Set-Membership Constrained Widely Linear Beamforming Algorithms

R. Zelenovsky and R. C. de Lamare

***Abstract*—This paper proposes a widely linear (WL) constrained minimum variance beamforming algorithms based on the set-membership filtering (SMF) framework. The proposed SMF-WL algorithms have the advantages of the widely linear processing concept and keep the computational cost low with the SMF technique. We present two versions: one using least-mean square (LMS) and another using recursive least squares (RLS) recursions. It is shown that the proposed algorithms have better steady state and convergence performances with a lower computational cost when compared with existing methods.**

***Index Terms*—adaptive beamforming, widely linear processing, set-membership filtering, antenna array.**

I. Introduction

Some modern electronic systems like radar, sonar, wireless communication use antenna array and depend on adaptive signal processing techniques [1], [2]. A great deal of research has been done on modified versions of the optimal linearly constrained minimum variance (LCMV) solution [1], [2], [3]. With standard LCMV algorithms, the second-order statistics of the incoming data are extensively used. Widely-linear (WL) processing [4], [5], [6] can improve the performance of the LCMV based algorithms when the data are second-order non-circular. However, the use of the WL techniques results in a greater computational cost as compared to their conventional linear counterparts due to the requirement of a larger number of parameters in the design.

To face the extra computational cost of the WL solution other techniques have been used. For example, the reduced-rank algorithms [7]-[13]. In this case, a projection matrix is used to reduce the dimension of the problem. The advantage is appreciable, especially, when a large number of antennas is used. However, the algorithm demands two inter-related adaptations: the reduced-rank filter and the projection matrix. Another way to reduce the computational cost is using the set-membership filtering (SMF) techniques [17], [15], [16], [18], [19]. In this case, an update is conducted if the estimation error or the array output is greater than a predetermined bound. The complexity of the bound computation is very low and is conducted all the time, but the costly information evaluation is done much less frequently, so the whole computation cost is very low. Another advantage is the additional flexibility that allows a designer to choose between a high performance and a low computational cost.

This paper develops constrained adaptive algorithms by combining widely linear processing and set-membership filtering techniques. The first algorithm is based on a least-mean square strategy and is called set-membership widely-linear least-mean square (SMF-WL-LMS) algorithm. The second algorithm resorts to a least-squares type of adaptation and is termed as set-membership widely-linear recursive least squares (SMF-WL-RLS) algorithm. We study the performance of the proposed algorithms via simulations and compare them against existing algorithms in the literature.

This paper is structured as follows. Section II presents the system and signal models of a sensor array processing system and the mathematical model employed. Section III states the beamforming design problem that we are interested in solving. Section IV presents the main concepts of set-membership widely-linear processing. Section V details the derivation of the proposed SMF-WL-LMS and SMF-WL-RLS algorithms. Section VI is devoted to the presentation and discussion of the simulation results, whereas Section VII gives the conclusions of this work.

II. System and Signal Models

We consider a sensor array processing system equipped with a uniform linear array (ULA) with M elements and K narrow-band sources in the far field, as depicted in Fig. 1. The number of array elements is greater than the number of sources ($M > K$). Each source has an unknown direction of arrival (DOA). The vector of DOAs is expressed by $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_K]^T$ and the steering vector of user l is represented by

$$\boldsymbol{a}(\theta_l) = \left[1, e^{-2\pi j 0 \frac{d_s}{\lambda_c}\cos\theta_l}, \ldots, e^{-2\pi j (M-1)\frac{d_s}{\lambda_c}\cos\theta_l}\right]^T \in \mathbb{C}^M \tag{1}$$

where $d_s = \lambda_c/2$ is the inter-element spacing, λ_c is the wavelength and ^{T} denotes the transpose operation.

