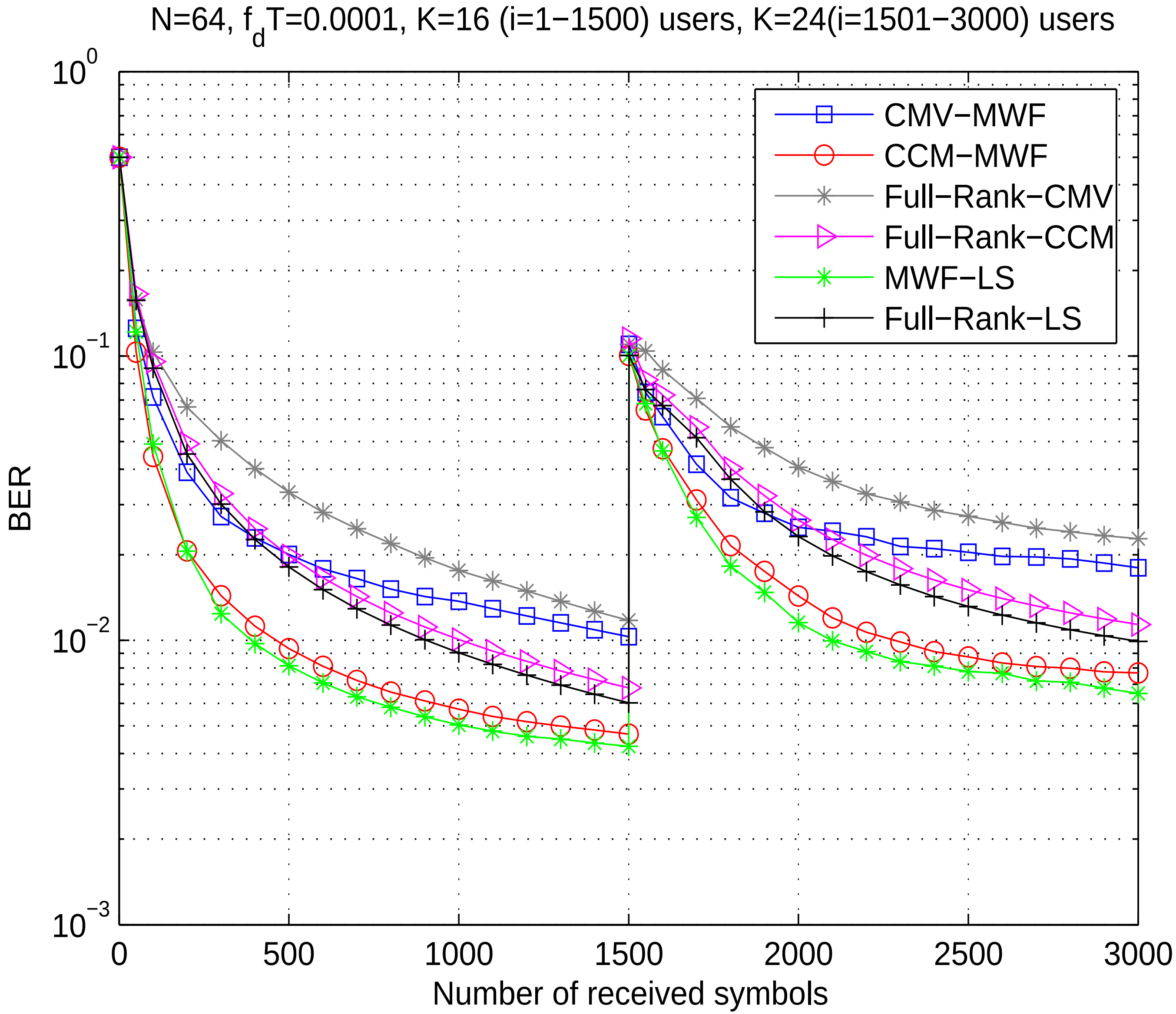
– We use 3-path channels with powers $p_{k,l}$ given by 0, -3 and -6 dB. In each run the spacing between paths is obtained from a discrete uniform random variable between 1 and 2 chips.

– Power distribution amongst the users : Follows a log-normal distribution with associated standard deviation of 1.5 dB.

– All LS type estimators use $\lambda = 0.998$ to ensure good performance and all experiments are averaged over 200 runs.

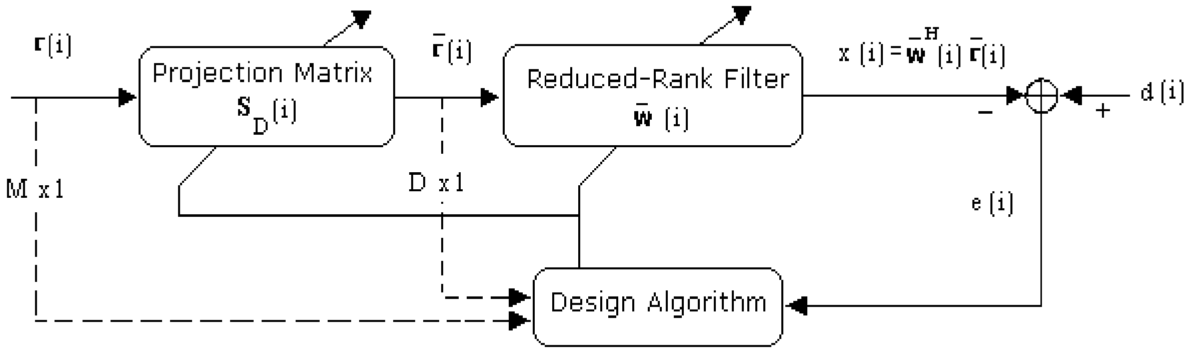
Applications : Interference Suppression for CDMA

– BER convergence performance at $E_b/N_0 = 12$ dB.



Techniques based on joint and iterative optimisation of filters

– Rank reduction is performed by joint and iterative optimisation (JIO) of projection matrix $\boldsymbol{S}_D(i)$ and reduced-rank filter $\bar{\boldsymbol{w}}(i)$.



– Design criteria : MMSE, LS, LCMV, etc

– Adaptive algorithms : LMS, RLS, etc

– Highlights : rank D does not scale with system size, very fast convergence, proof of global convergence established, very simple.

MMSE Design of JIO Scheme

– The MMSE expressions for the filters $\mathbf{S}_D(i)$ and $\bar{\mathbf{w}}(i)$ can be computed through the following cost function :

$$J = E\Big[\big|d(i) - \bar{\mathbf{w}}^H(i)\mathbf{S}_D^H(i)\mathbf{r}(i)\big|^2\Big]$$

– By fixing the projection $\mathbf{S}_D(i)$ and minimizing the cost function with respect to $\bar{\mathbf{w}}(i)$, the reduced-rank filter weight vector becomes

$$\bar{\mathbf{w}}(i) = \bar{\mathbf{R}}^{-1}(i)\bar{\mathbf{p}}(i)$$

where
$\bar{\mathbf{R}}(i) = E[\mathbf{S}_D^H(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{S}_D(i)] = E[\bar{\mathbf{r}}(i)\bar{\mathbf{r}}^H(i)]$,
$\bar{\mathbf{p}}(i) = E[d^*(i)\mathbf{S}_D^H(i)\mathbf{r}(i)] = E[d^*(i)\bar{\mathbf{r}}(i)]$.

MMSE Design of JIO Scheme

– Fixing $\bar{\mathbf{w}}(i)$ and minimizing the cost function with respect to $\mathbf{S}_D(i)$, we get

$$\mathbf{S}_D(i) = \mathbf{R}^{-1}(i)\mathbf{P}_D(i)\mathbf{R}_w^{-1}(i)$$

where
$\mathbf{R}(i) = E[\mathbf{r}(i)\mathbf{r}^H(i)]$,
$\mathbf{P}_D(i) = E[d^*(i)\mathbf{r}(i)\bar{\mathbf{w}}^H(i)]$ and
$\mathbf{R}_w(i) = E[\bar{\mathbf{w}}(i)\bar{\mathbf{w}}^H(i)]$.

– The associated MMSE is

$$\text{MMSE} = \sigma_d^2 - \bar{\mathbf{p}}^H(i)\bar{\mathbf{R}}^{-1}(i)\bar{\mathbf{p}}(i)$$

where $\sigma_d^2 = E[|d(i)|^2]$.

MMSE Design of JIO Scheme

– The filter expressions for $\bar{\mathbf{w}}(i)$ and $\mathbf{S}_D(i)$ are functions of one another and thus it is necessary to iterate (8) and (9) with an initial guess to obtain a solution.

– Unlike prior art, the JIO scheme provides an iterative exchange of information between the reduced-rank filter and the transformation matrix.

– The key strategy lies in the joint optimization of the filters.

– The rank D or model order must be set by the designer to ensure appropriate or adjusted on-line.

Adaptive JIO implementation : LMS algorithm

Initialize all parameter vectors, dimensions

For each data vector $i = 1, \ldots, Q$ do :

– Perform dimensionality reduction :

$$\bar{\boldsymbol{r}}(i) = \boldsymbol{S}_D^H(i)\boldsymbol{r}(i)$$

– Estimate parameters

$$\boldsymbol{S}_D(i+1) = \boldsymbol{S}_D(i) + \eta(i)e^*(i)\boldsymbol{r}(i)\bar{\boldsymbol{w}}^H(i)$$

$$\bar{\boldsymbol{w}}(i+1) = \bar{\boldsymbol{w}}(i) + \mu(i)e^*(i)\bar{\boldsymbol{r}}(i)$$

where $e(i) = d(i) - \bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{r}(i)$.

Applications : Interference Suppression for CDMA

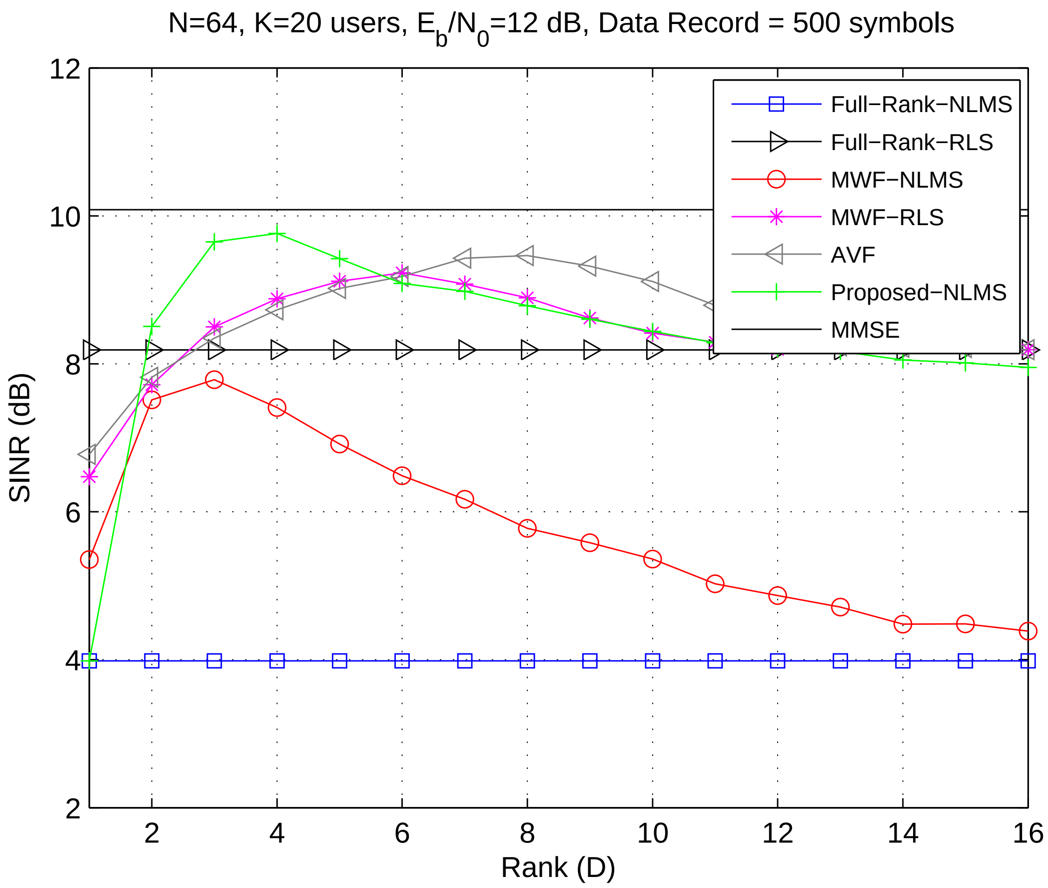
– We consider the uplink of a symbol synchronous BPSK DS-CDMA system with K users, N chips per symbol and L propagation paths.

– Initialization : for all simulations, we use $\bar{\mathbf{w}}(0) = \mathbf{0}_{D,1}$, $\mathbf{S}_D(0) = [\mathbf{I}_D \ \mathbf{0}_{D,M-D}]^T$.

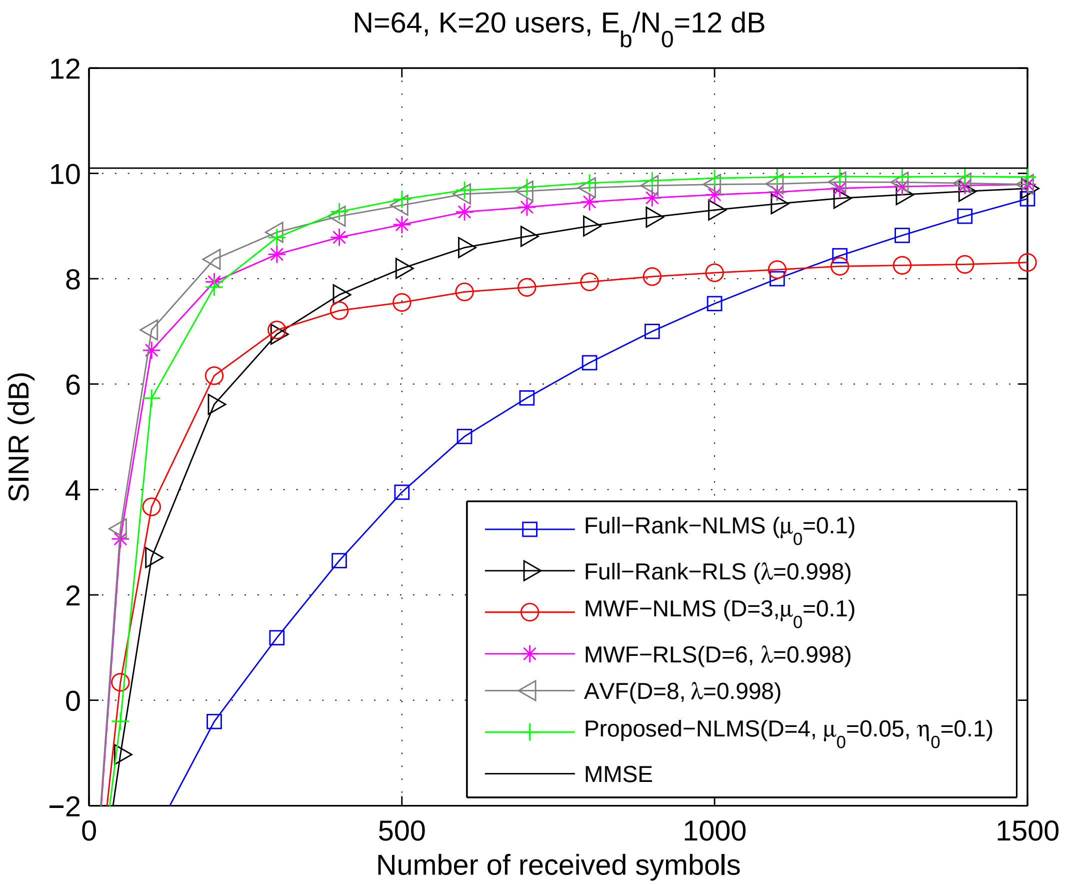
– We assume $L = 9$ as an upper bound on the channel delay spread, use 3-path channels with relative powers given by 0, -3 and -6 dB, where in each run the spacing between paths is obtained from a discrete uniform random variable between 1 and 2 chips and average the experiments over 200 runs.