The received vector from the linear array can be modeled as

$$\boldsymbol{x} = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s} + \boldsymbol{n} \in \mathbb{C}^M \tag{2}$$

where $\boldsymbol{A}(\boldsymbol{\theta}) = [\boldsymbol{a}(\theta_1), \ldots, \boldsymbol{a}(\theta_K)] \in \mathbb{C}^{M\times K}$ is the matrix with all steering vectors, $\boldsymbol{s} \in \mathbb{C}^K$ is the data vector from the K sources and the vector $\boldsymbol{n} \in \mathbb{C}^M$ models the complex noise of each sensor which is assumed to be zero-mean Gausian with covariance matrix $\sigma^2\boldsymbol{I}$.

III. Problem Statement

The output of the array is the scalar $y = \boldsymbol{w}^H\boldsymbol{x}$ where $\boldsymbol{w} = [w_1, \ldots, w_M]^T \in \mathbb{C}^M$ is the weight vector and $\boldsymbol{x} = [x_1, \ldots, x_M]^T \in \mathbb{C}^M$ is the snapshot from the array. Assuming that the signal of interest has a DOA equal to θ_k, the LCMV solution is the weight vector $\boldsymbol{w}$ that solves the following equation:

$$\begin{aligned} &\text{minimize } E[|\boldsymbol{w}^H\boldsymbol{x}|^2] = \boldsymbol{w}^H\boldsymbol{R}_x\boldsymbol{w} \\ &\text{subject to } \boldsymbol{w}^H\boldsymbol{a}(\theta_k) \ = 1 \end{aligned} \tag{3}$$

where $\boldsymbol{R}_x = E[\boldsymbol{x}\boldsymbol{x}^H] \in \mathbb{C}^{M\times M}$ is the covariance matrix of the received data and ^{H} indicates the Hermitian operator . This

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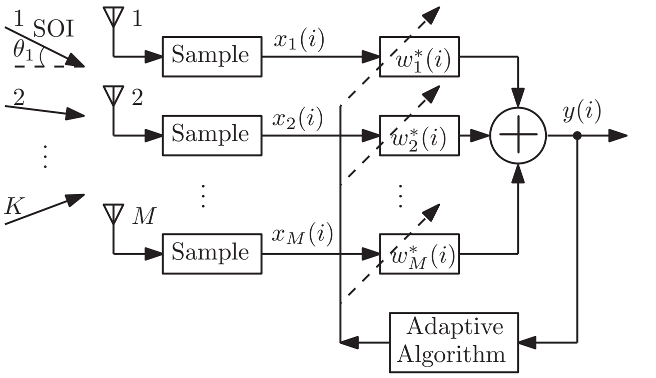

Fig. 1. Array processing system with a ULA with M sensors and K signals.

problem has a well-known solution. For example [3] and [20] give the following answer:

$$\boldsymbol{w}_{\text{opt}} = \frac{\boldsymbol{R}_x{}^{-1}\boldsymbol{a}(\theta_k)}{\boldsymbol{a}^H(\theta_k)\boldsymbol{R}_x{}^{-1}\boldsymbol{a}(\theta_k)} \tag{4}$$

Using adaptive techniques the filter weights $\boldsymbol{w}$ can be estimated with well known algorithms such as least-mean square (LMS) or recursive least squares (RLS). The convergence and tracking performance depend on M and on the eigenvalue spread of $\boldsymbol{R}_x$.

IV. Set-Membership Widely-Linear Processing Techniques

It has been shown in [4], [5] that widely-linear processing algorithms can be used to improve the performance of the LCMV solution when non-circular data are processed. In some cases such as BPSK the improper modulation results in non-circularity. That means, the covariance matrix $\boldsymbol{R} = [\boldsymbol{x}\boldsymbol{x}^H]$ cannot describe all the second-order statistics of the received signal $\boldsymbol{x}$. This is reason why the pseudo-covariance matrix $\boldsymbol{R} = [\boldsymbol{x}\boldsymbol{x}^T]$ is taken in account. A simple way to do that is to use a transformation τ that augments the original vector with its own complex conjugate as indicated by

$$\boldsymbol{x} \xrightarrow{\tau} \boldsymbol{x}_a : \boldsymbol{x}_a = [\boldsymbol{x}^T, \boldsymbol{x}^H]^T \in \mathbb{C}^{2M} \tag{5}$$

where $\xrightarrow{\tau}$ denotes the bijective transformation for the WL algorithm. For the two complex vectors $\boldsymbol{w}$ and $\boldsymbol{x}$ and their augmented versions $\boldsymbol{w}_a$ and $\boldsymbol{x}_a$ it is important to note that $\boldsymbol{w}_a^H\boldsymbol{x}_a = \boldsymbol{x}_a^T\boldsymbol{w}_a^* = 2\mathbb{R}\{\boldsymbol{x}^H\boldsymbol{w}\}$ where * indicates the complex conjugate. It is clear that because of the bijective transformation, the WL filter has to deal with vectors with double the size when compared with the linear one. This means more computational cost and may also result in a slow convergence.

To overcome the extra computational cost resulting from WL technique a set-membership filtering (SMF) solution may be used [7], [9], [10]. In this case, a new solution $\boldsymbol{w}_a$ is computed only when the output error or the array output is greater than a previously predetermined bound δ. That means, the solution $\boldsymbol{w}_a$ is updated only when the bound condition $|y|^2 \leq |\delta|^2$ is not matched. The SMF algorithm has two steps: 1) information evaluation and computation of the bound and 2) update of the weight $\boldsymbol{w}_a$ if the bound is exceeded. The information evaluation does not usually require much complexity and if the update of the $\boldsymbol{w}_a$ is not so frequent, the whole computational cost is substantially reduced.

The parameter space (space solution) consists of all possible values of $\{\boldsymbol{w}_a\}$. At each time instant i several values of $\{\boldsymbol{w}_a\}$ are consistent with the bound: $|y(i)|^2 \leq |\delta(i)|^2$. For that reason, the solution to the proposed SMF-WL is a set in this parameter space and it receives the name: *constraint set*. We use S_i to represent this set as:

$$S_i = \{\boldsymbol{w}_a \in \mathbb{C}^{2M} : |\boldsymbol{w}_a^H\boldsymbol{x}_a(i)|^2 \leq |\delta_i|^2\}. \tag{6}$$

Some elements of S_i may be consistent with the bound even for different instants of time i, that means, for different data $\boldsymbol{x}_a(i)$. Therefore, we define the *feasibility set*, designed by F_i as the intersection of the *constraint set* for a period of time $(l = 1, ..., i)$, stated as

$$F_i = \bigcap_{l=1}^{i} S_l. \tag{7}$$

The *feasibility set* is defined for all possible data pairs $D = \{\theta_k, \boldsymbol{x}_a\}$, where θ_k is the DOA of the signal of interest. The goal is to develop an adaptive algorithm that updates the parameter $\boldsymbol{w}_a$, while keeping it within the *feasibility set*. In an ideal case, F_i would include all solutions $\boldsymbol{w}_a$ that satisfy the constraint with $i \rightarrow \infty$. In pratice, the larger the observation data space $\boldsymbol{x}_a$, the smaller is the *feasibility set*. By taking this into account, as a practical set for the proposed algorithm, we define the *membership set* M_i as:

$$M_i = \bigcap_{l=1}^{i} S_l. \tag{8}$$

with i kept under an adequate upper limit, that means, i is finite. Of course F_i is the limiting set for M_i. In other words, M_i converge to F_i as the data pairs $\{\theta_k, \boldsymbol{x}_a\}$ traverse D completely.

V. Proposed Algorithms

The block diagram for the proposed SMF-WL solutions is presented in Fig. 2. The received data vector $\boldsymbol{x}(i) \in \mathbb{C}^M$ is augmented by the operator $\xrightarrow{\tau}$ resulting in the augmented data $\boldsymbol{x}_a(i) \in \mathbb{C}^{2M}$. The output $y(i)$ is generated with the use of the augmented vector of weights $\boldsymbol{w}_a(i) \in \mathbb{C}^{2M}$ in the following way: $y(i) = \boldsymbol{w}_a^H(i)\boldsymbol{x}_a(i)$. The adaptive algorithm using the criterion defined by the SMF updates the weights $\boldsymbol{w}_a(i)$.