– The system has a power distribution amongst the users for each run that follows a log-normal distribution with associated standard deviation equal to 1.5 dB.

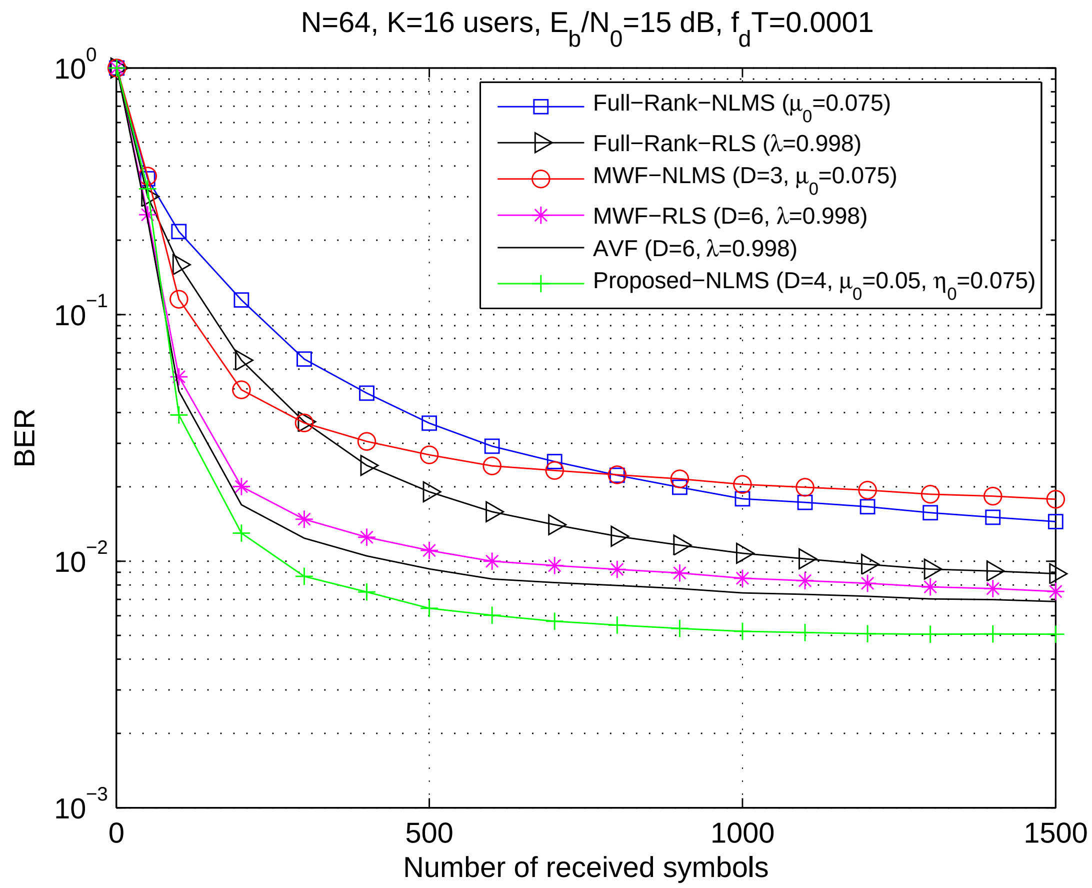
Applications : Interference Suppression for CDMA



Applications : Interference Suppression for CDMA



Applications : Interference Suppression for CDMA



LCMV Design of JIO Scheme

– Main differences in approach : the filters $\mathbf{S}_D(i)$ and $\bar{\mathbf{w}}(i)$ are jointly optimized and certain key quantities are assumed statistically independent.

– The LCMV expressions for the filters $\mathbf{S}_D(i)$ and $\bar{\mathbf{w}}(i)$ can be computed via the proposed optimization problem

$$\text{minimize } E\Big[|\bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{r}(i)|^2\Big] = \bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{R}\boldsymbol{S}_D(i)\bar{\boldsymbol{w}}(i)$$
$$\text{subject to } \bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{a}(\theta_k) = 1$$

– Solution $\rightarrow$ method of Lagrange multipliers

$$\mathcal{L}(\boldsymbol{S}_D(i), \bar{\boldsymbol{w}}(i), \lambda) = E\Big[|\bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{r}(i)|^2\Big] + 2\Re[\lambda(\bar{\boldsymbol{w}}^H(i)\mathbf{S}_D^H(i)\boldsymbol{a}(\theta_k) - 1)],$$

LCMV Design of JIO Scheme

– By fixing $\bar{\boldsymbol{w}}(i)$, minimizing $\mathcal{L}(\boldsymbol{S}_D(i), \bar{\boldsymbol{w}}(i), \lambda)$ with respect to $\boldsymbol{S}_D(i)$ and solving for λ, we get

$$\boldsymbol{S}_D(i) = \frac{\boldsymbol{R}^{-1}\boldsymbol{a}(\theta_k)\bar{\boldsymbol{w}}^H(i)\boldsymbol{R}_w^{-1}}{\bar{\boldsymbol{w}}^H(i)\boldsymbol{R}_w^{-1}\bar{\boldsymbol{w}}(i)\boldsymbol{a}^H(\theta_k)\boldsymbol{R}^{-1}\boldsymbol{a}(\theta_k)},$$

where
$\boldsymbol{R} = E[\boldsymbol{r}(i)\boldsymbol{r}^H(i)]$ and
$\boldsymbol{R}_w = E[\bar{\boldsymbol{w}}(i)\bar{\boldsymbol{w}}^H(i)]$.

– A simplified expression for $\boldsymbol{S}_D(i)$ obtained analytically with the exploitation of the constraint is given by

$$\boldsymbol{S}_D(i) = \frac{\boldsymbol{P}(i)\boldsymbol{a}(\theta_k)\bar{\boldsymbol{a}}^H(\theta_k)}{\boldsymbol{a}^H(\theta_k)\boldsymbol{P}(i)\boldsymbol{a}(\theta_k)}$$

LCMV Design of JIO Scheme

– By fixing $\boldsymbol{S}_D(i)$, minimizing the Lagrangian with respect to $\bar{\boldsymbol{w}}(i)$ and solving for λ, we arrive at the expression for $\bar{\boldsymbol{w}}(i)$

$$\bar{\boldsymbol{w}}(i) = \frac{\bar{\boldsymbol{R}}^{-1}(i)\bar{\boldsymbol{a}}(\theta_k)}{\bar{\boldsymbol{a}}^H(\theta_k)\bar{\boldsymbol{R}}^{-1}(i)\bar{\boldsymbol{a}}(\theta_k)},$$

where
$\bar{\boldsymbol{R}}(i) = \boldsymbol{S}_D^H(i)E[\boldsymbol{r}(i)\boldsymbol{r}^H(i)]\boldsymbol{S}_D(i) = E[\bar{\boldsymbol{r}}(i)\bar{\boldsymbol{r}}^H(i)]$,
$\bar{\boldsymbol{a}}(\theta_k) = \boldsymbol{S}_D^H(i)\boldsymbol{a}(\theta_k)$.

– The associated MV is

$$\text{MV} = \frac{1}{\bar{\boldsymbol{a}}^H(\theta_k)\bar{\boldsymbol{R}}^{-1}(i)\bar{\boldsymbol{a}}(\theta_k)}$$

LCMV Design of JIO Scheme

- The filter expressions $\bar{\boldsymbol{w}}(i)$ and $\boldsymbol{S}_D(i)$ are not closed-form solutions.
- They are functions of each other. Therefore, it is necessary to iterate the expressions with initial values to obtain a solution.
- Existence of multiple solution (which are identical with respect to the MMSE and symmetrical).
- Global convergence to the optimal reduced-rank LCMV filter (eigen-decomposition with known covariance matrix) has been established.
- The key strategy lies in the joint optimization of the filters.
- The rank D must be adjusted by the designer to ensure appropriate performance or can be estimated via another algorithm.

Adaptive LCMV version : LMS algorithm

Initialize all parameter vectors, dimensions

For each data vector $i = 1, \ldots, Q$ do :

– Perform dimensionality reduction :

$$\bar{\boldsymbol{r}}(i) = \boldsymbol{S}_D^H(i)\boldsymbol{r}(i)$$

– Estimate parameters

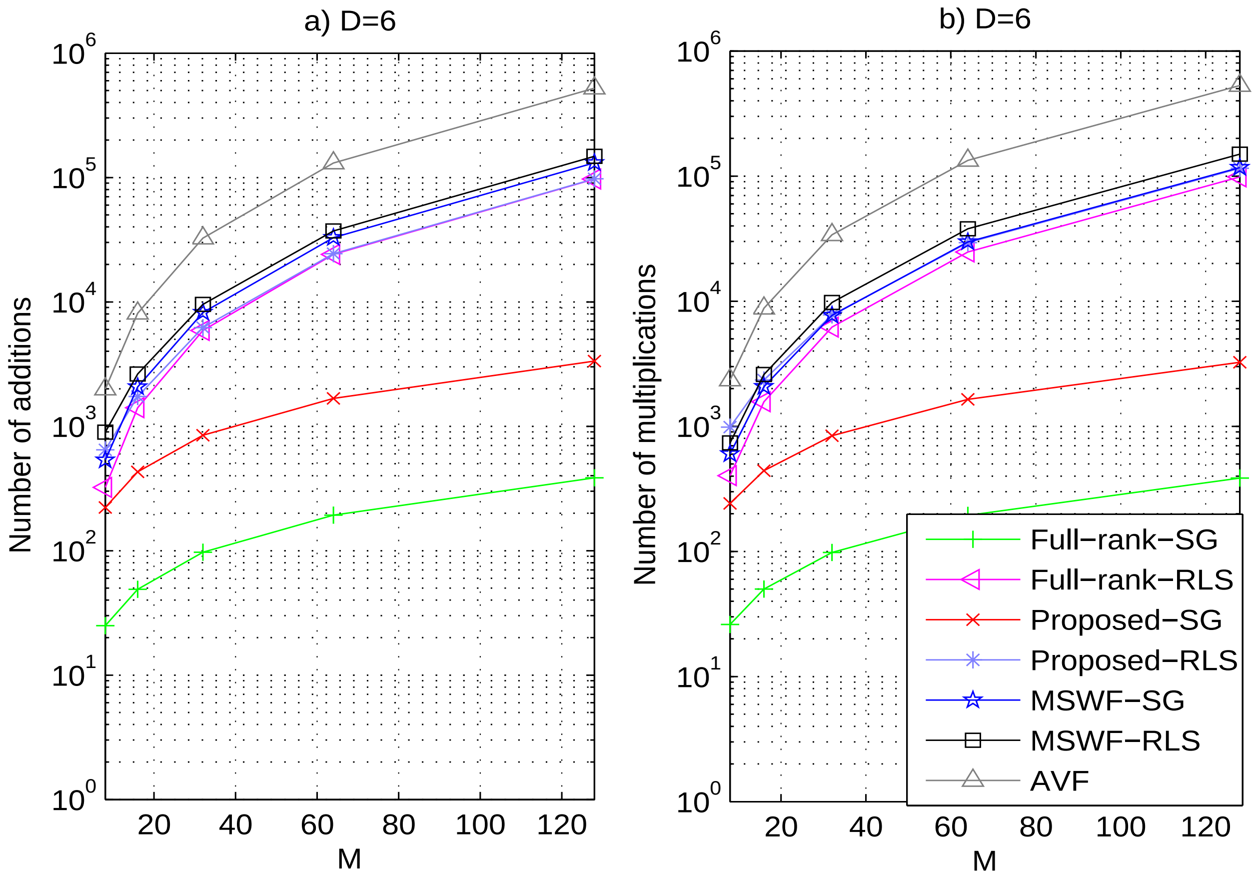
$$\boldsymbol{S}_D(i+1) = \boldsymbol{S}_D(i) - \mu_s x^*(i)\Big[\boldsymbol{r}(i)\bar{\boldsymbol{w}}^H(i) - \boldsymbol{a}(\theta_k)\bar{\boldsymbol{w}}^H(i)\boldsymbol{a}^H(\theta_k)\boldsymbol{r}(i)\Big]$$

$$\bar{\boldsymbol{w}}(i+1) = \bar{\boldsymbol{w}}(i) - \mu_w x^*(i)\Big[\boldsymbol{I} - \Big(\bar{\boldsymbol{a}}^H(\theta_k)\bar{\boldsymbol{a}}(\theta_k)\Big)^{-1}\bar{\boldsymbol{a}}(\theta_k)\bar{\boldsymbol{a}}^H(\theta_k)\Big]\bar{\boldsymbol{r}}(i)$$

Complexity of LCMV-JIO ALgorithms

Algorithm	Additions	Multiplications
Full-rank-SG [1]	$3M+1$	$3M+2$
Full-rank-RLS [1]	$3M^2-2M+3$	$6M^2+2M+2$
Proposed-SG	$3DM+2M$ $+2D-2$	$3DM+M$ $+5D+2$
Proposed-RLS	$3M^2-2M+3$ $+3D^2-8D+3$	$7M^2+2M$ $+7D^2+9D$
MSWF-SG [12]	DM^2-M^2 $+3D-2$	DM^2-M^2 $+2DM+4D+1$
MSWF-RLS [12]	$DM^2+M^2+6D^2$ $-8D+2$	DM^2+M^2 $+2DM+3D+2$
AVF [23]	$D((M)^2+3(M-1)^2)-1$ $+D(5(M-1)+1)+2M$	$D(4M^2+4M+1)$ $+4M+2$

Complexity of LCMV-JIO ALgorithms



Applications : LCMV Beamforming

– A smart antenna system with a ULA containing M sensor elements and half wavelength inter-element spacing is considered.