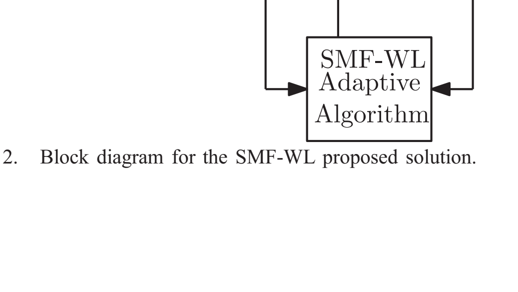

Fig. 2. Block diagram for the SMF-WL proposed solution.

In order to solve the beamforming problem for the user of interest which has a DOA equal to θ_k we propose the widely-linear LCMV optimization, stated as:

$$\begin{aligned} &\text{minimize } E[|y|^2] = E[|\boldsymbol{w}_a^H \mathbf{x}_a|^2] = \boldsymbol{w}_a^H \boldsymbol{R}_{ax} \boldsymbol{w}_a \\ &\text{subject to } \boldsymbol{w}_a^H \boldsymbol{a}_a(\theta_k) \ = 1 \end{aligned} \tag{9}$$

where the augmented steering vector in the direction of interest is $\boldsymbol{a}_a(\theta_k) = [\boldsymbol{a}^T(\theta_k), \boldsymbol{a}^H(\theta_k)]^T \in \mathbb{C}^{2M}$. The covariance matrix is augmented to $\boldsymbol{R}_{ax}$ and has a block structure as depicted below:

$$\boldsymbol{R}_{ax} = E[\boldsymbol{x}_a(i)\boldsymbol{x}_a^H(i)] = \begin{bmatrix} \boldsymbol{R}_x & \boldsymbol{R}_{cx} \\ \boldsymbol{R}_{cx}^* & \boldsymbol{R}_x^* \end{bmatrix} \in \mathbb{C}^{2M \times 2M} \tag{10}$$

where $\boldsymbol{R}_{cx} = E[\boldsymbol{x}\boldsymbol{x}^T]$. If the data are modelled by equation (2), then $\boldsymbol{R}_x = \boldsymbol{A}\boldsymbol{A}^H + \boldsymbol{I}\sigma_n^2$ and $\boldsymbol{R}_{cx} = \boldsymbol{A}\boldsymbol{A}^T$.

The problem presented in (9) can be solved [3], [20] using Lagrange Multipliers which transform the constrained optimization into an unconstrained one:

$$\mathcal{L}(\boldsymbol{w}_a(i), \lambda_l) = E\big[|\boldsymbol{w}_a^H(i)\boldsymbol{x}_a(i)|^2\big] + 2\Re[\lambda_l(\bar{\boldsymbol{w}}_a^H(i)\boldsymbol{a}_a(\theta_k) - 1] \tag{11}$$

where λ_l is a scalar Lagrange multiplier and the $\Re[.]$ selects the real part of its arguments. The solution to this problem is given by [3], [20] as:

$$\boldsymbol{w}_{\text{a-opt}} = \frac{\boldsymbol{R}_{ax}^{-1}\boldsymbol{a}_a(\theta_k)}{\boldsymbol{a}_a^H(\theta_k)\boldsymbol{R}_{ax}^{-1}\boldsymbol{a}_a(\theta_k)} \tag{12}$$