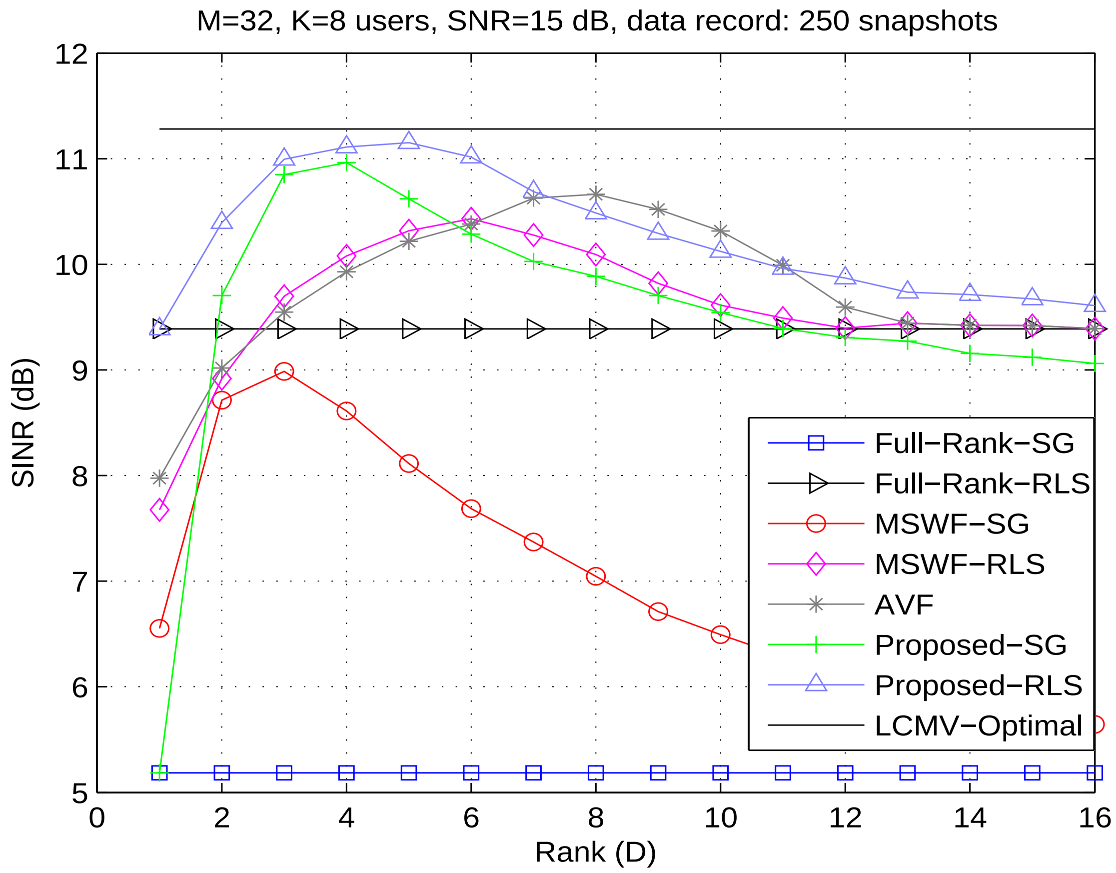
– Figure of merit : the SINR, which is defined as

$$\text{SINR}(i) = \frac{\bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{R}_s(i)\boldsymbol{S}_D(i)\bar{\boldsymbol{w}}(i)}{\bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{R}_I(i)\boldsymbol{S}_D(i)\bar{\boldsymbol{w}}(i)}$$

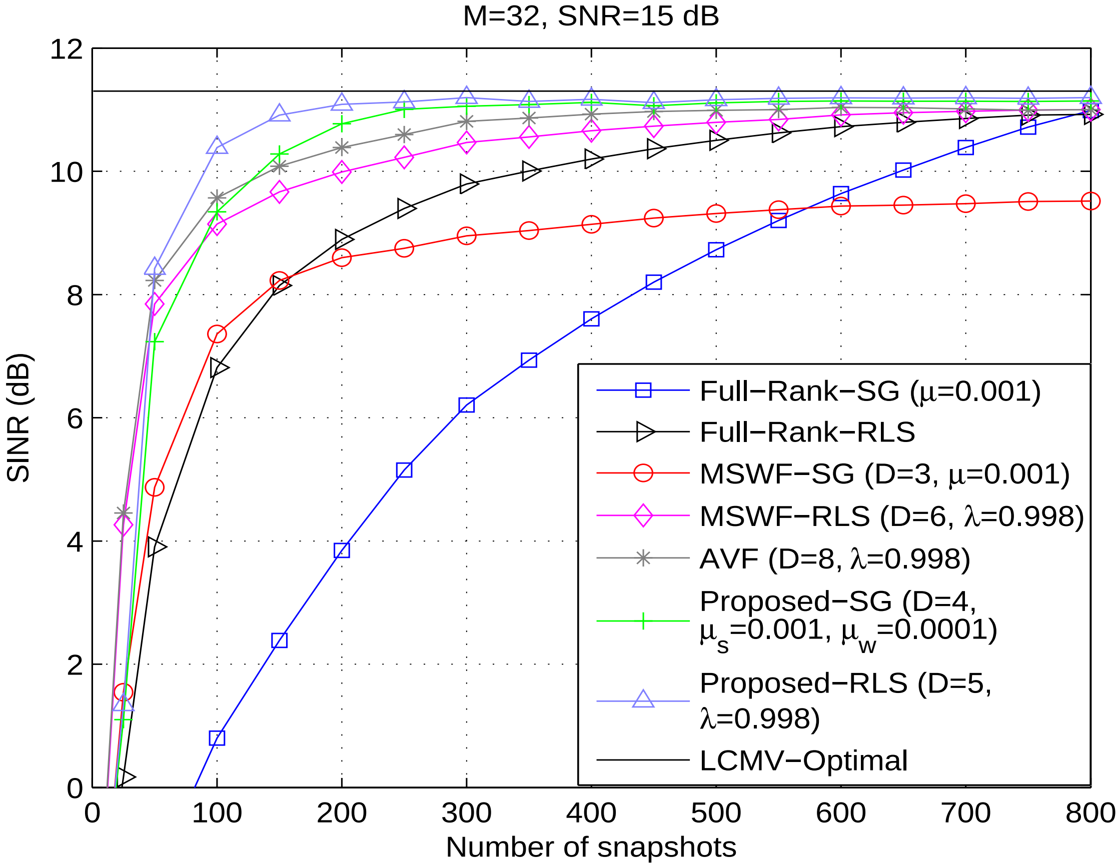
– The signal-to-noise ratio (SNR) is defined as $\text{SNR} = \frac{\sigma_d^2}{\sigma^2}$.

– Initialization : $\bar{\boldsymbol{w}}(0) = [1\ 0\ \ldots\ 0]$ and $\boldsymbol{S}_D(0) = [\boldsymbol{I}_D^T\ \boldsymbol{0}_{D\times(M-D)}^T]$, where $\boldsymbol{0}_{D\times M-D}$ is a $D \times (M - D)$ matrix with zeros in all experiments.

Applications : LCMV Beamforming



Applications : LCMV Beamforming



Applications : Direction of Arrival Estimation

– A smart antenna system with a ULA containing M sensor elements and half wavelength inter-element spacing is considered.

– We compare the proposed LCMV JIO method with an LS algorithm with the Capon, MUSIC, ESPRIT, AVF, and CG methods, and run $K = 1000$ iterations to get each curve.

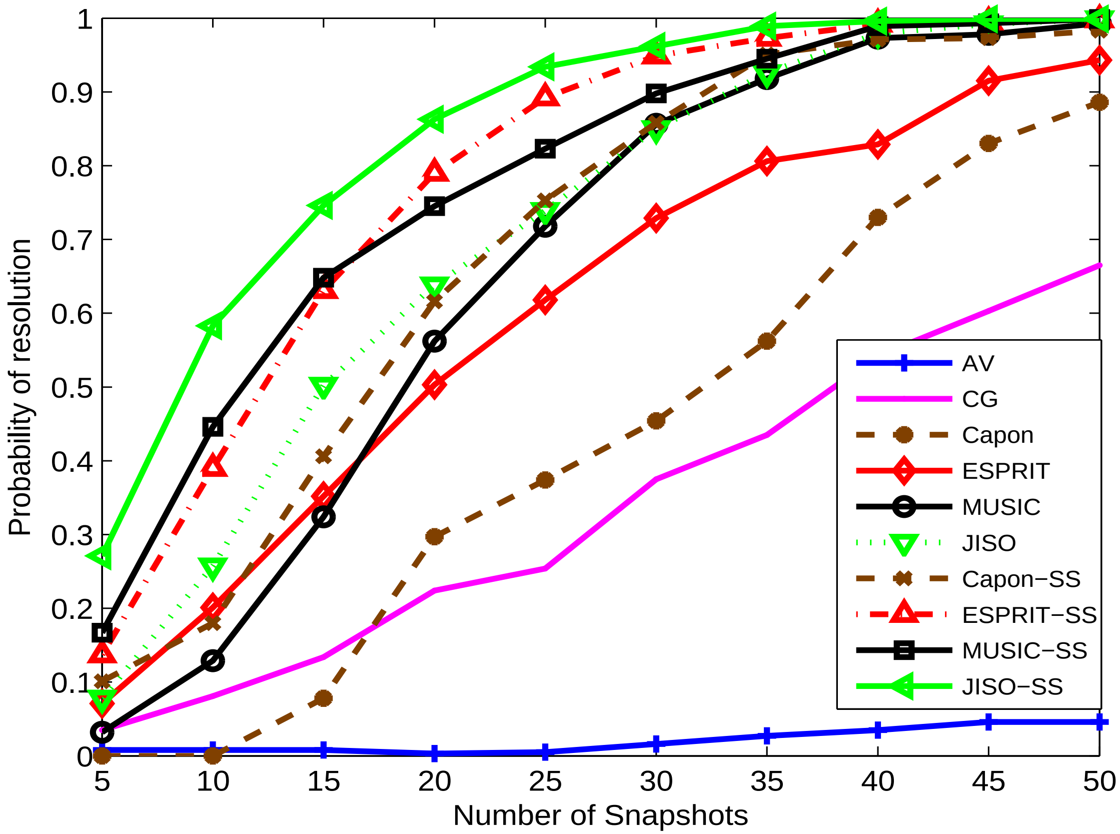
– The spatial smoothing (SS) technique is employed for each algorithm to improve the performance in the presence of correlated sources.

– The DOAs are considered to be resolved if $|\hat{\theta}_{\text{JISO}} - \theta_k| < 1^o$.

– The probability of resolution is used as a figure of merit and plotted against the number of snapshots.

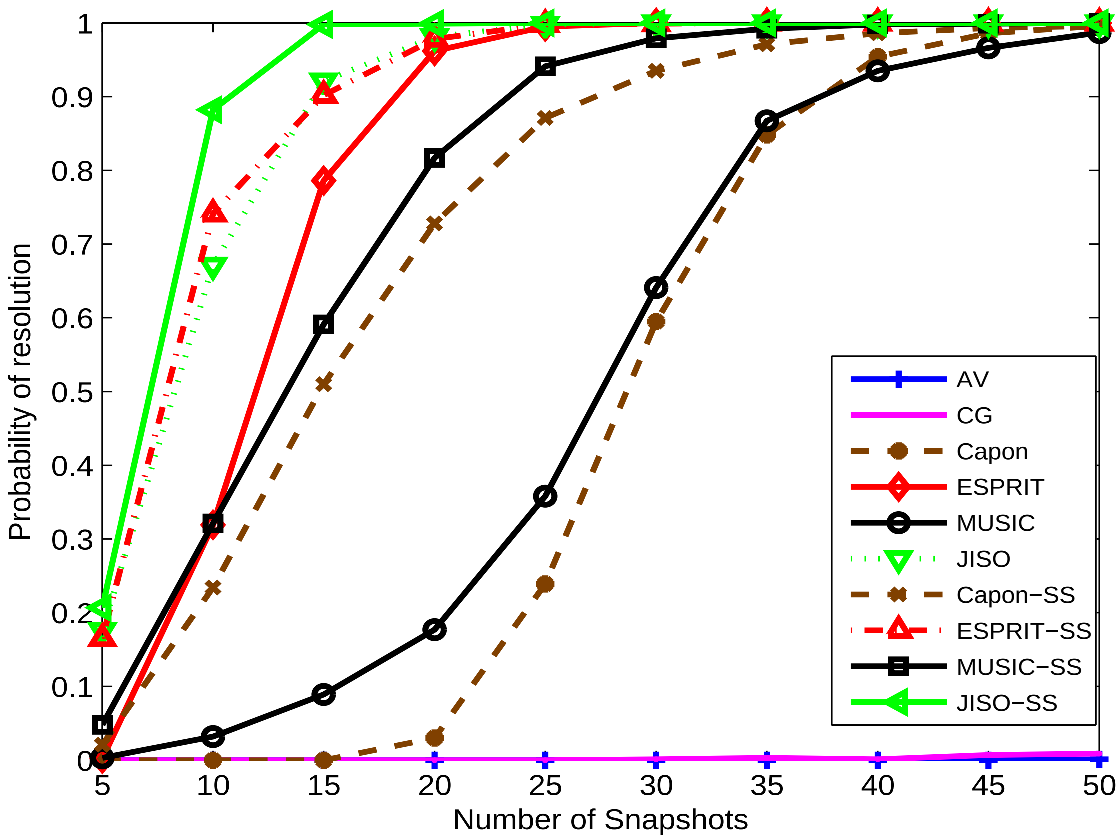
Applications : Direction of Arrival Estimation

Parameters : Probability of resolution versus number of snapshots (separation 3^o, SNR$= -2$dB, $q = 2$, c$= 0.9$, $m = 30$, $r = 6$, $\delta = 5 \times 10^{-4}$, $\alpha = 0.998$, $n = 26$)



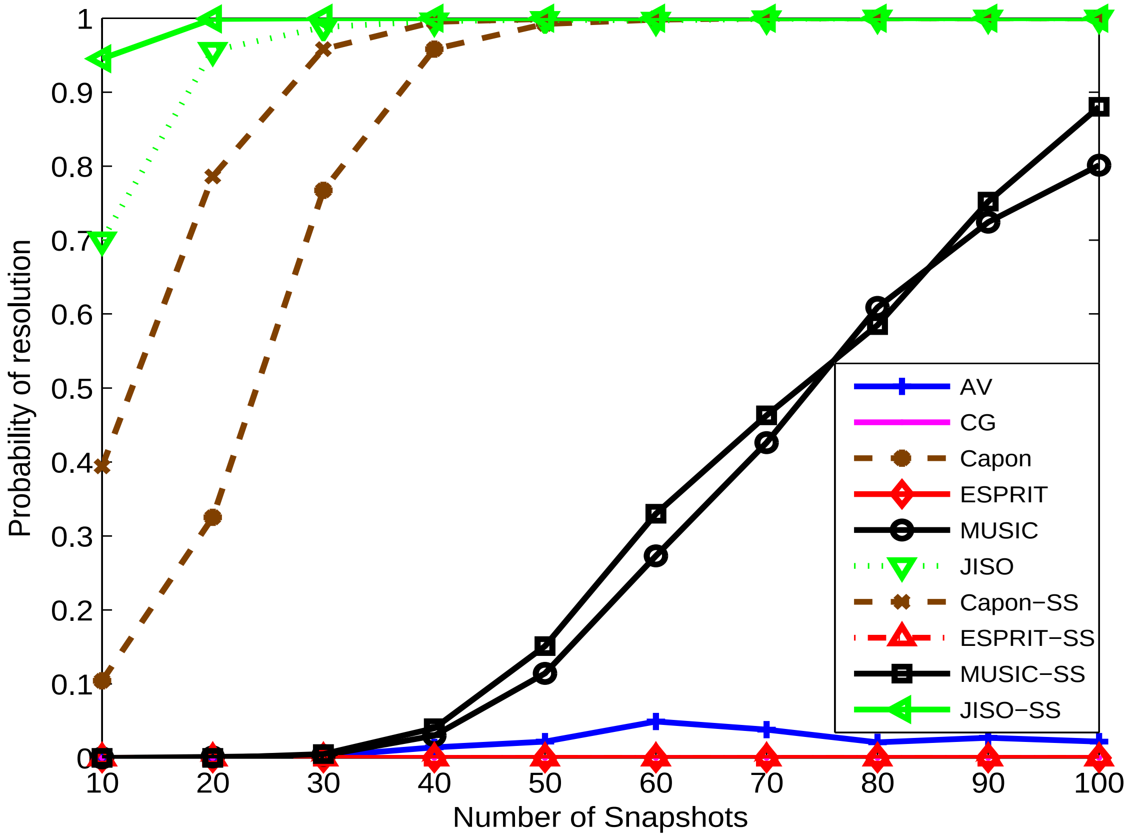
Applications : Direction of Arrival Estimation