A. The Proposed SMF-WL-LMS Algorithm

In this section we present a low-complexity set membership (SMF) widely linear (WL) least mean squares (LMS) algorithm. The instantaneous gradients of (11) with respect to $\boldsymbol{w}_a^*$ and λ_l are

$$\begin{aligned} \nabla\mathcal{L}_{\boldsymbol{w}_a^*} &= \boldsymbol{R}_{ax}\boldsymbol{w}_a + 2\boldsymbol{a}_a(\theta_k)\lambda_l^* \\ \nabla\mathcal{L}_{\lambda_l} &= 2(\boldsymbol{w}_a^H + 2\boldsymbol{a}_a(\theta_k) - 1). \end{aligned} \tag{13}$$

To construct the LMS adaptation [3] we use the gradient as $\boldsymbol{w}_a(i+1) = \boldsymbol{w}_a(i) - \mu\nabla\mathcal{L}_{\boldsymbol{w}_a^*}$ where μ is the step size. Solving the equations and using the constraint, the result for the WL-LMS solution is given by

$$\boldsymbol{w}_a(i+1) = \boldsymbol{w}_a(i) - \mu y^*(i)\big(\boldsymbol{I} - \frac{\boldsymbol{a}_a(\theta_k)\boldsymbol{a}_a^H(\theta_k)}{\boldsymbol{a}_a^H(\theta_k)\boldsymbol{a}_a(\theta_k)}\big)\boldsymbol{x}_a(i) \tag{14}$$

It is clear that the widely-linear solution has doubled the dimension of the problem, that is $\boldsymbol{w}_a \in \mathbb{C}^{2M}$. To mitigate this disadvantage, we propose the use of SMF techniques as depicted by the fluxogram in Fig 3.

As can be seen, the solution $\boldsymbol{w}_a$ is not updated at each snapshot, but only when $|y|^2 > |\delta|^2$. It is possible to choose between performance and computacional cost by tuning the bound parameter δ. The greater the update rate, the better is the performance and the higher is the computational cost. The choice of the bound parameter is not so simple. A static bound, may be a problem because of the risk of underbounding or overbounding. Therefore, we propose a variable bound as shown below:

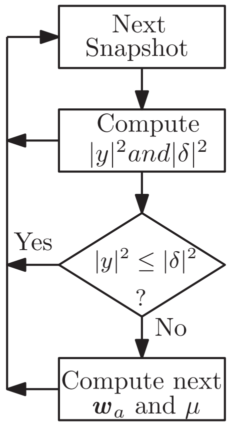

Fig. 3. Fluxogram for the SMF-WL proposed solution.

$$\delta(i) = \beta\delta(i-1) + (1-\beta)\sqrt{\alpha||\boldsymbol{w}_a||^2\hat{\sigma}_n^2} \tag{15}$$

where α and β are constants and $\hat{\sigma}_n^2$ is an estimate of the noise power. The positive constant α is used as a tuning parameter and has a direct impact on the update rate and convergence. The positive constant β is usually close to 1, but lower than 1, in general $\beta = 0.99$, and it guarantees the proper time-average of the evolution of the solution $\boldsymbol{w}_a$. The equation (15) provides a smooth evaluation for δ and avoids too high or too low values of $||\boldsymbol{w}_a||^2$.

Since the solution $\boldsymbol{w}_a$ is not computed for each snapshot, the convergence is a problem because it may become slow. This can be a big problem if $\boldsymbol{w}_a$ is not computed for a large period of time. To compensate that and to garantee a fast convergence, the step size μ is adjusted before each update of $\boldsymbol{w}_a$. The following equation for the step size μ offers a good trade of between convergence and misadjustment:

$$\mu(i+1) = \begin{cases} \dfrac{1 - \frac{\delta(i)}{|y_a(i)|}}{\boldsymbol{x}_a^H(i)\big(\boldsymbol{I} - \frac{\boldsymbol{a}_a(\theta_k)\boldsymbol{a}_a^H(\theta_k)}{\boldsymbol{a}_a^H(\theta_k)\boldsymbol{a}_a(\theta_k)}\big)\boldsymbol{x}_a(i)} & \text{if } |y|^2 > \delta^2, \\ 0 & \text{if } |y|^2 \leq \delta^2. \end{cases} \tag{16}$$

B. The Proposed SMF-WL-RLS Algorithm

In this section we present a low-complexity set-membership (SMF) widely linear (WL) recursive least squares (RLS) algorithm [3]. The proposed constrained problem in (9) is transformed into an unconstrained one with the method of Lagrange multipliers and can be written in the following way

$$\begin{aligned} \mathcal{L}(\boldsymbol{w}_a(i), \mu_l) = &\sum_{j=1}^{i} \alpha_l^{i-j}\big|\boldsymbol{w}_a^H(i)\boldsymbol{x}_a(j)\big|^2 \\ &+ 2\Re[\lambda_l\big(\boldsymbol{w}_a^H(i)\boldsymbol{a}_a(\theta_k) - 1\big)] \end{aligned} \tag{17}$$

where α_l is the forgetting factor. It must be positive and close to 1, but less than 1. By considering the multiplier λ_l constant, it is easy to compute the gradient of (17) with respect to $\boldsymbol{w}_a$. Equating this gradient to a zero vector and solving for λ_l, we obtain the filter solution as:

$$\boldsymbol{w}_a(i) = \frac{\hat{\boldsymbol{R}}_{ax}^{-1}(i)\boldsymbol{a}_a(\theta_k)}{\boldsymbol{a}_a^H(\theta_k)\hat{\boldsymbol{R}_{ax}}^{-1}(i)\boldsymbol{a}_a(\theta_k)} \quad (18)$$

where the $\hat{\boldsymbol{R}}_{ax}(i) = \sum_{j=1}^{i} \alpha_l^{i-j}(i)\boldsymbol{x}_a(j)\boldsymbol{x}_a^H(j)$ is the instantaneous input covariance matrix. Using the matrix inversion lemma [21], [3], $\boldsymbol{R}_{ax}^{-1}$ can be estimated as follows:

$$\boldsymbol{R}_{ax}^{-1}(i) = \alpha^{-1}(i)\boldsymbol{R}_{ax}^{-1}(i-1) - \alpha^{-1}(i)\boldsymbol{G}(i)\boldsymbol{x}_a^H(i)\boldsymbol{R}_{ax}^{-1}(i-1) \quad (19)$$

$$\boldsymbol{G}(i) = \frac{\boldsymbol{R}_{ax}^{-1}(i)\boldsymbol{x}_a(i)}{\alpha(i) + \boldsymbol{x}_a^H(i)\boldsymbol{R}_{ax}^{-1}\boldsymbol{x}_a(i)} \quad (20)$$

where $\boldsymbol{G}(i)$ is the gain vector at instant i. As can be seen, the inversion of the matrix $\boldsymbol{R}_{ax}(i)$ is replaced by the recursive process in (19) and (20). The recursion of (19) can be started with $\boldsymbol{R}_{ax}^{-1}(0) = C\boldsymbol{I}_{2M}$, where C is a positive constant and $\boldsymbol{I}_{2M}$ is the $2M \times 2M$ identity matrix.

As stated in the previous section, the solution $\boldsymbol{w}_a \in \mathbb{C}^{2M}$ has twice the computational complexity of the the linear solution. So, in order to minimize the problem, the SMF technique is used again, as shown in Fig 3. This is equivalent to performing an update whenever the condition $|y|^2 \leq \delta^2$ is not satisfied. The bound δ is the same as stated in (15). The forgetting factor needs to be updated to guarantee the speed of the convergence. To this end, we propose the following update for α:

$$\alpha(i) = \begin{cases} \frac{\boldsymbol{a}_a^H(\theta_k)\boldsymbol{R}_{ax}^{-1}(i)[\delta(i)\boldsymbol{a}_a(\theta_k) - \boldsymbol{x}_a(i)]}{\boldsymbol{a}_a^H(\theta_k)\boldsymbol{G}(i)\boldsymbol{x}_a(i)\boldsymbol{R}_{ax}^{-1}(i)[\delta(i)\boldsymbol{a}_a(\theta_k) - \boldsymbol{x}_a(i)]}, & \text{if } |y|^2 > \delta^2, \\ 0 & \text{if } |y|^2 \leq \delta^2. \end{cases} \quad (21)$$

In summary, the above solution offers a good convergence and a low computation cost.