Parameters : Probability of resolution versus number of snapshots (separation 3^o, SNR$= -5$dB, q$= 10$, $m = 50$, $r = 6$, $\delta = 5 \times 10^{-4}$, $\alpha = 0.998$, $n = 41$)

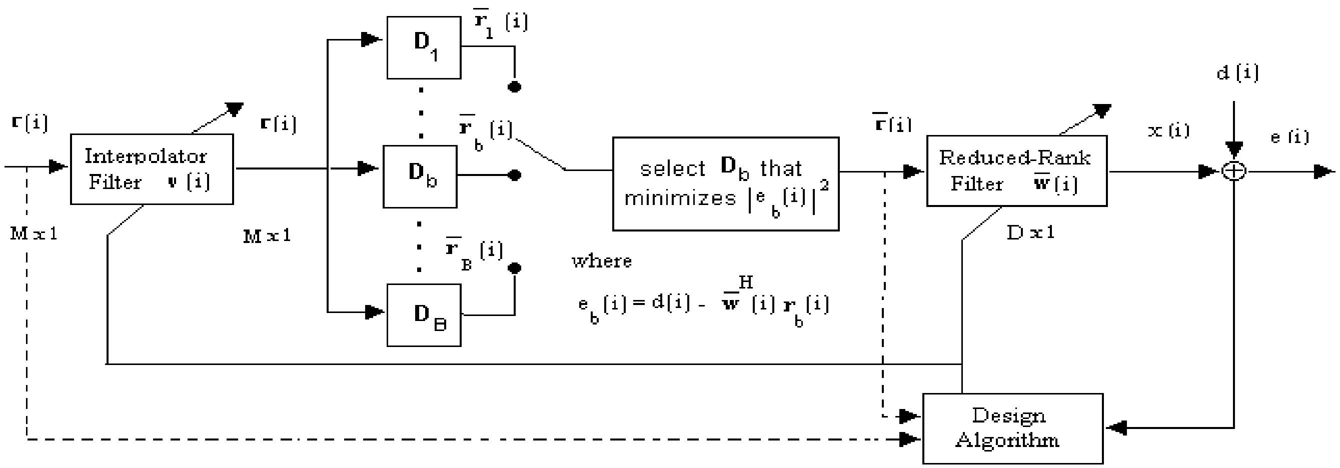


Applications : Direction of Arrival Estimation

Parameters : Probability of resolution versus snapshots (separation 3^o, SNR$=$ 0dB, $q_w = 9$, $m = 50$, $r = 6$, $\delta = 5 \times 10^{-4}$, $\alpha = 0.998$, $n = 41$). We assume an incorrect number of sources $q_w = 9$ instead of $q = 10$.



Reduced-rank processing based on joint and iterative interpolation, decimation and filtering(JIDF)



– Interpolated received vector : $\boldsymbol{r}_I(i) = \boldsymbol{V}^H(i)\boldsymbol{r}(i)$

– Decimated received vector for branch b : $\bar{\boldsymbol{r}}(i) = \boldsymbol{D}_b(i)\boldsymbol{V}^H(i)\boldsymbol{r}(i)$

– Selection of decimation branch $\boldsymbol{D}(i)$: Euclidean distance

– Expression of estimate as a function of $\boldsymbol{v}(i)$, $\boldsymbol{D}(i)$ and $\boldsymbol{w}(i)$:

$$x(i) = \bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{r}(i) = \bar{\boldsymbol{w}}^H(i)\boldsymbol{D}_b(i)\boldsymbol{V}^H(i)\boldsymbol{r}(i) = \bar{\boldsymbol{w}}^H(i)\boldsymbol{D}(i)\Re_o(i)\boldsymbol{v}^*(i)$$

– Joint optimisation of $\boldsymbol{v}(i)$, $\boldsymbol{D}(i)$ and $\bar{\boldsymbol{w}}(i)$

Reduced-rank processing based on joint and iterative interpolation, decimation and filtering(JIDF)

- Decimation schemes : Optimal, uniform, random, pre-stored.
- The decimation pattern $\boldsymbol{D}(i)$ is selected according to :

$$\boldsymbol{D}(i) = \boldsymbol{D}_b \quad \text{when} \quad \boldsymbol{D}_b(i) = \arg \min_{1 \leq b \leq B} |e_b(i)|^2$$

- Optimal decimator : combinatorial problem with B possibilities

$$\boldsymbol{B} = \underbrace{M \cdot (M-1) \ldots (M - M/L + 1)}_{M/L \ \text{terms}} = \frac{M!}{(M - M/L)!}$$

- Suboptimal decimation schemes :
  - Uniform (U) Decimation
  - Pre-Stored (PS) Decimation.
  - Random (R) Decimation.

Reduced-rank processing based on joint and iterative interpolation, decimation and filtering(JIDF)

– General framework for decimation schemes

$$\boldsymbol{D}_b = \begin{bmatrix} \underbrace{0 \ \ldots \ 0}_{r_1 \ zeros} & 1 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \underbrace{0 \ 0 \ \ldots \ 0}_{r_m \ zeros} & 1 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \underbrace{0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ldots 0}_{r_D \ zeros} & 1 & \underbrace{0 \ \ldots \ 0}_{(JM-r_D-1) \ zeros} \end{bmatrix}$$

where m $(m = 1, 2, \ldots, M/L)$ denotes the m-th row and r_m is the number of zeros given by the decimation strategy.

– Suboptimal decimation schemes :

a. Uniform (U) Decimation with $B = 1 \rightarrow r_m = (m-1)L$.

b. Pre-Stored (PS) Decimation. We select $r_m = (m-1)L + (b-1)$ which corresponds to the utilization of uniform decimation for each branch b out of B branches.

c. Random (R) Decimation. We choose r_m as a discrete uniform random variable between 0 and $M-1$.

MMSE Parameter Vector Design of JIDF

– The MMSE expressions for $\bar{\mathbf{w}}(i)$ and $\mathbf{v}(i)$ can be computed via the minimization of the cost function

$$J_{\text{MSE}}^{(\mathbf{v}(i),\mathbf{D}(i),\bar{\mathbf{w}}(i))} = E[|d(i) - \mathbf{v}^H(i)\mathfrak{R}_o^T(i)\mathbf{D}^T(i)\bar{\mathbf{w}}^*(i)|^2]$$

– Fixing the interpolator $\mathbf{v}(i)$ and minimizing the cost function with respect to $\bar{\mathbf{w}}(i)$ the interpolated Wiener filter weight vector is

$$\bar{\mathbf{w}}(i) = \boldsymbol{\alpha}(\mathbf{v}) = \bar{\mathbf{R}}^{-1}(i)\bar{\mathbf{p}}(i)$$

where
$\bar{\mathbf{R}}(i) = E[\bar{\mathbf{r}}(i)\bar{\mathbf{r}}^H(i)]$,
$\bar{\mathbf{p}}(i) = E[d^*(i)\bar{\mathbf{r}}(i)]$,
$\bar{\mathbf{r}}(i) = \mathfrak{R}(i)\mathbf{v}^*(i)$.

MMSE Parameter Vector Design of JIDF

– Fixing $\bar{\mathbf{w}}(i)$ and minimizing the cost function with respect to $\mathbf{v}(i)$ the interpolator weight vector is

$$\mathbf{v}(i) = \boldsymbol{\beta}(\bar{\mathbf{w}}) = \mathbf{R}_u^{-1}(i)\mathbf{p}_u(i)$$

where
$\mathbf{R}_u(i) = E[\mathbf{u}(i)\mathbf{u}^H(i)]$, $\mathbf{p}_u(i) = E[d^*(i)\mathbf{u}(i)]$ and $\mathbf{u}(i) = \Re^T(i)\bar{\mathbf{w}}^*(i)$.

– The associated MSE expressions are

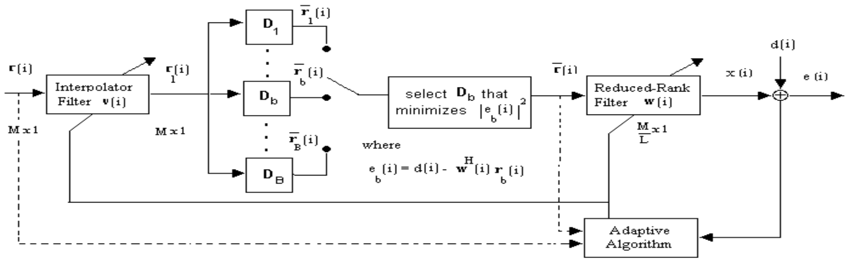
$$J(\mathbf{v}) = J_{\text{MSE}}(\boldsymbol{\alpha}(\mathbf{v}), \mathbf{v}) = \sigma_d^2 - \bar{\mathbf{p}}^H(i)\bar{\mathbf{R}}^{-1}(i)\bar{\mathbf{p}}(i)$$

$$J_{\text{MSE}}(\bar{\mathbf{w}}, \boldsymbol{\beta}(\bar{\mathbf{w}})) = \sigma_d^2 - \mathbf{p}_u^H(i)\mathbf{R}_u^{-1}(i)\mathbf{p}_u(i)$$

where $\sigma_d^2 = E[|d(i)|^2]$.

– The points of global minimum can be obtained by $\mathbf{v}_{\text{opt}} = \arg\min_{\mathbf{v}} J(\mathbf{v})$ and $\bar{\mathbf{w}}_{\text{opt}} = \boldsymbol{\alpha}(\mathbf{v}_{\text{opt}})$ or $\bar{\mathbf{w}}_{\text{opt}} = \arg\min_{\bar{\mathbf{w}}} J_{\text{MSE}}(\bar{\mathbf{w}}, \boldsymbol{\beta}(\bar{\mathbf{w}}))$ and $\mathbf{v}_{\text{opt}} = \boldsymbol{\beta}(\bar{\mathbf{w}}_{\text{opt}})$.

Adaptive JIDF implementation : LMS algorithms



Initialize all parameter vectors, dimensions, number of branches B and select decimation technique

For each data vector $i = 1, \ldots, Q$ do :

– Select decimation branch that minimizes $e_b(i) = d(i) - \boldsymbol{w}^H(i)\bar{\boldsymbol{r}}(i)$
– Make $\bar{\boldsymbol{r}}(i) = \bar{\boldsymbol{r}}_b(i)$ when $b = \arg\min_{1 \leq b \leq B} |e_b(i)|^2$
– Estimate parameters

$$\boldsymbol{v}(i+1) = \boldsymbol{v}(i) + \eta e^*(i)\boldsymbol{u}(i)$$

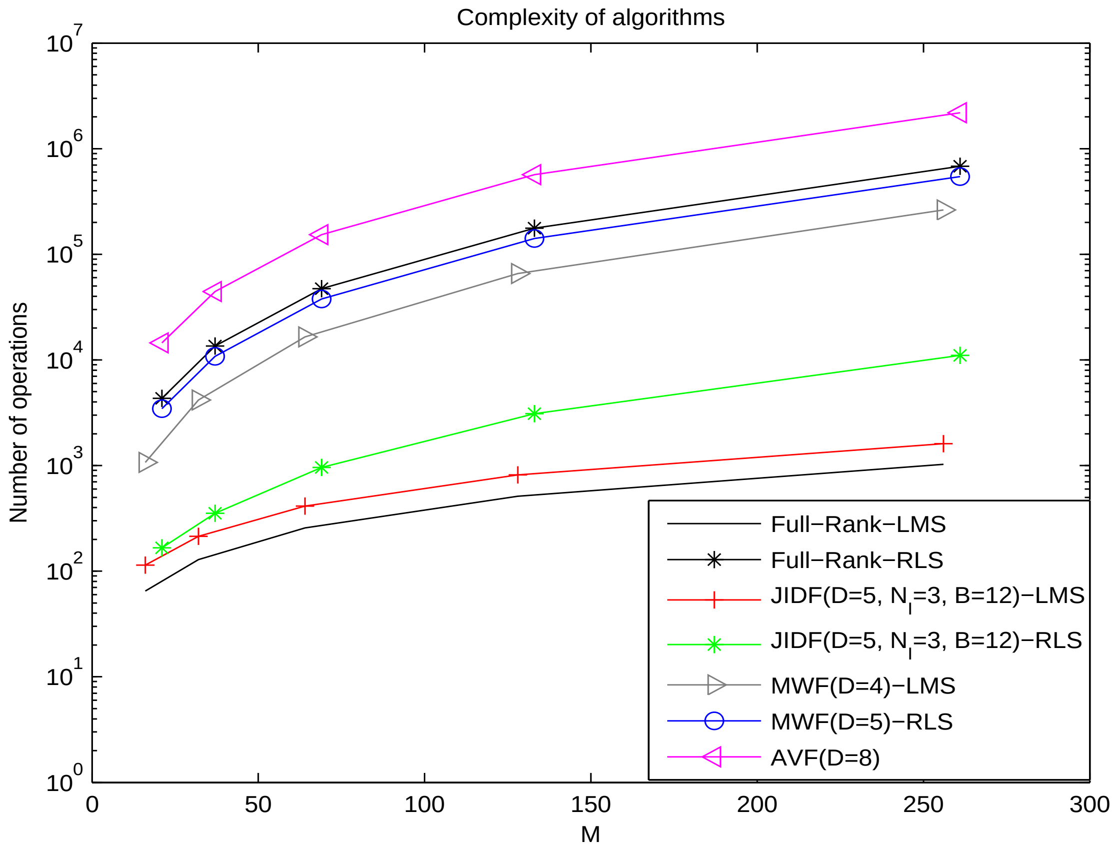
$$\bar{\boldsymbol{w}}(i+1) = \bar{\boldsymbol{w}}(i) + \mu e^*(i)\bar{\boldsymbol{r}}(i)$$

where $\boldsymbol{u}(i) = \Re^T(i)\bar{\boldsymbol{w}}^*(i)$ and $\bar{\boldsymbol{r}}(i) = \boldsymbol{D}(i)\boldsymbol{V}^H(i)\boldsymbol{r}(i)$.