VI. Simulations

In this section, we present computer simulations to illustrate the performance of the proposed algorithms: SMF-WL-LMS and SMF-WL-RLS. These algorithms are compared with LMS, WL-LMS, RLS, and WL-RLS algorithms. It is assumed that in all simulations there is only one desired user and its DOA is perfectly known. Therefore, the DOA of the user of interest was used to initialize the vector $\boldsymbol{w}_a(0)$. The environment has a uniform linear array with 8 elements, 1 desired user plus 2 interferers with the same power. Their DOAs are 20, 50, -30 degrees, and the first one is the user of interest. The noise is modeled as AWGN with zero mean and variance σ^2. Fig 4 evaluates the signal-to-interference-plus-noise (SINR) versus snapshots to show the quality of the convergence and the steady state performance. The SINR is calculated by

$$SINR(i) = \frac{\boldsymbol{w}_a^H(i)\boldsymbol{R}_{as}\boldsymbol{w}_a(i)}{\boldsymbol{w}_a^H(i)\boldsymbol{R}_{ai}\boldsymbol{w}_a(i)}, \quad (22)$$

where $\boldsymbol{R}_{as}$ is the augmented autocorrelation matrix of the desired signal and $\boldsymbol{R}_{ai}$ is the augmented cross-correlation matrix of the interferers plus noise. In Fig. 4, it is clear the good convergence and steady-state performance of the proposed algorithms, when compared with other algorithms. The SMF-WL-RLS presented a performance a little better than the SMF-WL-LMS. Is this figure, the optimums linear L_{opt} and widely linear WL_{opt} cases are shown as straight horizontal lines. The solutions, as expected, are asymptotic to the optimum cases.

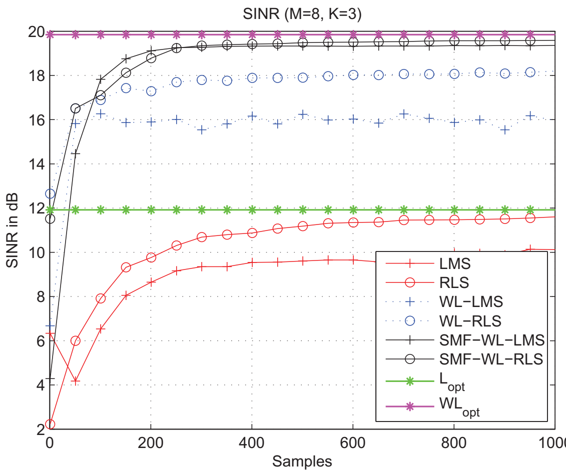

Fig. 4. SINR performance of algorithms against snapshots with 8 sensors, 1 user and 2 interferers (σ^2=0.1).

Fig. 5 shows the behaviour of the algorithms using a $SINR \times SNR$ plot. This figure shows, for each SNR value, the SINR after 1000 snapshots. To avoid flutuations, instead of using the very last value of SINR, we used the average over the last 50 values of SINR. It can be seen the good performance of the proposed algorithms. The optimum cases, L_{opt} and WL_{opt}, are shown as almost straight lines.

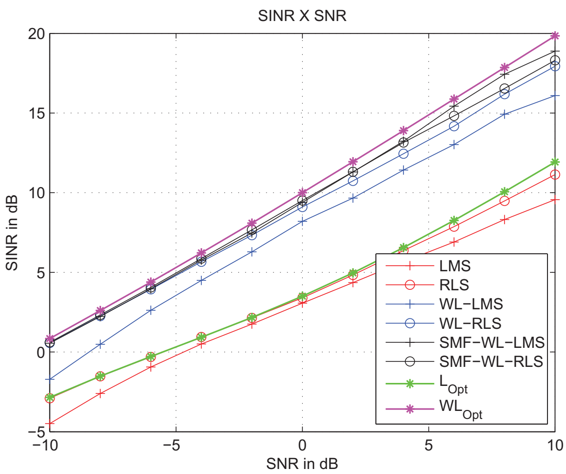

Fig. 5. Performance SINR versus noise SNR.

Fig. 6 shows the compromise between the update rate and the performance for the SMF-WL-LMS. Since the update rate is directly linked to the computational cost, this figure illustrates the correlation between the performance and the computational cost. The percentage rate was computed over the 500 snapshots. Of course, the high update rate at the beginning has a big contribution. It is clear that an update rate near 20% is enough for a good performance while it offers a great computational cost reduction. Update rates over 30% may be inefficient. Simulations have shown that at steady state the update rate can be as low as 10%. The SMF-WL-RLS algorithm has similar results.

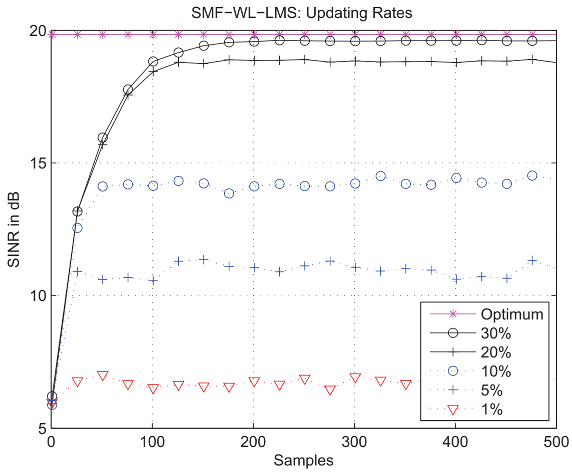

Fig. 6. Performance versus update rate with 8 sensors, 1 user and 2 interferers (σ^2=0.1).

To illustrate the beamformer pattern, Fig. 7 shows the case with one user and 2 interferers using the SMF-WL-LMS. Vertical dashed lines indicate the DOAs of the interferers and the solid line the DOA of interest. Three patterns are shown, namely, the pattern due to initialization of $\boldsymbol{w}_a(0)$, the optimum pattern, and the pattern at the end of simulation, i. e., the pattern after convergence. It is obvious that the final pattern generated by the proposed algorithm is very close to the optimum.

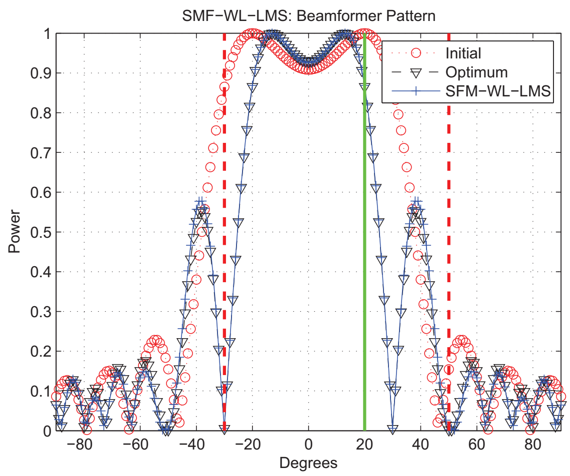

Fig. 7. Beam pattern for 8 sensors, 1 user and 2 interferers (σ^2=0.1).

VII. Conclusions

We have proposed two novel algorithms using widely linear and set-membership filtering techniques: SMF-WL-LMS and SMF-WL-RLS. The second-order statistics are fully exploited with the widely linear techniques, resulting in a better steady-state and convergence performances. Nevertheless, widely-linear processing increases the computational cost and in order to address this problem we have used the SMF technique. By tuning the SMF parameters it is possible to choose a trade-off between performance and computational cost.

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