Complexity of JIDF Algorithms

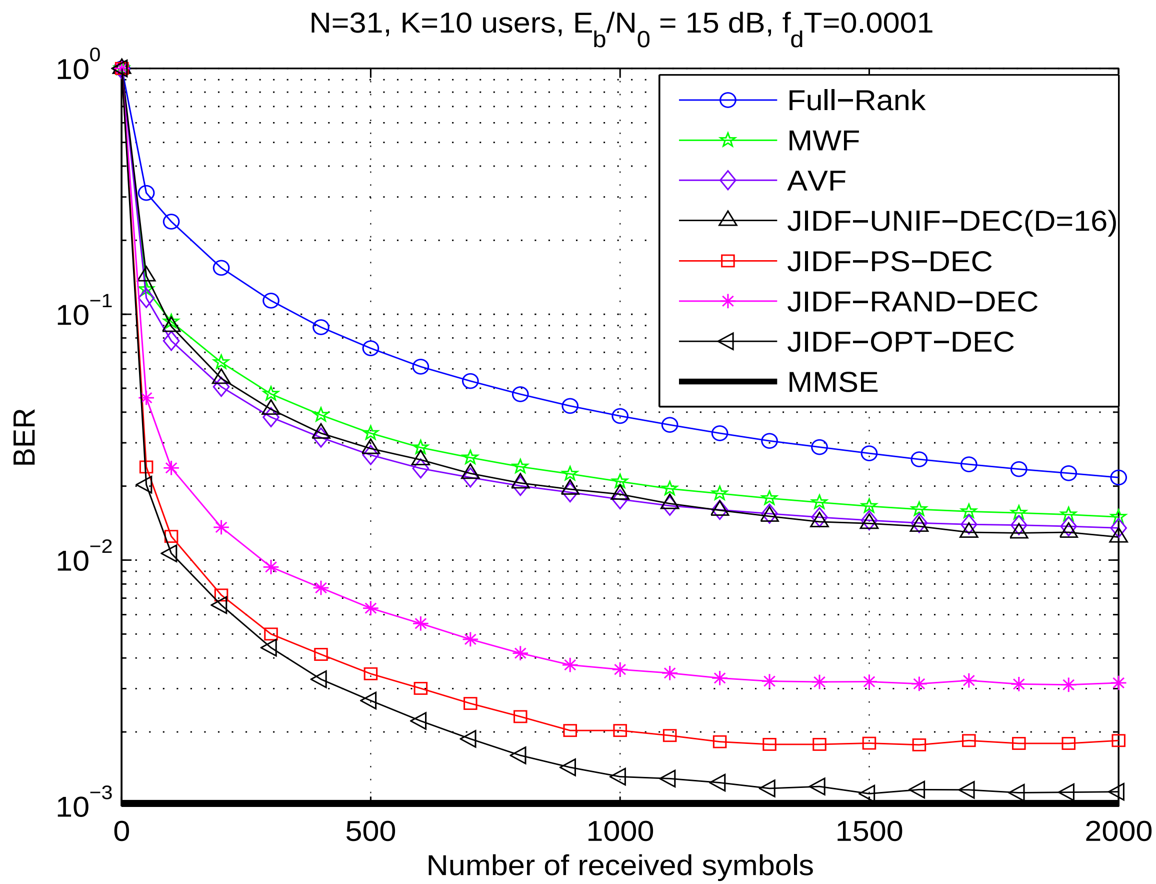
	Number of operations per symbol	
Algorithm	Additions	Multiplications
Full-rank-LMS	$2M$	$2M+1$
Full-rank-RLS	$3(M-1)^2+M^2+2M$	$6M^2+2M+2$
JIDF-LMS	$(B+1)(D)+2N_I$	$(B+2)D$
JIDF-RLS	$3(D-1)^2+3(N_I-1)^2$ $+(D-1)N_I+N_IM+(D)^2$ $+N_I^2+(B+1)D+2N_I$	$6(D)^2+6N_I^2$ $+DN_I+2$ $+(B+2)D+N_I$
MWF-LMS	$D(2(\bar{M}-1)^2+\bar{M}+3)$	$D(2\bar{M}^2+5\bar{M}+7)$
MWF-RLS	$D(4(\bar{M}-1)^2+2\bar{M})$	$D(4\bar{M}^2+2\bar{M}+3)$
AVF	$D((M)^2+3(M-1)^2)-1$ $+D(5(M-1)+1)+2M$	$D(4(M)^2+4M+1)$ $+4M+2$

Complexity of JIDF Algorithms



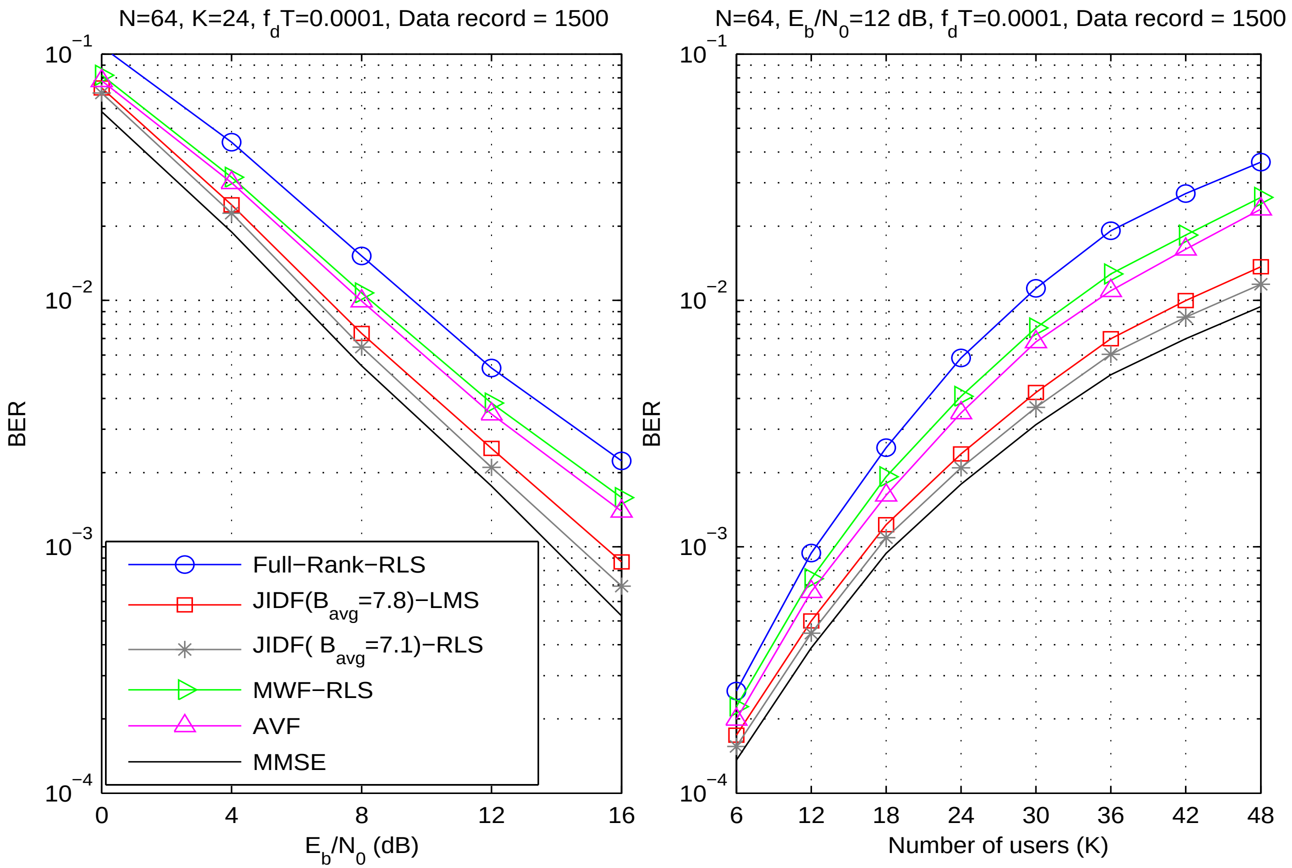
Applications : Interference suppression for CDMA

Parameters : Uplink scenario, QPSK symbols, K users, N chips per symbol and L propagation paths, receiver filter has $M = N + L_p - 1$ taps.



Applications : Interference suppression for CDMA

Parameters : Uplink scenario, QPSK symbols, K users, N chips per symbol and L propagation paths, receiver filter has $M = N + L_p - 1$ taps.



Techniques based on joint and iterative optimisation of basis functions

– Consider the $M \times D$ transformation matrix expressed as

$$\boldsymbol{S}_D(i) = [\phi_1(i), \cdots, \phi_d(i), \cdots \phi_D(i)]$$

where $\{\phi_d(i) |\ d = 1, \ldots, D\}$ are the M-dimensional basis vectors.

– In order to start the development, let us express the reduced-rank input vector as

$$\begin{aligned}\bar{\mathbf{r}}(i) &= \mathbf{S}_D^H(i)\mathbf{r}(i)\\ &= \begin{bmatrix} \mathbf{r}^T(i) & & & \\ & \mathbf{r}^T(i) & & \\ & & \ddots & \\ & & & \mathbf{r}^T(i) \end{bmatrix}_{D\times MD} \begin{bmatrix} \phi_1(i) \\ \phi_2(i) \\ \vdots \\ \phi_D(i) \end{bmatrix}^*_{MD\times 1}\\ &= \mathbf{R}_{\text{in}}(i)\mathbf{t}(i)\end{aligned}$$

where the projection matrix is transformed into a vector form, and $\mathbf{t}(i)$ is called parameter vector in what follows.

Techniques based on joint and iterative optimisation of basis functions

– Let us now design the parameter vector using the cost function

$$\mathbf{J}_{\text{MSE}}(\bar{\mathbf{w}}(i),\mathbf{t}(i)) = E[|d(i) - \bar{\mathbf{w}}^H(i)\mathbf{R}_{\text{in}}(i)\mathbf{t}(i)|^2]$$

– The MMSE solution of the reduced-rank filter in the generic scheme has the same form as before, i.e.

$$\bar{\mathbf{w}}(i) = \bar{\mathbf{R}}^{-1}(i)\bar{\mathbf{p}}(i)$$

where
$\bar{\mathbf{R}}(i) = E[\mathbf{S}_D^H(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{S}_D(i)] = E[\bar{\mathbf{r}}(i)\bar{\mathbf{r}}^H(i)]$,
$\bar{\mathbf{p}}(i) = E[d^*(i)\mathbf{S}_D^H(i)\mathbf{r}(i)] = E[d^*(i)\bar{\mathbf{r}}(i)]$.

Techniques based on joint and iterative optimisation of basis functions

– The MMSE expression for the parameter vector $\mathbf{t}(i)$ is

$$\mathbf{t}(i) = \mathbf{R}_w^{-1}(i)\mathbf{p}_w(i)$$

where $\mathbf{R}_w(i) = E[\mathbf{R}_{\text{in}}^H(i)\bar{\mathbf{w}}(i)\bar{\mathbf{w}}^H(i)\mathbf{R}_{\text{in}}(i)]$ and $\mathbf{p}_w(i) = E[d(i)\mathbf{R}_{\text{in}}^H(i)\bar{\mathbf{w}}(i)]$.

– The associated MMSE is

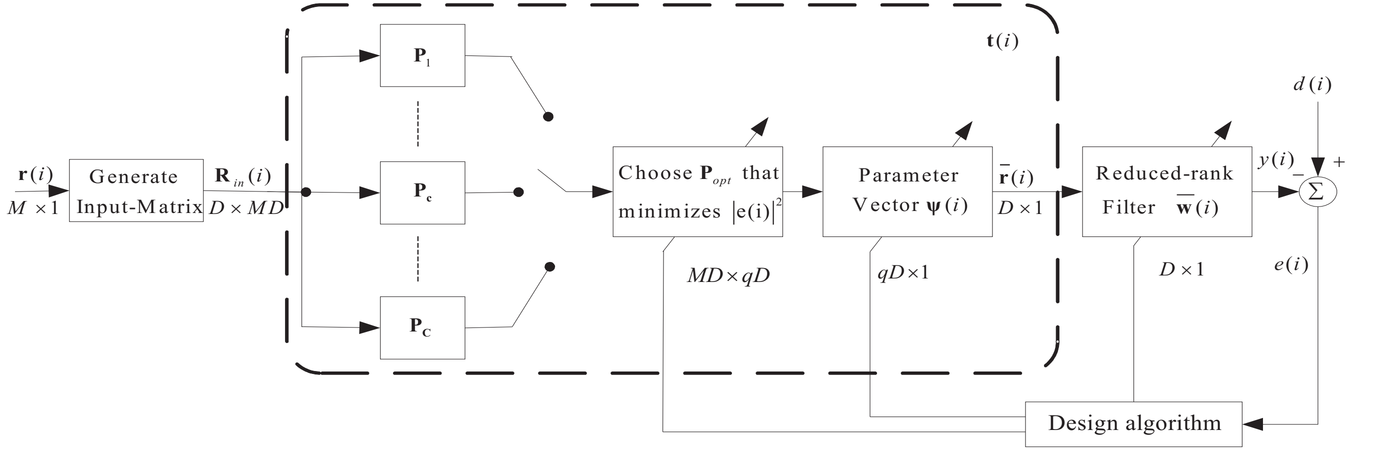
$$\mathbf{MMSE}_{\text{g}} = \sigma_d^2 - \bar{\mathbf{p}}^H\bar{\mathbf{R}}^{-1}\bar{\mathbf{p}}$$

– However, in this generic scheme, a D-dimensional reduced-rank filter and an MD-dimensional parameter vector are required to be adapted for each iteration.

– In applications such as DS-UWB systems where the received signal size M is large, the complexity of updating the parameter vector or the projection matrix is very high.

– In order to reduce the complexity of this generic scheme, we will introduce constraints in the design of the transformation matrix in order to obtain a cost-effective structure.

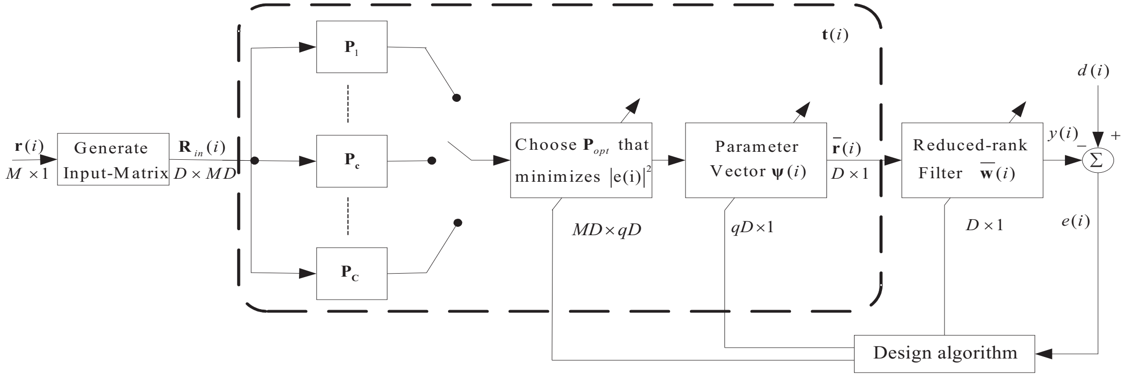
Techniques based on joint and iterative optimisation of basis functions



– The proposed switched approximation of adaptive basis functions (SAABF) constrains the structure of the MD-dimensional parameter vector $\mathbf{t}(i)$. , using a multiple-branch framework.

– The SAABF scheme uses a structure with C branches for determining the best position of the basis function vectors.

Techniques based on joint and iterative optimisation of basis functions



– For each branch, the mapping matrix $\boldsymbol{S}_{D,c}(i)$ is constructed by a set of adaptive basis function vectors as given by

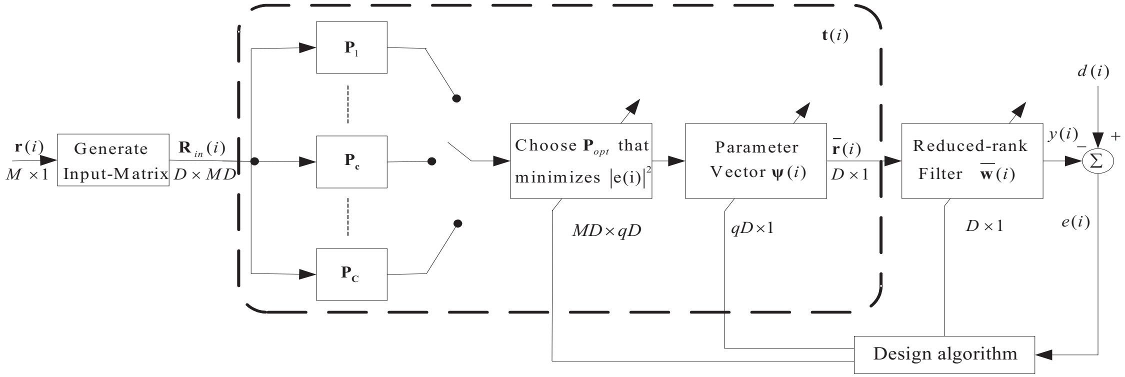
$$\boldsymbol{S}_{D,c}(i)(i) = [\boldsymbol{\phi}_{c,1}(i), \cdots, \boldsymbol{\phi}_{c,d}(i), \cdots \boldsymbol{\phi}_{c,D}(i)]$$

where $c = [1, 2, \cdots, C]$, $d = [1, 2, \cdots, D]$ and the M-dimensional basis function vector is

$$\boldsymbol{\phi}_{c,d}(i) = [\underbrace{0, \cdots, 0}_{z_{c,d}}, \underbrace{\boldsymbol{\varphi}_d^T(i)}_{q}, \underbrace{0, \cdots, 0}_{M-q-z_{c,d}}]^T$$

where $z_{c,d}$ is the number of zeros before the $q \times 1$ function $\boldsymbol{\varphi}_d(i)$, which is called the inner function in what follows.

Techniques based on joint and iterative optimisation of basis functions



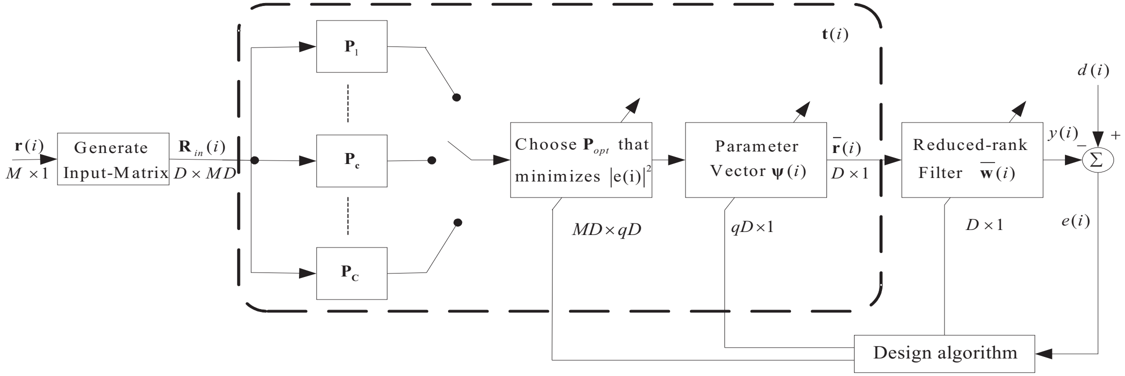
– At each time instant, the output signal of each branch or mapping matrix can be expressed as :

$$y_c(i) = \bar{\mathbf{w}}^H(i)\boldsymbol{S}^H_{D,c}(i)\mathbf{r}(i) = \bar{\mathbf{w}}^H(i)\mathbf{R}_{\text{in}}(i)\mathbf{t}_c(i),$$

where the $MD \times 1$ vector $\mathbf{t}_c(i)$ is

$$\mathbf{t}_c(i) = \left[\phi^T_{c,1}(i), \phi^T_{c,2}(i), \cdots, \phi^T_{c,D}(i)\right]^H$$

Techniques based on joint and iterative optimisation of basis functions

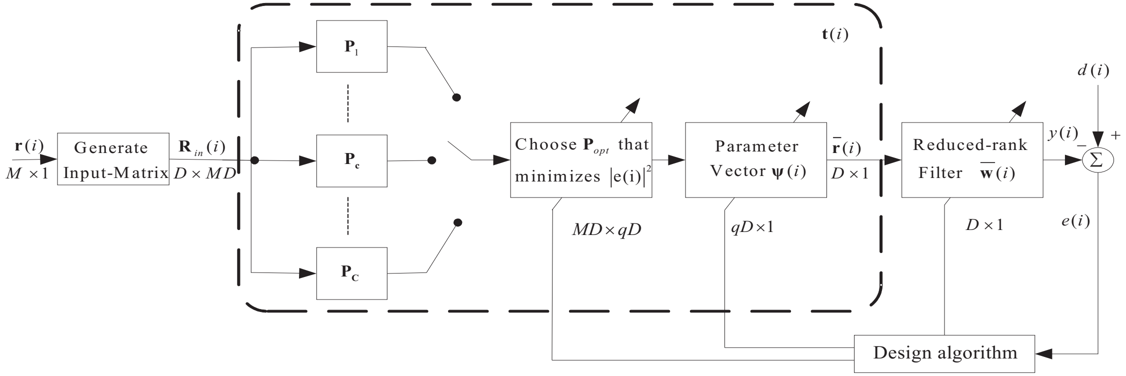


– For each basis function, we rearrange the expression as

$$\phi_{c,d}(i) = \begin{bmatrix} \mathbf{0}_{z_{c,d}\times q} \\ \mathbf{I}_q \\ \mathbf{0}_{(M-q-z_{c,d})\times q} \end{bmatrix}_{M\times q} \varphi_d(i) = \mathbf{Z}_{c,d}\varphi_d(i)$$

where the matrix $\mathbf{Z}_{c,d}$ consists of zeros and ones. With an $q \times q$ identity matrix in the middle, the zero matrices have the size of $z_{c,d} \times q$ and $(M - q - z_{c,d}) \times q$, respectively.

Techniques based on joint and iterative optimisation of basis functions

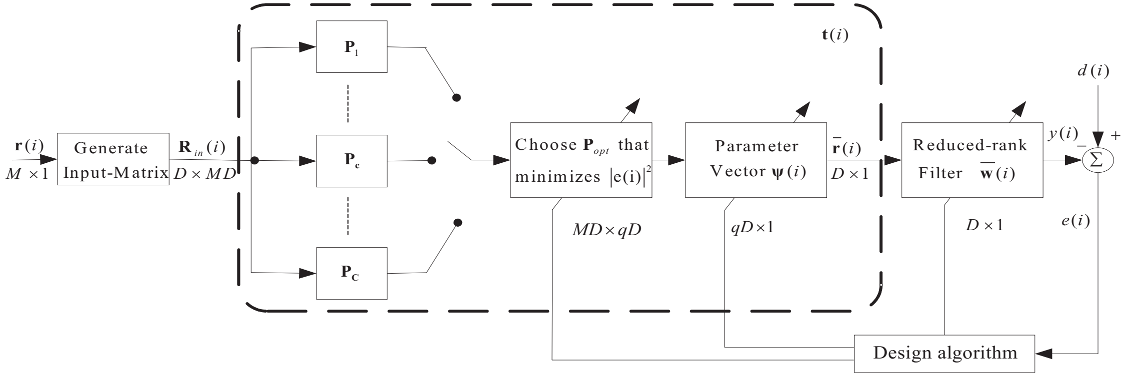


– With this kind of arrangement, we rewrite the expression of $\mathbf{t}_c$ as :

$$\mathbf{t}_c(i) = \begin{bmatrix} \mathbf{Z}_{c,1} & & & \\ & \mathbf{Z}_{c,2} & & \\ & & \ddots & \\ & & & \mathbf{Z}_{c,D} \end{bmatrix} \begin{bmatrix} \varphi_1(i) \\ \varphi_2(i) \\ \vdots \\ \varphi_D(i) \end{bmatrix}^* = \mathbf{P}_c \psi(i),$$

where the $MD \times qD$ block diagonal matrix $\mathbf{P}_c$ is called position matrix which determines the positions of the q-dimensional inner functions.

Techniques based on joint and iterative optimisation of basis functions



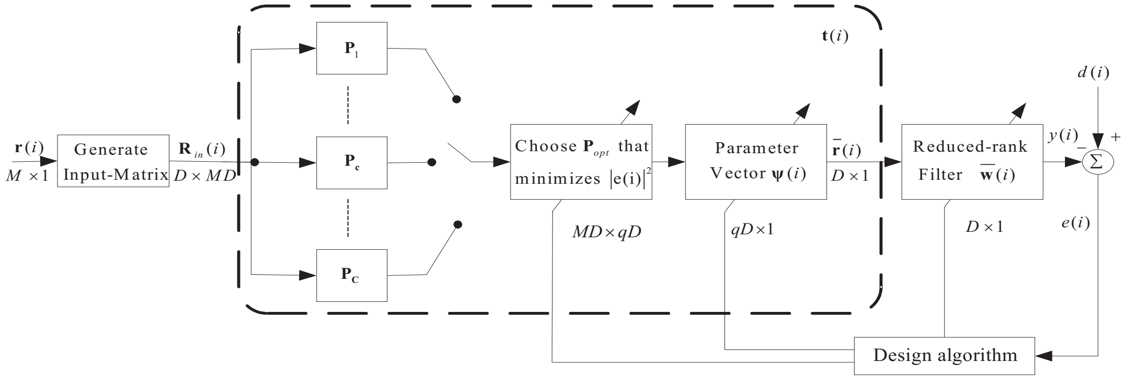
– The parameter $\boldsymbol{\psi}(i)$ denotes the qD-dimensional parameter vector which is constructed by the inner functions.

– For each mapping matrix, we have a unique position matrix $\mathbf{P}_c$.

– The dimension of the parameter vector $\mathbf{t}(i)$ is shortened from MD to qD and only a qD-dimensional parameter vector will be updated for the rank reduction.

– The adaptation of the instantaneous position matrix, the parameter vector and the reduced-rank filter involves a discrete parameter optimization for choosing the instantaneous position matrix and a continuous filter design for adapting the parameter vector and the reduced-rank filter.

Discrete Parameter Optimization of SAABF



– In order to calculate the error signal, we find the output signal of each branch and express it as

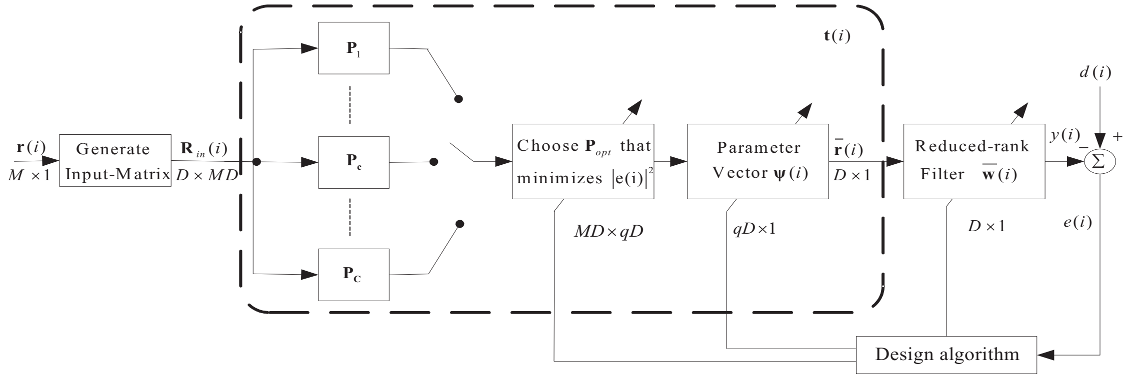
$$y_c(i) = \bar{\mathbf{w}}^H(i)\mathbf{R}_{\mathsf{in}}(i)\mathbf{P}_c\psi(i)$$

the corresponding error signal is $e_c(i) = d(i) - y_c(i)$. Hence, the selection rule can be expressed as

$$c_{\mathsf{opt}} = \arg \min_{c \in \{1,\ldots,C\}} |e_c(i)|^2$$

$$\mathbf{P}(i) = \mathbf{P}_{c_{\mathsf{opt}}}$$

Discrete Parameter Optimization of SAABF

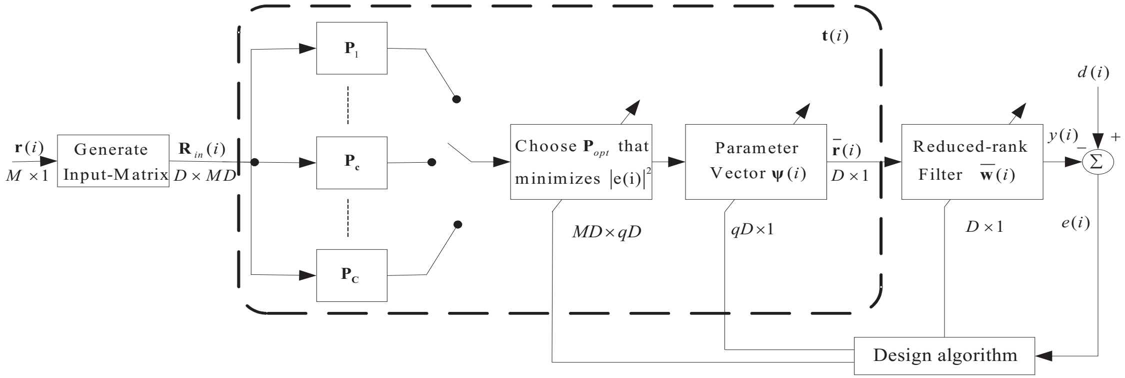


– In the SAABF scheme, the position matrices are distinguished by the values of $z_{c,d}$.

– An exhaustive approach has been considered for the selection of $z_{c,d}$, in which all the possibilities of the positions should be tested. We then choose a structure for the projection matrix which corresponds to the minimum squared error.

– However, in applications such as UWB systems, the number of possible positions is $(M - q)^D$, when M is much larger than q and D, say $M = 120$ and $q = D = 4$, it becomes impractical to compare such a huge number of possibilities.

Discrete Parameter Optimization of SAABF



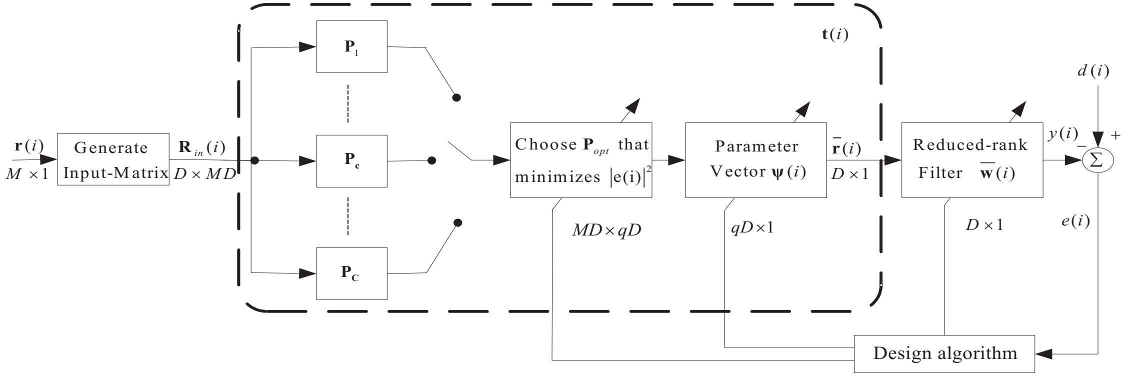
– Hence, we constrain the number of possibilities or equivalently, we set a small value of C that enables us to find the sub-optimum position matrix for each time instant, and the sub-optimum solution enables the SAABF scheme to obtain required performance.

– It turns out that a deterministic way to set the values of $z_{c,d}$ was the most practical. Assuming that q and D are much smaller than M, we set

$$z_{c,d} = \lfloor \frac{M}{D} \rfloor \times (d-1) + (c-1)q,$$

where $c = 1, \ldots, C$ and $d = 1, \ldots, D$.

Discrete Parameter Optimization of SAABF



– Bearing in mind the matrix form shown, we implement this deterministic approach to generate the position matrices. The first $MD \times qD$ position matrix $\mathbf{P}_1$ can be expressed as

$$\mathbf{P}_1 = \begin{bmatrix} \mathbf{I}_q & & & \\ \mathbf{0}_{M-q} & & & \\ & \mathbf{0}_{\lfloor \frac{M}{D} \rfloor} & & \\ & \mathbf{I}_q & & \\ & \mathbf{0}_{M-q-\lfloor \frac{M}{D} \rfloor} & & \\ & & \ddots & \\ & & & \mathbf{0}_{\lfloor \frac{M}{D} \rfloor D} \\ & & & \mathbf{I}_q \\ & & & \mathbf{0}_{M-q-\lfloor \frac{M}{D} \rfloor D} \end{bmatrix},$$

where all the zero and identity matrices have q columns and the subscripts denote the number of rows of these matrices.

LS Parameter Vector Design of SAABF

– After determining the position matrix $\mathbf{P}(i)$, the LS design of the reduced-rank filter and the parameter vector can be designed by minimizing the following cost function

$$\mathbf{J}_{\mathsf{LS}}(\bar{\mathbf{w}}(i), \boldsymbol{\psi}(i)) = \sum_{j=1}^{i} \lambda^{i-j} |d(j) - \bar{\mathbf{w}}^H(i)\mathbf{R}_{\mathsf{in}}(j)\mathbf{P}(i)\boldsymbol{\psi}(i)|^2, \qquad (1)$$

where λ is a forgetting factor. Since this cost function is a function of $\bar{\mathbf{w}}(i)$ and $\boldsymbol{\psi}(i)$, the LS solutions can be obtained as follows.

– Firstly, we calculate the gradient of with respect to $\bar{\mathbf{w}}(i)$

$$\mathbf{g}_{\mathsf{LS}\bar{w}^*(i)} = -\bar{\mathbf{p}}_{w_{\mathsf{LS}}}(i) + \bar{\mathbf{R}}_{w_{\mathsf{LS}}}(i)\bar{\mathbf{w}}(i), \qquad (2)$$

where $\bar{\mathbf{p}}_{w_{\mathsf{LS}}}(i) = \sum_{j=1}^{i} \lambda^{i-j} d^*(j)\bar{\mathbf{r}}(j)$ and $\bar{\mathbf{R}}_{w_{\mathsf{LS}}}(i) = \sum_{j=1}^{i} \lambda^{i-j}\bar{\mathbf{r}}(j)\bar{\mathbf{r}}^H(j)$.

LS Parameter Vector Design of SAABF

– Assuming that $\boldsymbol{\psi}(i)$ is fixed, the LS solution of the reduced-rank filter is

$$\bar{\mathbf{w}}_{\mathsf{LS}}(i) = \bar{\mathbf{R}}_{w_{\mathsf{LS}}}^{-1}(i)\bar{\mathbf{p}}_{w_{\mathsf{LS}}}(i).$$

– Secondly, we examine the gradient of the cost function with respect to $\boldsymbol{\psi}(i)$, which is

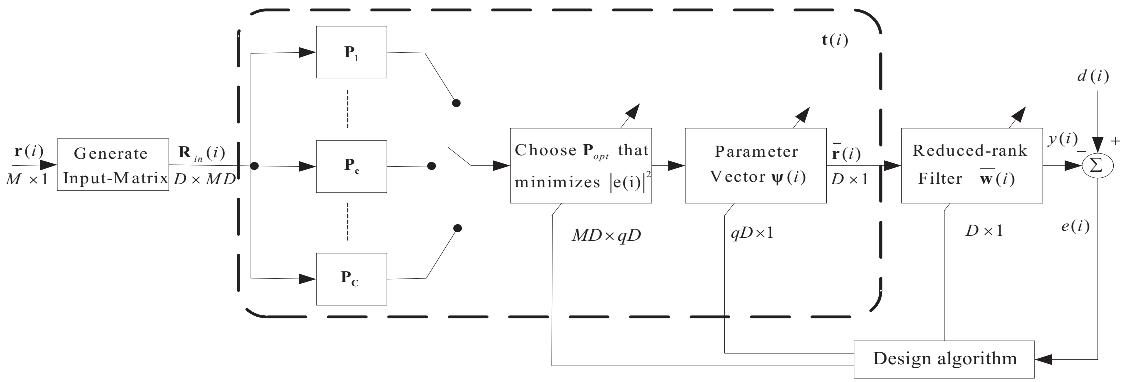
$$\mathbf{g}_{\mathsf{LS}\psi^*(i)} = -\mathbf{p}_{\psi_{LS}}(i) + \mathbf{R}_{\psi_{\mathsf{LS}}}(i)\boldsymbol{\psi}(i),$$

where the vector $\mathbf{p}_{\psi_{\mathsf{LS}}}(i) = \sum_{j=1}^{i} \lambda^{i-j} d(j)\mathbf{r}_{\psi}(j)$ and the matrix $\mathbf{R}_{\psi_{\mathsf{LS}}}(i) = \sum_{j=1}^{i} \lambda^{i-j}\mathbf{r}_{\psi}(j)\mathbf{r}_{\psi}^{H}(j)\boldsymbol{\psi}(i)$, and $\mathbf{r}_{\psi}(j) = \mathbf{P}^{H}(j)\mathbf{R}_{\mathsf{in}}^{H}(j)\bar{\mathbf{w}}(j)$.

– With the assumption that $\bar{\mathbf{w}}(i)$ is fixed, the LS solution of the parameter vector is

$$\boldsymbol{\psi}_{\mathsf{LS}}(i) = \mathbf{R}_{\psi_{\mathsf{LS}}}^{-1}(i)\mathbf{p}_{\psi_{\mathsf{LS}}}(i).$$

Adaptive version of SAABF : LMS algorithms



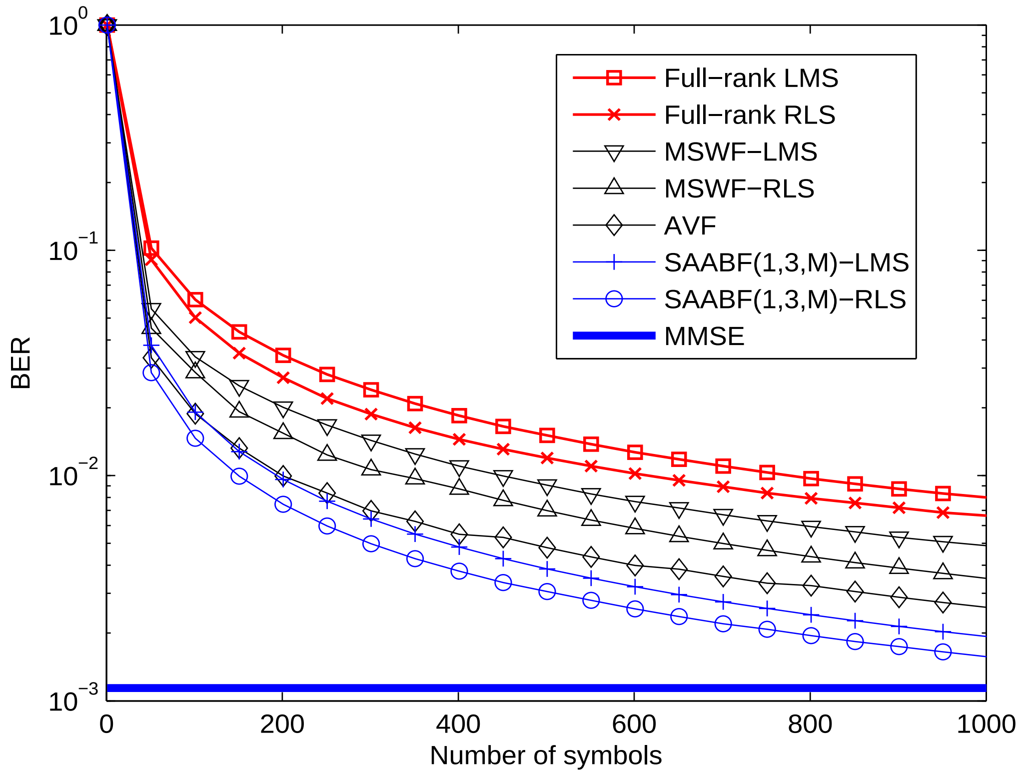
Step 1 :	Initialization :
	$\boldsymbol{\psi}(0)$=ones(qD, 1) and $\bar{\mathbf{w}}(0)$=zeros(D, 1)
	Set values for μ_w and μ_ψ
	Generate the position matrices $\mathbf{P}_1, \ldots, \mathbf{P}_C$
Step 2 :	For i=0, 1, 2, ...
	(1) Compute the error signals $e_c(i)$ for each branch,
	(2) Select the branch $c_{\text{opt}} = \arg\min_{c\in\{1,\ldots,C\}} \lvert e_c(i)\rvert^2$,
	(3) Set the instantaneous position matrix $\mathbf{P}(i)$=$\mathbf{P}_{c_{\text{opt}}}$,
	(4) Update $\bar{\mathbf{w}}(i+1)$: $\bar{\mathbf{w}}(i+1) = \bar{\mathbf{w}}(i) + \mu_w \mathbf{R}_{\text{in}}(i)\mathbf{P}(i)\boldsymbol{\psi}(i)e^*(i)$
	(5) Update $\boldsymbol{\psi}(i+1)$: $\boldsymbol{\psi}(i+1) = \boldsymbol{\psi}(i) + \mu_\psi \mathbf{P}^H(i)\mathbf{R}_{\text{in}}^H(i)\bar{\mathbf{w}}(i+1)e(i)$.

Applications : UWB communications

– We apply the proposed generic and SAABF schemes to the downlink of a multiuser BPSK DS-UWB system and evaluate their performance against existing reduced-rank and full-rank methods.
– In all numerical simulations, the pulse shape adopted is the RRC pulse with the pulse-width 0.375ns.
– The spreading codes are generated randomly with a spreading gain of 24 and the data rate of the communication is approximately 110Mbps.
– The standard IEEE 802.15.4a channel model for the NLOS indoor environment is employed.
– We assume that the channel is constant during the whole transmission.
– The sampling rate at the receiver is assumed to be 8GHz that is the same as the standard channel model and the observation window length M for each data symbol is set to 120 samples.

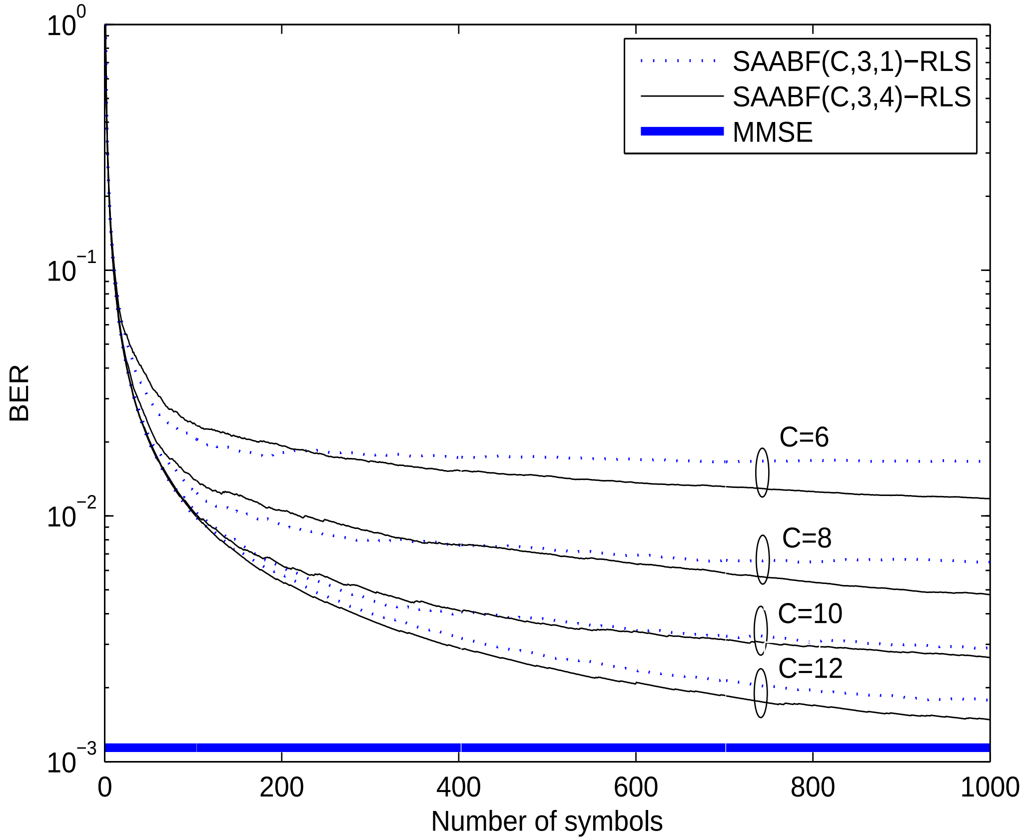
Applications : UWB communications

Parameters :BER performance of different algorithms for a SNR=16dB and 3 users. The following parameters were used : full-rank LMS ($\mu = 0.075$), full-rank RLS ($\lambda = 0.998$, $\delta = 10$), MSWF-LMS ($D = 6$, $\mu = 0.075$), MSWF-RLS ($D = 6$, $\lambda = 0.998$), AVF ($D = 6$), SAABF (1,3,M)-LMS ($\mu_w = 0.1$, $\mu_\psi = 0.2$, 2 iterations) and SAABF (1,3,M)-RLS ($\lambda = 0.998$, $\delta = 0.1$, 1 iteration).

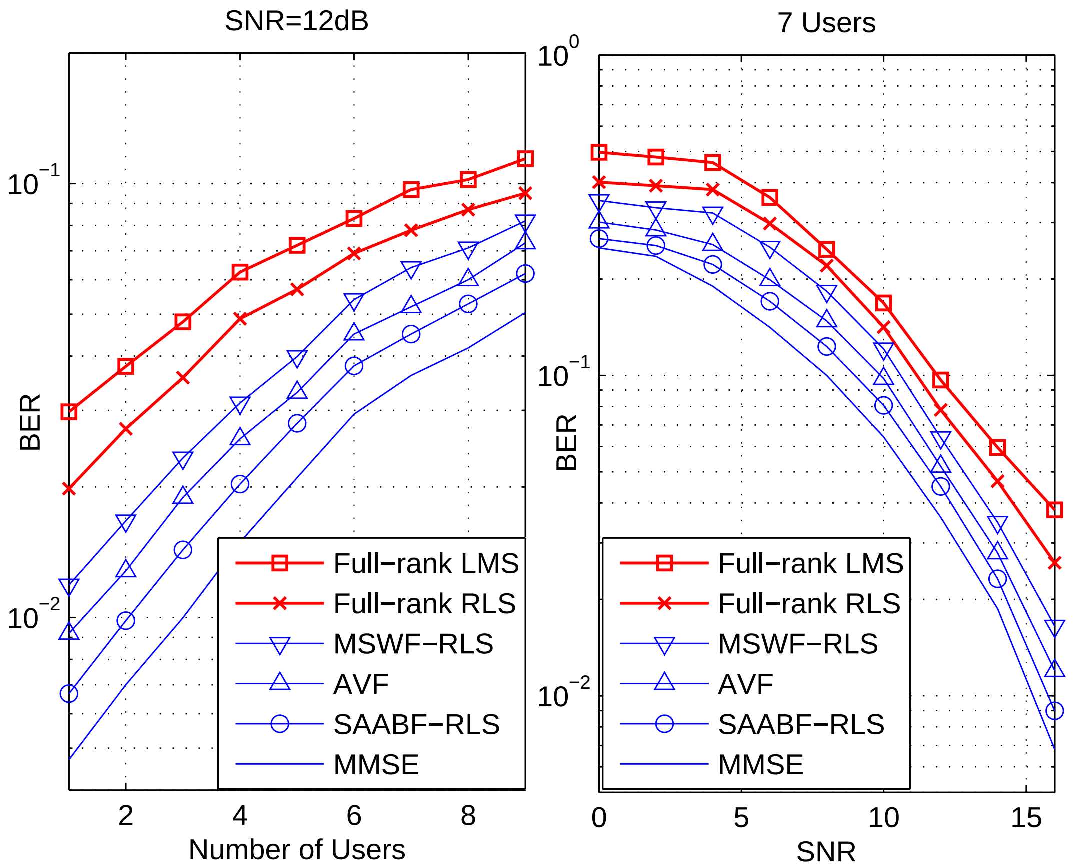


Applications : UWB communications

Parameters :BER performance of the proposed SAABF scheme versus the number of training symbols for a SNR=16dB. The number of users is 3 and the following parameters were used : SAABF-RLS ($\lambda = 0.98$, $\delta = 10$).



Applications : UWB communications



Model-order selection techniques

- Basic principle : to determine the best fit between observed data and the model used.
- General approaches to model-order selection :
  - Setting of upper bounds on models with ”some” prior knowledge : one of the most used in communications.
  - Akaike's information theoretic criterion : works well but requires some computations.
  - Minimum description length (MDL) : also works well but requires some computations.
  - Adaptive filtering approach : use for dynamic lengths adaptive algorithms, work well and have lower complexity than prior art.

Model-order selection techniques

- Approaches used for reduced-rank techniques :
  - Testing of orthogonality conditions between columns of transformation matrix $\boldsymbol{S}_D(i)$ [12] : used with the MSWF for selecting the rank D.
  - Cross-validation of data [23] : used with the AVF, works but can be complex since the algorithms sometimes selects D quite large. This can be a problem if M is large and D approaches it.
  - Use of a priori values of least-squares type cost functions with lower and upper bounds : works very well and it is simple to use and design [12, 17, 29]. It can be easily extended when the designer has multiple parameters with orders to adjust.

Model-order selection with LCMV JIO algorithm

– Consider the exponentially weighted *a posteriori* least-squares type cost function described by

$$\mathcal{C}(\boldsymbol{S}_D(i-1), \bar{\boldsymbol{w}}^{(D)}(i-1)) = \sum_{l=1}^{i} \alpha^{i-l} \Big| \bar{\boldsymbol{w}}^{H,\ (D)}(i-1)\boldsymbol{S}_D(i-1)\boldsymbol{r}(l)|^2,$$

where α is the forgetting factor and $\bar{\mathbf{w}}^{(D)}(i-1)$ is the reduced-rank filter with rank D.

– For each time interval i, we can select the rank D_{opt} which minimizes $\mathcal{C}(\boldsymbol{S}_D(i-1), \bar{\boldsymbol{w}}^{(D)}(i-1))$ and the exponential weighting factor α is required as the optimal rank varies as a function of the data record.

– The key quantities to be updated are the projection matrix $\boldsymbol{S}_D(i)$, the reduced-rank filter $\bar{\boldsymbol{w}}(i)$, the associated reduced-rank steering vector $\bar{\boldsymbol{a}}(\theta_k)$ and the inverse of the reduced-rank covariance matrix $\bar{\boldsymbol{P}}(i)$ (for the proposed RLS algorithm).

Model-order selection with LCMV JIO algorithm

– Let us define the following extended projection matrix $\boldsymbol{S}^{(D)}(i)$ and the extended reduced-rank filter weight vector $\bar{\boldsymbol{w}}^{(D)}(i)$ as follows :

$$\boldsymbol{S}^{(D)}(i) = \begin{bmatrix} s_{1,1} & \cdots & s_{1,D_{\text{min}}} & \cdots & s_{1,D_{\text{max}}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ s_{M,1} & \cdots & s_{M,D_{\text{min}}} & \cdots & s_{M,D_{\text{max}}} \end{bmatrix} \text{ and } \bar{\boldsymbol{w}}^{(D)}(i) = \begin{bmatrix} w_1 \\ \vdots \\ w_{D_{\text{min}}} \\ \vdots \\ w_{D_{\text{max}}} \end{bmatrix}$$

– $\boldsymbol{S}^{(D)}(i)$ and $\bar{\boldsymbol{w}}^{(D)}(i)$ are updated along with the associated quantities $\bar{\boldsymbol{a}}(\theta_k)$ and $\bar{\boldsymbol{P}}(i)$ for the maximum allowed rank D_{max}.

– The rank adaptation algorithm determines the rank that is best for each time instant i using the cost function.

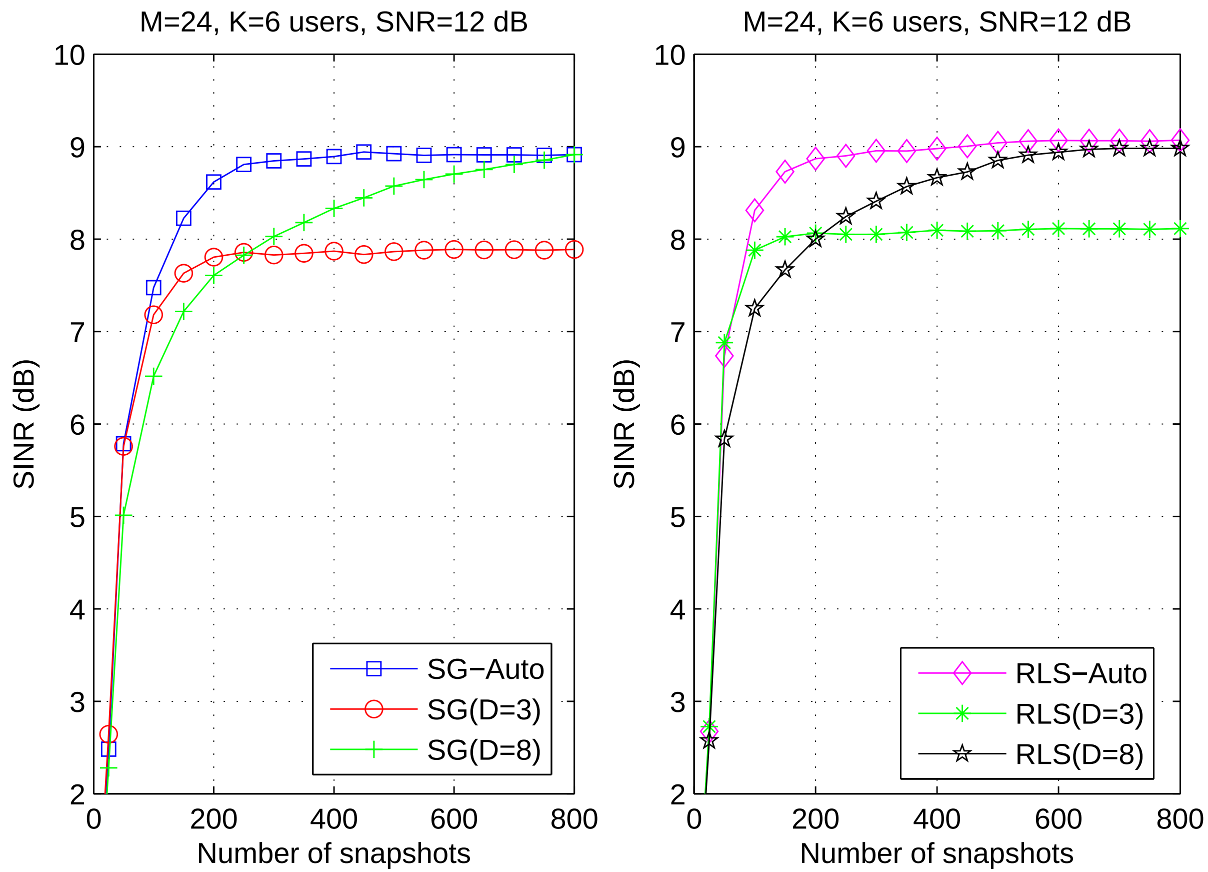
– The proposed rank adaptation algorithm is then given by

$$D_{\text{opt}} = \arg \min_{D_{\text{min}} \leq d \leq D_{\text{max}}} \mathcal{C}(\boldsymbol{S}_D(i-1), \bar{\boldsymbol{w}}^{(D)}(i-1))$$

where d is an integer, D_{min} and D_{max} are the minimum and maximum ranks allowed for the reduced-rank filter, respectively.

Model-order selection with LCMV JIO algorithm

SINR performance of LCMV (a) SG and (b) RLS algorithms against snapshots with $M = 24$, $SNR = 12$ dB with automatic rank selection.



Model-order selection with JIDF algorithm

– Consider the following exponentially weighed *a posteriori* least-squares type cost function

$$\mathcal{C}(\bar{\mathbf{w}}^{(D)}, \mathbf{v}^{(N_I)},\ \mathbf{D}) = \sum_{l=1}^{i} \alpha^{i-l} \Big| d(l) - \bar{\mathbf{w}}^{H,\ (D)}(l)\mathbf{D}(l)\mathfrak{R}_o(l)\mathbf{v}^{*,\ (N_I)}(l)\Big|^2,$$

where α is the forgetting factor, $\tilde{\mathbf{w}}^{(D)}(i-1)$ is the reduced-rank filter with rank D and $\mathbf{v}^{(N_I)}(i)$ is the interpolator filter with rank N_I.

– For each time interval i and a given decimation pattern and B, we can select D and N_I which minimizes $\mathcal{C}(\bar{\mathbf{w}}^{(D)}, \mathbf{v}^{(N_I)},\ \mathbf{D})$.

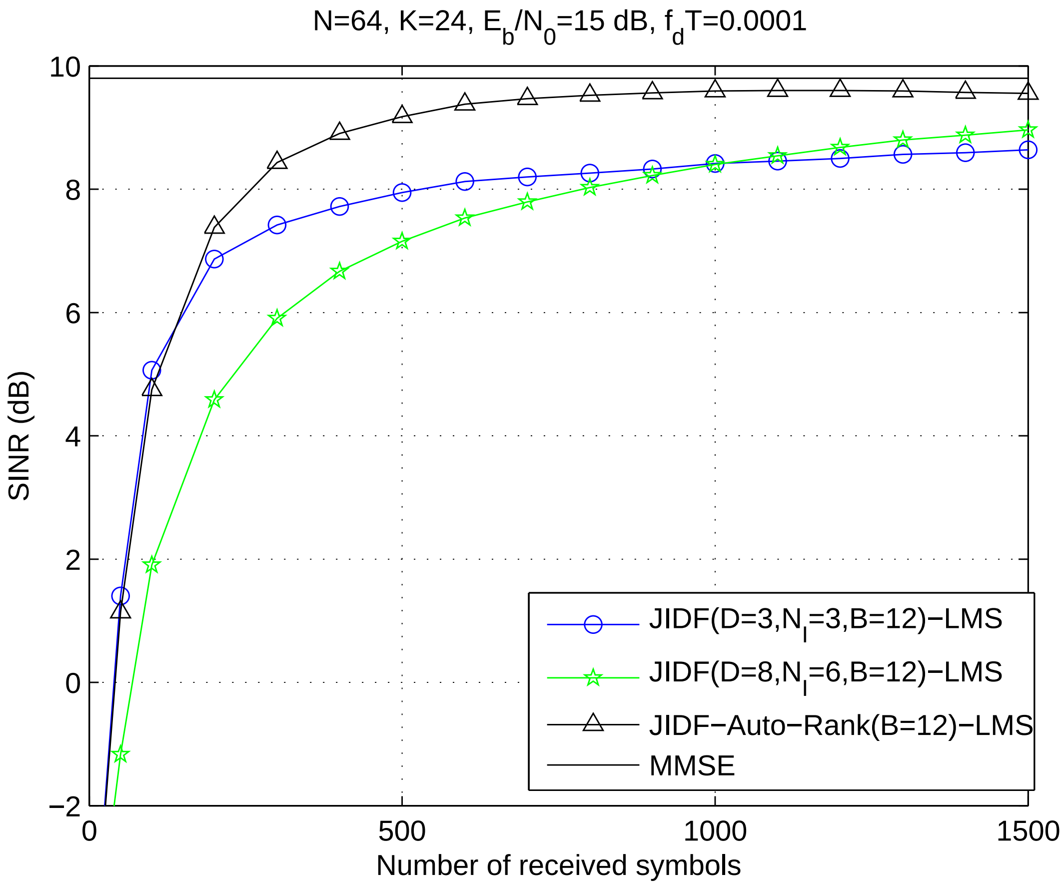
– The rank adaptation algorithm that chooses the best lengths D_{opt} and $N_{I_{\text{opt}}}$ for the filters $\mathbf{v}(i)$ and $\bar{\mathbf{w}}(i)$, respectively, is given by

$$\{D_{\text{opt}}, N_{I_{\text{opt}}}\} = \arg \min_{\substack{N_{I_{\text{min}}} \leq n \leq N_{I_{\text{max}}} \\ D_{\text{min}} \leq d \leq D_{\text{max}}}} \mathcal{C}(\bar{\mathbf{w}}^{(d)}, \mathbf{v}^{(n)},\ \mathbf{D})$$

where d and n are integers, D_{min} and D_{max}, and $N_{I_{\text{min}}}$ and $N_{I_{\text{max}}}$ are the minimum and maximum ranks allowed for $\bar{\mathbf{w}}(i)$ and $\mathbf{v}(i)$, respectively.

Model-order selection with JIDF algorithm

SINR performance against rank (D) for the analyzed schemes using LMS and RLS algorithms.



Applications, perspectives and future work

– Applications : interference suppression, beamforming, channel estimation, echo cancellation, target tracking, wireless sensor networks, signal compression, radar, control, seismology and bio-inspired systems, etc.

– Perspectives :

- Work in this field is not widely explored.
- Many unsolved problems when dimensions become large : estimation, tracking, general acquisition.

– Future work :

- Information theoretic study of very large observation data : performance limits as M goes to infinity.
- Investigation of tensor-based reduced-rank schemes.
- Development of vector and matrix-based parameter estimates as opposed to current scalar parameter estimation of existing methods.
- Distributed reduced-rank processing.

Concluding remarks

– Reduced-rank signal processing is a set of powerful techniques that allow the processing of large data vectors, enabling a substantial reduction in training with low complexity.

– A survey on reduced-rank techniques, detailing eigen-decomposition methods and the MSWF, was presented along with some critical comments on their suitability for practical use.

– A family of reduced-rank algorithms based on joint and iterative optimisation (JIO) of filters was presented.

– A recently proposed reduced-rank scheme that employs joint interpolation, decimation and filtering (JIDF) was also briefly described.

– Techniques based on approximations of basis functions (SAABF) were discussed and algorithms were devised for an UWB application.

– Several applications have been envisaged as well as a number of future investigation topics.

Questions ?

Vielen Dank !

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